HYPOTHESES

I

If there were no gravity, and if the air did not impede the motion of bodies, then any body will continue its given motion with uniform velocity in a straight line.

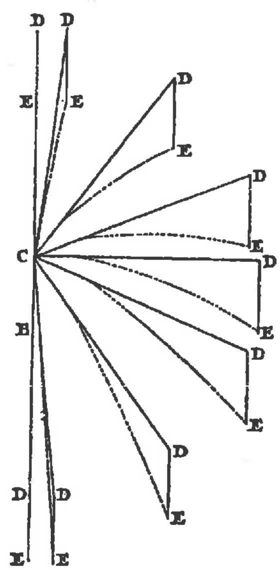
II

By the action of gravity, whatever its sources,[1] *it happens that bodies are moved by a motion composed both of a uniform motion in one direction or another and of a motion downward due to gravity.*

III

These two motions can be considered separately, with neither being impeded by the other.

LET C be a heavy body which, starting from rest, crosses the distance CB by the force of gravity in a certain time F [Fig. 7]. And let the same body be imagined to undergo another motion by which, assuming that gravity does not exist, it crosses the straight line CD with a uniform motion in the same time F. When the force of gravity is added, the body will not move from C to D in the time F



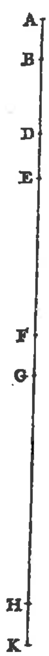
[Fig. 7]

but rather to some point E vertically below D such that the distance DE equals the distance CB. And thus the uniform motion and the motion due to gravity each make a contribution, and neither impedes the other. In what follows later we will define the line in which the body moves with this composite motion when the uniform motion is neither straight up or down but in an oblique direction. But when the uniform motion CD occurs downward on the perpendicular, it is obvious that the line CD is increased by the straight line DE when the motion due to gravity is added. Likewise, when the uniform motion CD is directed upward, CD is decreased by the straight line DE, so that, for example, after the time F the body will always be found at the point E. Thus, if we consider the two motions separately in each case, as we said, and if we recognize that neither motion is in any way impeded by the other, then from this we can discover the cause and the laws of acceleration of heavy falling bodies. And first we will show the following two things.

PROPOSITION I

In equal times equal amounts of velocity are added to a falling body, and in equal times the distances crossed by a body falling from rest are successively increased by an equal amount.

[Fig. 8]



Let there be a heavy body at rest at A [Fig. 8]. In the first unit of time it falls through the distance AB; and when it has arrived at B, it has acquired a velocity by which it next could cross the distance BD with a uniform velocity in the second unit of time. But we know that in the second unit of time it will cross a distance greater than BD because it would travel the distance BD only if all the action of gravity had ceased at B. Actually it moves with a motion composed of the uniform motion by which it would have crossed the distance BD and of a motion characteristic of falling bodies by which it necessarily falls through a distance equal to AB. Hence by adding DE, which is equal to AB, to BD, we know that in the second unit of time the body will arrive at E.

But if we inquire what velocity the body has at E at the end of the second unit of time, we find that this ought to be double the velocity which it had at B at the end of the first unit of time. For we said that it is moved by a motion composed of a uniform motion equal to the velocity acquired at B and of a motion due to gravity, which clearly is the same in the second unit of time as in the first. Hence a velocity ought to be added to the falling body in the second unit of time which is equal to the velocity added in the first unit of time. Thus, since it conserves the whole velocity acquired at the end of the first unit of time, it is clear that at the end of the second unit of time it has twice, or double, the velocity which it acquired at the end of the first unit of time.

Now if, after having arrived at E, the body were to be moved with a uniform velocity equal to what it has acquired at E, it is clear that in a third unit of time equal to each of the first two it would cross the distance EF, which is double the distance BD. For we said that the latter was crossed with half this velocity by a uniform motion in an equal time. But by adding again the action of gravity, in the third unit of time the body will cross the distance EF and also the distance FG, which is equal to AB or to DE. And thus at the end of the third unit of time the body will be found at G. It will have here a velocity which is triple that which it had at B at the end of the first unit of time. For in addition to the velocity acquired at E, which we said was double that acquired at B, in the third unit of time of the fall a velocity is added which again is equal to the velocity at the end of the first unit of time. Hence at the end of the third unit of time both velocities add up to triple the velocity found at the end of the first unit of time.

