116 NEWTON TO OLDENBURG[1]

23 JUNE 1673

From the original in the Library of the Royal Society of London.
In reply to Letters 112 and 113

Sr

I received your letters wth M. Hugens kind present,[2] wch I have viewed wth great satisfaction, finding it full of very subtile & usefull speculations very worthy of ye Author. I am glad yt we are to expect another discours of ye *vis centrifuga*, which speculation may prove of good use in naturall Philosophy & Astronomy as well as mechanicks.[3] Thus for instance if the reason why the same side of ye Moon is ever towards ye earth be ye greater conatus of ye other side to recede from it; it will follow (upon supposition of ye Earths motion about ye Sun) that ye greatest distance of ye sun from ye earth is to ye greatest distance of ye Moon from ye earth, not greater then 10000 to 56 & therefore the parallax of ye Sun not less then $\frac{56}{10000}$ of ye Parallax of ye Moon: Because were the sun's distance less in proportion to yt of ye Moon, she would have a greater conatus from ye sun then from ye earth. I thought also sometime that ye moons libration might depend upon her conatus from ye Sun & Earth compared together, till I apprehended a better cause.[4]

In ye Demonstration of ye 8th Proposition,[5] *De descensu gravium*, there seems to be an illegitimate supposition, namely yt ye flexures at *B* & *C* do not hinder ye motion of ye descending body. For in reality they will hinder it, so yt a body wch descends from *A* shall not acquire so great velocity when arrived to *D* as one wch descends from *E*. If this supposition be made becaus a body descending by a curve line meets with no such opposition, & this Proposition is laid down in order to ye contemplation of motion in curve lines: then it should have been shown that though rectilinear flexures do hinder, yet ye infinitely little flexures which are in curves, though infinite in number, do not at all hinder the motion.

The rectifying curve lines by that way wch M. Hugens calls Evolution,[6] I have been sometimes considering also, & have met wth a way of resolving it wch seems more ready & free from ye trouble of calculation then that of M. Hugens. If he please I will send it him. The Problem also is capable of being improved by being propounded thus more generally.

Curvas[7] invenire quotascunque quarum longitudines cum propositæ alicujus Curvæ longitudine, vel cum area ejus ad datam lineam applicata, comparari possunt.

AD. Hence, since the times are respectively equal to these latter times, it is clear that the time of descent through AC will have the same ratio to the time of descent through AD as AC has to AD. Q.E.D.

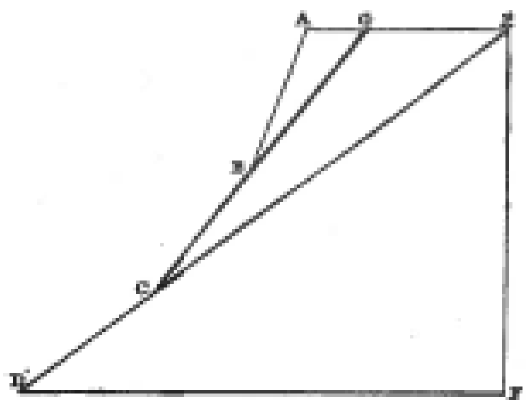
In the same way it is also shown that the time of descent through AC is related to the time of fall through the perpendicular AB as AC is related to the length AB.

PROPOSITION VIII

If from the same height a body descends by a continuous motion through any number of contiguous planes having any inclinations whatsoever, it will always acquire at the end the same velocity; namely, a velocity equal to that which would be acquired by falling perpendicularly from the same height.

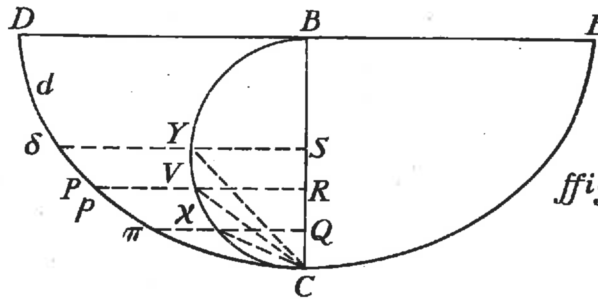
Let AB, BC, and CD be contiguous planes [Fig. 13] whose terminus A has a height above the horizontal line DF, drawn through the lower terminus D, equal to the perpendicular EF. And let a body descend through these planes from A to D. Now I say that at D it will have the same velocity which it would have at F by falling from E.

