

F. Ptolemy's Model of Planetary Motion

1. Turn now from Swerdlow's reconstruction to the finished model Ptolemy gives in the *Almagest*
 - a. Deferent and epicycle just as with Hipparchus
 - b. Angular location of planet on epicycle corresponds to direction from earth O to mean sun for outer planets
 - c. Angular location of center of epicycle always tied to mean sun for inner planets
2. The distinctive feature of the model: the center of the epicycle is not moving with uniform angular velocity on its circle, the deferent, but is instead doing so with respect to a point E located symmetrically with respect to O on the opposite side of the center M of the deferent
 - a. Others after Ptolemy called this point "the equant" -- the center of equal angular motion of the center of the epicycle
 - b. The introduction of the equant was regarded as Ptolemy's great contribution
 - c. Note the abandonment of uniform circular motion: the center of the epicycle is speeding up and slowing down on the deferent, with half of the total change in apparent motion a real change in motion, and half only apparent!
 - d. This deviation from uniform circular motion was a major source of objection to Ptolemy over the next 13 centuries, including Copernicus
3. Given the tie to the mean Sun, Ptolemy's model had five basic parameters -- called "elements" -- per planet: ratio of radii of deferent and epicycle, eccentricity, direction of line of apsides with respect to the stars, period of one complete revolution on deferent (tropical or sidereal period), and period of one complete revolution on epicycle (synodic period)
 - a. Two other elements needed to fix a starting point in time: location of planet on epicycle and epicycle on deferent at some (epochal) time -- e.g. a time of some specific vernal equinox (or accession to a throne)
 - b. But (given tie to mean sun) five basic elements plus start time needed to "represent" complex motion displaying many more (apparent) degrees of freedom, or at least degrees of irregularity, in the patterns of retrograde motion
 - c. An extraordinary discovery, which he surely took to be a "secret of the universe" type of discovery
4. The empirical basis for the standard Ptolemaic planetary model
 - a. The mean speeds on the deferent and epicycle from the tropical and synodic periods, i.e. the mean times of return to the same location along the zodiac and with respect to Earth and Sun
 - b. The orientation of the line of apsides (relative to the stars of along the zodiac) from the locations of the maximum and minimum apparent angular speeds: apogee where apparently slowest
 - c. The total eccentricity from the ratio of the maximum and minimum apparent angular speeds -- i.e. the distance from the earth to the point of presumed equiangular motion

- d. The pattern of variation in timing and width of retrograde motions then requires the center of the deferent to be located midway between the earth and that point: bisection of eccentricity
 - e. Specifically, given Ptolemy's method of deriving the ratio of the deferent and epicycle radii, do not given consistent values of ratio from one retrograde loop to the next unless the center is located (very nearly) midway between the two points
 - f. This implies that, of the total change in the apparent angular speed from minimum to maximum, half is only merely apparent and half is real
5. Not only a "model", but complete computational procedure for taking preferred observations to determine the elements and for then calculating geocentric longitudes
 - a. Computationally tractable; even more so with tables
 - b. Model helps indicate which observations to use to determine elements
 6. Mercury the only planet that posed a special problem for Ptolemy, requiring a feature beyond the basic epicycle-eccenter-equant model
 - a. Not clear what data led Ptolemy to augment the model for Mercury
 - b. One thing that makes Mercury different is that its (elliptical) orbit has a large eccentricity (0.2 versus 0.09 for Mars, 0.007 for Venus, and all others less than 0.06) -- i.e. Mercury orbit most elliptical (though even then its minor axis is only 2 percent shorter than its major axis)
 - c. Another thing: Mercury difficult to observe, because of short time during which it can be observed and atmospheric refraction
 - d. Ptolemy could easily have been misled by data; note, though, that he let data dictate the model, and his ultimate model has a trajectory approximating an ellipse
 7. Ptolemy's solution for Mercury: a moving center for deferent, somewhat akin to way his first newly discovered anomaly of moon handled
 - a. Motion on the two smaller circles constrained so as to counterbalance one another
 - b. The only oddity in an otherwise uniform account of the longitudes of the five planets
 - c. But even this less glaring than it may at first seem, for Mercury nearest the moon and hence not so odd that its motion involves features of the moon's
- G. The Achievement of Ptolemaic Astronomy
1. Achieved what he set out to: a complete, computationally tractable account of the motion of the sun, moon, and five planets, covering all principal irregularities to a high degree of approximation
 - a. First successful account in history
 - b. No real improvement on its accuracy until the first decade of the 17th century, with Kepler
 2. Focused on oppositions, stationary points, patterns of retrograde loops, maximum elongations, principal anomalies etc., where it had extraordinary success
 - a. Complete prediction of latitude and longitude for all time, past and future, including (incorrect) effects of precession of the equinoxes