In the same way it can be shown that in the fourth unit of time the body ought to cross both the distance GH, which is triple BD, and the distance HK, which is equal to AB; and the velocity at K, at the end of the fourth unit of time, will be quadruple what it was at B at the end of the first unit of time. Therefore, it is clear that whatever distances we take to be crossed successively in equal times, these distances will each increase by an amount equal to BD, and simultaneously the velocities will also be increased equally in equal times.

From this it will not be difficult to prove the following proposition which Galileo asked that we accept as in a sense being self-evident.[4] For the demonstration which he tried to give later and which appears in the later edition of his works does not seem to me to be too strong. The proposition is the following.

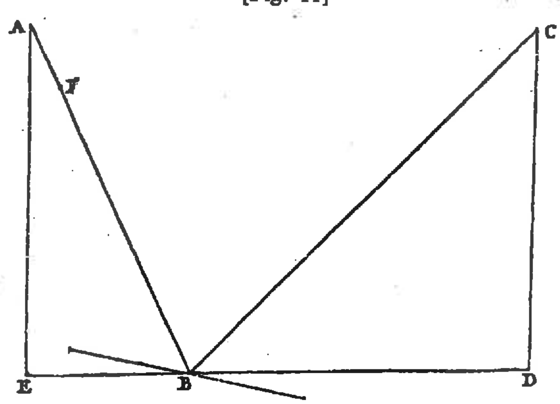
PROPOSITION VI

The velocities acquired by bodies falling through variably inclined planes are equal if the elevations of the planes are equal.

The elevation of the plane will be called its height on the perpendicular.

Let AB and CB [Fig. 11] be sections of inclined planes extended to the horizontal plane, and let their heights AE and CD be equal. Then let a body fall from A through the plane AB, and a body fall from C through the plane CB. Now I say that in each case the same degree of velocity will be acquired at the point B.

[Fig. 11]



For if we were to assume that the body falling through CB were to acquire less velocity than the body falling through AB, it would follow that the body falling through CB would acquire exactly the same velocity as a body falling only through FB, where FB is less than AB. But the body falling through CB acquires a velocity by which it could ascend again through the whole of BC [Proposition 4]. Therefore, if the body falling

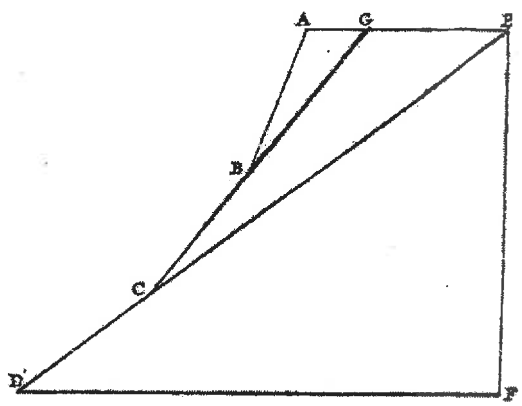
4. Galileo, *Discourses Concerning Two New Sciences.* Third Day. (*Le Opere di Galileo Galilei* 8:205.) ". . . the Author requires and takes as true one single assumption; that is, [Postulate] I assume that the degrees of speed acquired by the same moveable over different inclinations of planes are equal whenever the heights of those planes are equal." [S. Drake's translation, 162.]

PROPOSITION VIII

If from the same height a body descends by a continuous motion through any number of contiguous planes having any inclinations whatsoever, it will always acquire at the end the same velocity; namely, a velocity equal to that which would be acquired by falling perpendicularly from the same height.

Let AB, BC, and CD be contiguous planes [Fig. 13] whose terminus A has a height above the horizontal line DF, drawn through the lower terminus D, equal to the perpendicular EF. And let a body descend through these planes from A to D. Now I say that at D it will have the same velocity which it would have at F by falling from E.

