# On the Motion of Projectiles

**We have considered properties existing in equable motion, and those in naturally accelerated motion over inclined planes of whatever slope. In the studies on which I now enter, I shall try to present certain leading essentials [*symptomata*] , and to establish them by firm demonstrations, bearing on a moveable [*Mobili*] when its motion is compounded from two movements; that is, when it is moved equably and is also naturally accelerated. Of this kind appear to be those which we speak of as projections, the origin of which I lay down as follows.**

**I mentally conceive of some moveable projected on a horizontal plane, all impediments being put aside. Now it is evident from what has been said elsewhere at greater length that equable motion on this plane would be perpetual if the plane were of infinite extent; but if we assume it to be ended, and [situated] on high, the moveable (which I conceive of as being endowed with heaviness [*gravitate*]), driven to the end of this plane and going on further, adds on to its previous equable and indelible motion that downward tendency [*propensionem*] which it has from its own heaviness. Thus there emerges a certain motion, compounded from equable horizontal and from naturally accelerated downward [motion], which I call "projection." We shall demonstrate some of its *accidentia*, of which the first is this:**

> ***Proposition 1*, *Theorem 1*. When a projectile is carried in motion compounded from equable horizontal and from naturally accelerated downward [motions], it describes a semiparabolic line in its movement.**

*remo veduto questa parte che resta intorno al Moto de i Proietti; che sarà, se così gli piace, nel seguente giorno.*

Salu. *Non mancherò d'esser con lei.*

Finisce la terza Giornata.

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# GIORNATA QUARTA.

Salu. *ATtempo arriua ancora il S. Simplicio, però sensa interpor quiete venghiamo al Moto, & ecco il Testo del nostro Autore.*

## DE MOTV PROIECTORVM.

Quæ in Motu æquabili contingunt accidentia, itemque in Motu naturaliter acceleraro super quascunque planorum inclinationes, supra consideravimus. In hac, quam modo aggredior, contemplatione, præcipua quædam symptomata, eaque scitu digna in medium afferre conabor, eademque firmis demonstrationibus stabilire, quæ Mobili accidunt dum motu ex duplici latione composito, æquabili nempe, & naturaliter accelerata, movetur: hujusmodi autem videtur esse Motus ille, quem de Projectis dicimus: cujus generationem talem constituo.

Mobile quoddam super planum horizontale projectum mente concipio omni secluso impedimento: jam constat ex his quæ fusius alibi dicta sunt illius motum æquabilem, & perpetuum super ipso plano futurum esse, si planum in infinitum extendatur: si vero terminatum, & in sublimi positum intelligamus, mobile, quod gravitate præditum concipio, ad plani terminum delatum, ulterius progrediens, æquabili, atque indelebili priori lationi superaddet illam, quam à pro-

propria gravitate habet deorsum propensionem, indeque motus quidam emerget compositus ex æquabili horizontali, & ex deorsum naturaliter accelerato: quem Projectionem voco. Cujus accidentia nonnulla demonstrabimus; quorum primum sit.

## THEOR. I. PROPOS. I.

Projectum dum fertur motu composito ex horizontali æquabili, & ex naturaliter accelerato deorsum, lineam semiparabolicam describit in sua latione.

Sagr. *E' forza S. Salu. in gratia di me, & anco credo io del S. Simpl. far qui vn poco di pausa; auuenga che io non mi son tanto inoltrato nella Geometria ch' io habbia fatto studio in Apollonio, se non in quanto sò ch'ei tratta di queste Parabole e dell' altre sezzioni coniche, senza la cognizione delle quali, e delle lor passioni, non credo che intendersi possano le dimostrazioni di altre proposizioni à quelle aderenti. E perche già nella bella prima proposizione ci vien proposto dall' Autore douersi dimostrare la linea descritta dal Projetto esser Parabolica, mi vò imaginando, che, non douendosi trattar d'altro che di tali linee, sia assolutamente necessario hauere vna perfetta intelligenza, se non di tutte le passioni di tali Figure dimostrate da Apollonio, almeno di quelle, che per la presente scienza son necessarie.*

Salu. *V. S. si humilia molto, volendosi far nuouo di quelle cognizioni, le quali non è gran tempo che ammesse come ben saputo: allora dico che nel trattato delle Resistenze hauemmo bisogno della notizia di certa proposizione d'Apollonio, sopra la quale ella non mosse difficoltà.*

Sagr. *Può essere ò che io la sapessi per ventura, ò che io la supponesse per vna volta, tanto che ella mi bisognò in tutto quel trattato: mà qui doue mi imagino d'hauere à sentir tutte le dimostrazioni circa tali linee, non bisogna, come si dice, beuer grosso, buttando via il tempo e la fatica.*

***Sagredo*: It cannot be denied that the reasoning is novel, ingenious and conclusive, being argued *ex supposition*; that is, by assuming that the transverse motion is kept always equable, and that the natural downward [motion] likewise maintains its tenor of always accelerating according to the squared ratio of the times; also that such motions, or their speeds, in mixing together do not alter, disturb, or impede one another. In this way, the line of the projectile, continuing its motion, will not degenerate into some other kind [of curve]. But this seems to me impossible; for the axis of our parabola is vertical, just as we assume the natural motion of heavy bodies to be, and it goes to the end of the center of the earth. Yet the parabolic line goes ever widening from its axis, so that no projectile would ever end at the center [of the earth], or if it did, as it seems it must, then the path of the projectile would become transformed into some other line, quite different from the parabolic.**

***Simplicio*: To these difficulties I add some more. One is that we assume the [initial] plane to be horizontal, which would be neither rising nor falling, and to be a straight line – as if every part of such a line could be at the same distance from the center, which is not true. For as we move away from its midpoint towards its extremities, this [line] departs ever farther from the center [of the earth], and hence it is always rising. One consequence of this is that it is impossible that the motion is perpetuated, or even remains equable through any distance; rather it would be always growing weaker. Besides, in my opinion, it is impossible to remove the impediment of the medium so that this will not destroy the equability of the transverse motion and the rule of acceleration for falling heavy things. All these difficulties make it highly improbable that anything demonstrated from such fickle assumptions can ever be verified in actual experiments.**