III. De Motu, Version 3 (the "Augmented" Version) -- Two Extended Scholia

A. The "A Priori Proof" of Copernicanism

1. The "Copernican Scholium," added following Theorem 4, makes a series of claims, falling into three consecutive parts; the first part pivots on the claim that the Copernican system can be proved:

   "Moreover the whole space of the planetary heavens either rests (as is commonly believed) or moves uniformly in a straight line, and hence the communal center of gravity of the planets (by Law 4) either rests or moves along with it. In both cases (by Law 3) the motions of the planets among themselves [*inter* se] are the same, and their common centre of gravity rests in relation to the whole of space, and so can certainly be taken for the still centre of the whole planetary system. Hence truly the Copernican system is proved a priori. For if the common centre of gravity is calculated for any position of the planets it either falls in the body of the Sun or will always be very close to it."

   a. Desire to prove Copernicanism, and not worries about multiple centers, may have prompted Laws 3 and 4 in the first place
   b. Regardless, Laws 3 and 4 entail that the appropriate single point to refer the motion of the planetary system is its "communal centre of gravity"
   c. And calculation shows this point to be very near the sun

2. The standard objection to this line of reasoning, voiced by Wilson (and supported by Whiteside), is that it presupposes key unstated elements of the law of gravity

   The term 'center of gravity' is odd -- 'center of mass' would be preferable -- but it is possible that the "notion of center of gravity played a heuristic role, leading Newton's thought from familiar experiences with connected systems of weights to the idea of the solar system as a group of interconnected bodies. That such mutual interaction actually occurs is so far an unsubstantiated assumption..."

   "The other important assumption in the computation is that the attractions exercised at a given distance from these several bodies are as the masses of the attracting bodies."

   a. In other words, the supposed proof turns on calculating the center of mass of the planetary system, which itself requires first a determination of the masses of the bodies, not to mention the concept of mass itself
   b. But the only way of determining these masses is from the strength of the centripetal force "field" around each body being proportional to its mass, an inference licensed only by the law of gravity

3. There is another way of construing Newton's reasoning here, starting from concerns about multiple centers and the need for a single center to refer the motion to
   a. As indicated in Problem 5, Newton had identified a^3/P^2 as a measure of the accelerative strength of the inverse-square centripetal acceleration field around any body known to have such a field
   b. This quantity is a constant for each such body: C_h, C_j, C_s, C_e, the respective constants for the sun, Jupiter, Saturn, and the earth

4. Consider now just the case of the sun and Jupiter -- a two attractive center problem, with the strength of each attraction given

a. By Law 4 there must be a "center of gravity" around which both are revolving

b. And the only way in which Law 4 cannot be violated is if they revolve in a tandem balance with respect to one another, the sun revolving about this center with a radius r_h, and Jupiter with a radius r_j, where r_h/r_j a constant

5. The question then is, what is the (non-varying) ratio between r_h and r_j -- i.e. what are the comparative radii by which each revolves about the proper center of reference

   a. Assuming circular motion, Jupiter's centripetal acceleration is as r_j/P^2, so that $C_h/(r_{jh})^2$ is as r_j/P^2

   b. Similarly, in the case of the sun: $C_j/(r_{jh})^2$ is as r_h/P^2

   c. But then $C_h/C_j = r_j/r_h$!! -- a conclusion that has been reached without any consideration (or notion) of mass

6. But C_h is known to be much larger than C_j, for the diameter of the orbits of Jupter's satellites in astronomical units are known to sufficient accuracy to assure that C_h/C_j is a very large number

   a. But then r_h must be small compared to r_j; hence proper center of reference is very near the Sun

   b. The argument generalizes to Saturn and earth individually paired with the sun, both of which have smaller a^3/P^2 than that of Jupiter

   c. And the conclusion continues to hold even for a "displacement" of the sun corresponding to 6 times the strongest known planetary attraction, that of Jupiter's

7. Thus indeed a proof, not just that the Tychonic system is false, but that a main claim of the Copernican system is basically true

   a. A much stronger inference is being drawn from Kepler's 3/2 power rule than the one I am proposing that Newton was trying to draw in the original "Moon test" -- an inference made possible by Law 4!!

   b. Of course, there are loose ends in the reasoning, and hence some may say that this is not yet a proof, but it nevertheless is an extremely promising line of reasoning toward a proof

B. The Imperfectibility of Astronomy

1. The second part of the "Copernican Scholium" draws a corollary from the proof, leading to the conclusion that astronomy is imperfectible

   "By reason of this deviation of the Sun from the centre of gravity the centripetal force does not always tend to that immobile center, and hence the planets neither move exactly in ellipses nor revolve twice in the same orbit. So that there are as many orbits to a planet as it has revolutions, as in the motion of the Moon, and the orbit of any one planet depends on the combined motion of all the planets, not to mention the action of all these on each other. But to consider simultaneously all these causes of motion and to define these motions by exact laws allowing of convenient calculation exceeds, unless I am mistaken, the force of any human mind. Ignoring those minutiae, the simple orbit and the mean among all errors will be the ellipse of which I have already treated."

   a. The "proof" of Copernicanism has as a corollary that the movements of the planets are exceedingly complex, and hence orbital astronomy is probably not perfectible

   b. Nothing comparable to the above statement anywhere in the *Principia*!