[Fig. 13]

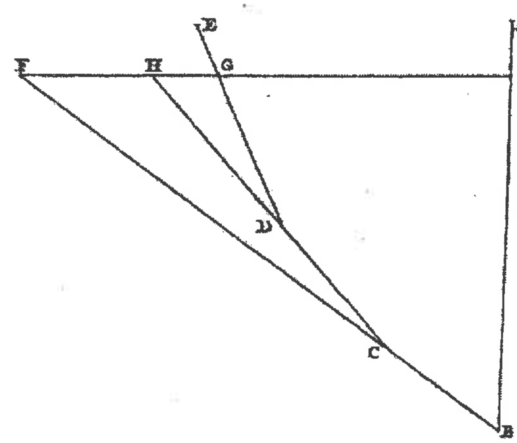


PROPOSITION IX

If, after falling, a body converts its motion upwards, it will rise to the same height from which it came, no matter how many contiguous plane surfaces it may have crossed, and no matter what their inclinations are.

Let a body fall from the height AB [Fig. 14]. From the point B let the planes BC, CD, and DE be inclined upwards such that their extremity E has the same height as the point A. Now I say that if a body, after falling through AB, converts its motion so that it continues to be moved through these inclined planes, it will rise up to the point E.

[Fig. 14]

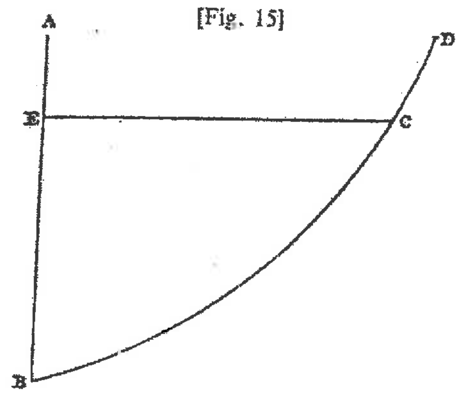


PROPOSITION X

If a body falls perpendicularly or through any surface, and if it later moves upwards by the acquired impetus through any other surface, then it will always have the same velocity at points of equal height in its descent and ascent.

Let a body fall from the height AB [Fig. 15] and then continue its motion through the surface BCD, in which the point C has the same height as the point E in AB. Now I say that the same velocity is present in the body at C as was present in it at E.

[Fig. 15]

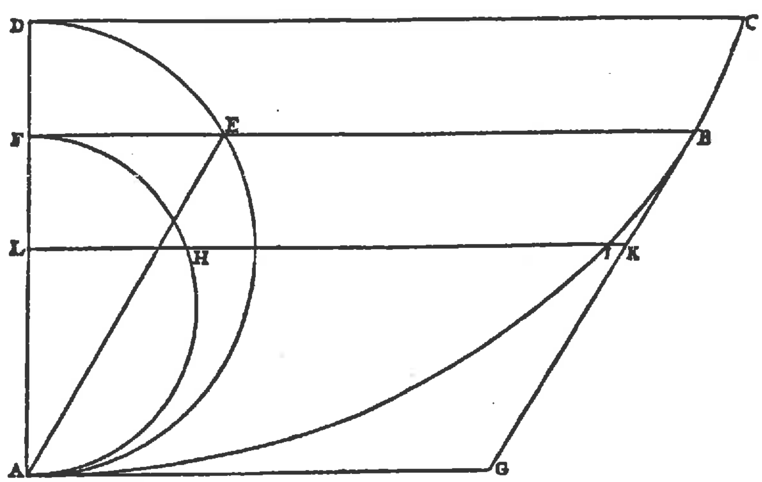


PROPOSITION XXV

On a cycloid whose axis is erected on the perpendicular and whose vertex is located at the bottom, the times of descent, in which a body arrives at the lowest point at the vertex after having departed from any point on the cycloid, are equal to each other; and these times are related to the time of a perpendicular fall through the whole axis of the cycloid with the same ratio by which the semicircumference of a circle is related to its diameter.

