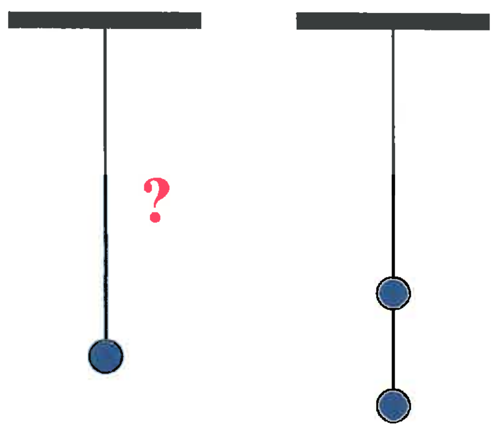
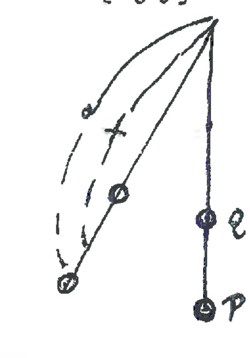
WHAT IS THE LENGTH OF A PENDULUM?

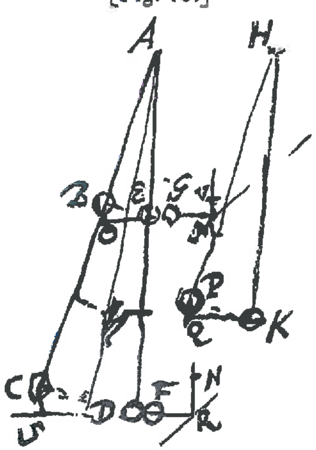
Huygens's Solution for the Center of Oscillation



[Fig. 3.]



[Fig. 16.]



Generalized Galilean Principles:

1. "If any number of weights begin to move by the force of their own gravity, their center of gravity cannot rise higher than its location at the beginning of the motion."

2. "Abstracting from the air and every other impediment, the center of gravity of a pendulum crosses through equal arcs in descending and ascending."

Length of a one-second pendulum with a 2 inch diameter spherical bob is 0.1 lines longer than the distance to the center of the bob.

Center of Oscillation

Prop. III. If any magnitudes all descend or ascend, albeit through unequal intervals, the heights of descent or ascent of each, multiplied by the magnitude of itself, yield a sum of products equal to that which results from the multiplication of the height of descent or ascent of the center of gravity of all the magnitudes times all the magnitudes.

Prop. IV. Assume that a pendulum is composed of many weights, and beginning from rest, has completed any part of its whole oscillation. Imagine next that the common bond between the weights has been broken and that each weight converts its acquired velocity upwards and rises as high as it can. Granting all this, the common center of gravity will return to the same height which it had before the oscillation.

***Solution for the length of a compound pendulum*:**

$$\ell = \frac{\sum m_i \ell_i^2}{\sum m_i \ell_i} = \frac{\sum m_i \ell_i^2}{\ell_{cg} \sum m_i} \quad \text{or} \quad = \frac{\int y^2 dm}{\int y dm}$$

Leibniz (1686): Forces are proportional, jointly, to bodies (of the same specific gravity or solidity) and to the heights which produce their velocity or from which their velocities can be acquired. More generally, since no velocities may actually be produced, the forces are proportional to the heights which might be produced by these velocities. They are not generally proportional to their own velocities.... Many errors have arisen from this latter view.... I believe this error is also the reason why a number of scholars have recently questioned Huygens's law for the center of oscillation of a pendulum, which is completely true.