[Fig. 13]



For when CB is extended it cuts AE at G, and likewise DC, when extended, cuts AE at E. Now a body descending through AB will acquire the same velocity at B as a body descending through GB [Proposition 6]. Hence it is clear that, if the change of direction at B has no effect, the body

Newton on the Isochronism of Cycloidal Pendulums



ffig. 1

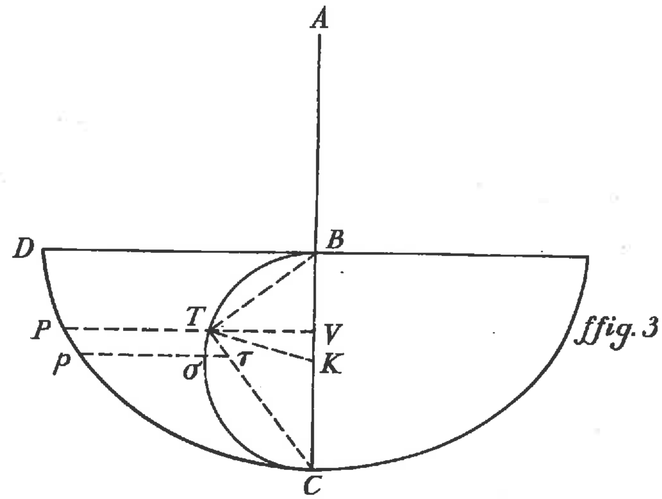
***Claim*: The accelerations at D and any point P along the arc DC of the cycloid are everywhere proportional to the remaining length of the arc to C, that is, to the arcs DC and PC.**

For the tangents along the arc are everywhere parallel to the chords of the generating circle and the acclerations are everywhere as the sines of the angles of inclination of the tangents; therefore the ratio of the accelerations at D and P are as BC to YC, and the arc lengths DC and PC are (by Wren) twice the lengths of these chords.

***Claim*: If the acceleration everywhere along the arc DC is as the arc length remaining to C, then the times of descent from any starting point along DC are the same.**

Since a_D/a_P=DC/PC, the spaces described in the small increment of time required to complete Dd and Pp are in the same ratio – i.e. Dd/Pp=DC/PC; but then dC/pC= DC/PC, and hence a_d/a_p= a_D/a_P; thus the accelerations remain in the same ratio at all times all the way down to C where "both simultaeously dwindle to nothing."

Newton on the Law of the Cycloidal Pendulum



***Claim*: Arc lengths along the generating circle can represent time – i.e. t_{Dp}/t_{DC} = Bσ/arc-BC**

The velocity varies as $\sqrt{BV} \propto BT$ (since $BT^2=BV\times BC$). Now let Pp be a small element so that Pp and Tτ can be taken as straight and parallel to one another. But then Tστ and TKB are similar triangles (respective sides perpendicular to one another), so that BT×Tσ = TK×Tτ. TK is given, and and so is Tτ=Pp. Therefore since BT and Tσ vary inversely with one another, as do the velocity at p and the increment of time from P to p do, Bσ can represent the total time from D to p.

***Claim*: But then t_{DP}/t_{BV} = arc-BT/BT, and t_{DC}/t_{BC} = arc-BC/BC – i.e. time of descent along the cycloid = $(\pi/2)\sqrt{(2BC/g)}$**

Since $BT^2=BV\times BC$, time to fall vertically can be represented by BT. At the outset descent from D and from B vertically are the same, and hence the two repesentations of time are commensurate with one another: they are "so constituted among themselves that they initially show equal times of descent of equal distances, and afterwards they will correctly show times of descent of equal distances gradually becoming unequal."