Let ABC [Fig. 35] be a cycloid whose vertex A is located at the bottom and whose axis AD is erected on the perpendicular. Select any point on the cycloid, for example B, and let a body descend by its natural impetus through the arc BA, or through a surface so curved. Now I say that the time of this descent is related to the time of a fall through the axis DA as the semicircumference of a circle is related to its diameter. When this has been demonstrated, it will also be established that the times of descent through all arcs of the cycloid terminating at A are equal to each other.

[Fig. 35]

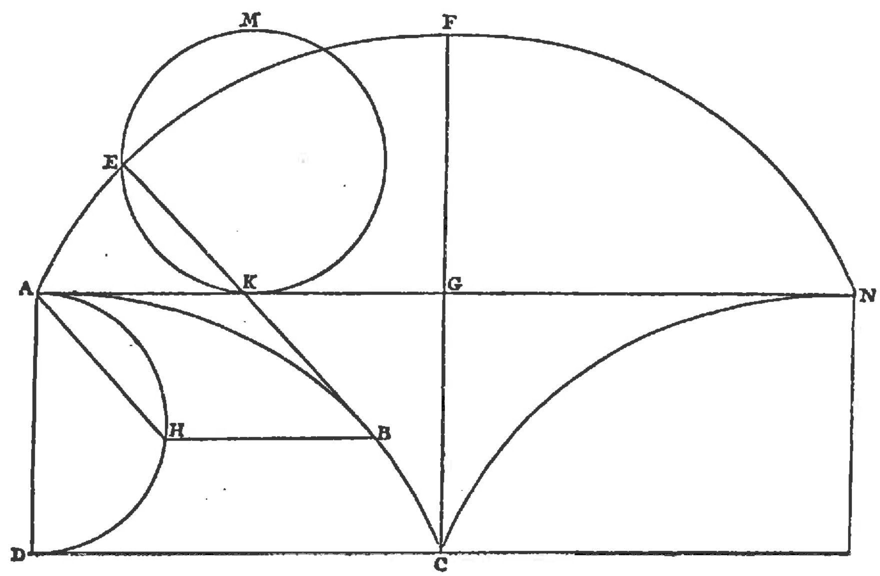


PROPOSITION V

If a straight line is tangent to a cycloid at its apex, and if on that line as a base another cycloid similar and equal to the first is constructed starting from the point of the apex just mentioned, then any straight line which is tangent to the lower cycloid will meet the arc of the higher cycloid at right angles.

Let the straight line AG be a tangent to the cycloid ABC at the apex A [Fig. 42]. On AG as a base construct another similar cycloid AEF whose apex is F. Moreover let the line BK be a tangent to the cycloid ABC at B. Now I say that the extension of BK will meet the cycloid AEF at right angles.

[Fig. 42]



Let the generative circle AHD be drawn around AD, which is the axis of the cycloid ABC. Let BH, which is parallel to the base, meet the circle at H, and draw HA. Since BK is tangent to the cycloid at B, it follows that it is parallel to HA [Proposition 15, Part II]. Thus AHBK is a parallelogram, and so AK is equal to HB, i.e., to the arc AH [Proposition 14, Part II]. Next construct the circle KM equal to the generative circle AHD, making it tangent to the base AG at K, and extend BK to the point E. Now since AH is parallel to BKE, and since the angle EKA thus equals KAH, it is clear that the extension of BK cuts an arc from the circle KM equal to the arc which AH cuts from the circle AHD. Thus the arc KE is equal to the arc AH, that is, to the line HB and to the line KA. From this it follows, according to the properties of a cycloid, that since the generative circle touches the base at K, the point describing the cycloid would be at E. Thus the line KE meets the cycloid at E at right angles [Proposition 15, Part II]. But KE is the extension of BK. Therefore it is clear that BK when extended meets the cycloid at right angles. Q.E.D.