b. He ends up handling it via a (circularly) moving eccenter

4. Ptolemy then discovered a still further anomaly -- i.e. discrepancy between theory and observation -- when Moon in octants that is not handled by this moving eccenter
   a. Just as with first anomaly he discovered, found that irregularity correlated with relative earth-moon-sun positions
   b. Handles it by requiring motion on epicycle to be measured from the "mean," not the true apogee
5. One feature of his lunar theory stood out as a shortcoming right down to the time of Copernicus
   a. Theory predicts that the distance between the earth and the moon varies much more widely than parallax and apparent diameter measurements indicate
   b. Specifically, model calls for variation from 64 to 34 earth-radii, which would produce almost a factor of 2 variation in the apparent diameter of the Moon, while the actual variation in apparent diameter is closer to 10 percent
6. In spite of this and other lesser shortcomings, the account of lunar motion the other major advance of *Almagest* over all prior Greek astronomy
   a. Major advances: two newly discovered anomalies, and realization that they correlate with earth-moon-sun positions
   b. Each discovered via deviations from less refined model, leading to further refinement in described motion
   c. Notice that Ptolemy here refining his theory on the basis of systematic deviations from simpler versions of it -- a sequence of successive approximations
7. (An aside: the motion of the moon turns out to be the most difficult of that of any of the naked-eye observable bodies in our planetary system
   a. For this reason we will give little emphasis to it until we get to the *Principia*
   b. And even there, as you will see, it proved more complicated than Newton could handle)

E. The Problem of Planetary Motion for Ptolemy
1. The *Almagest* presents only Ptolemy's finished account, not the way in which he reached it
   a. This gives the impression that Ptolemy simply came up with a hypothetical model and then accepted it because it gave good results
   b. Noel Swerdlow and Jim Evans have each made a compelling case to the contrary, reconstructing steps by which Ptolemy reached his model for Mars from passing remarks in the *Almagest*
   c. Swerdlow's reconstruction presented here (paper in Supplementary Readings); paper by Jim Edwards assigned for next class offers a parallel account
2. From what Ptolemy says, Hipparchus had made the same move in trying to account for the variations in the retrograde loops as he had for the sun
   a. Put the center of the epicycle on a circle eccentric with respect to earth -- an eccenter – to account for the first or zodiacal inequality

b. The "equation of center" – d_1 in Swerdlow's diagram in the Appendix -- gives an optical correction for the center of the epicycle
c. Oppositions of the planet provide direct observation of this center, and hence establish the magnitude and direction of the eccentricity
d. But oppositions do not give any information about the distance of the center of the epicycle from the Earth

3. Hipparchus found that this model, while giving a good equation of center, gives totally inaccurate lengths of different retrograde loops, at least for Mars
   a. Where model predicts longest retrograde arcs, actual arcs are shortest, and vice versa
   b. This was the 300 year old problem that Ptolemy faced and solved
4. Ptolemy's solution: since model gives the direction, but not the distance of the center of the epicycle from the earth, let empirical data determine the distance
   a. Since the radius of that circle, the deferent, is in effect given (i.e. R=60), this amounts to letting data determine where its center is located
   b. In other words, do not presuppose that the center of the deferent is at the point of equiangular motion
   c. Instead, taking directions dictated by the point of equiangular motion as given, use the lengths of the retrograde loops to locate the center of the deferent
   d. Different combinations of data on retrograde loops for Mars consistently implied that the center of the deferent is very near the midpoint between the Earth and the point of equiangular motion
   e. Venus provided an independent way of locating the center of the deferent, from the variations in maximum elongations, and it too gave the midpoint
   f. Only the model-mediated determination for Venus is presented in the *Almagest*, with the text appearing simply to adopt the conclusion drawn from Venus for the three outer planets
5. The "***bisection of eccentricity***": use oppositions to determine total eccentricity and then locate the center of the deferent at the midpoint between O and E -- i.e. the observer and the "equant"
   a. An empirically driven refinement to Hipparchus's model, solving the 300 year problem of giving an account of the retrograde loops
   b. Ptolemy had mathematics not available to Hipparchus
   c. Given the insight that distance from O to the center of the epicycle is not empirically fixed by fixing the location of E, used that mathematics to infer the distance from (redundant) observations
   d. Observations consistently implied, to within reasonable approximation, a bisection of eccentricity (see Evans on qualifications to this claim)
   e. Conclude exactly a bisection: a conclusion that amounts to shifting the circular trajectory of the center of the epicycle by moving its center nearer the earth along the line of apsides