Exploratory Study of Fourth Graders Concurrently Investigating Perimeter, Area, Surface Area, and Volume in an Interactive Classroom

Qualifying Paper

Ph.D. Program in MSTE Education, Tufts University

Joan D. Martin

September 2007

Table of Contents

Tables

Figures

Abstract

This paper describes an exploratory study of how a fourth grade class investigated the relationships among perimeter, area, surface area, and volume of rectangular shapes. The five-session intervention involved 11 boys and 8 girls in a suburban public school. The students working independently and in partnerships were given particular tools (e.g., measuring strings, grid mats, rectangular prisms, pre-cut rectangular shapes, one-inch plastic tiles, and Klick-like puzzles*) to complete measurement tasks. The students demonstrated their understanding of the 2-D and 3-D attributes in pretests and posttests, discussions, reflections, prediction tasks, constructions of 2-D and 3-D rectangular shapes and arrays, and posters. The quantitative results indicate increased student performance and understanding, particularly for area, surface area, volume, and appropriate units of spatial measure. Qualitative findings suggest that the interaction of planned partnerships, the materials students used, and the tasks, activities, and reflections they completed, produced positive results.

* Cubes connected by an elastic band threaded through each cube such that the cubes can be manipulated into various positions. Klick is the commercial name.

Exploratory Study of Fourth Graders Concurrently Investigating Perimeter, Area, Surface Area, and Volume in an Interactive Classroom

Introduction

Freudenthal (1973) stated that “ geometry . . . is grasping space.. . .it is grasping that space in which the child lives, breathes and moves” (p. 403). In 1981, he further stated that “the mathematised [sic] spatial environment is geometry, the most neglected subject of mathematics teaching today” (Freudenthal, 1981, p. 145). In 1992, the Executive Committee of the International Commission on Mathematics Instruction (ICMI) amplified the message that geometry instruction needed attention. This international concern about the role of geometry in school mathematics, including didactical issues and curriculum design, caused the start of an ICMI Study on the teaching of geometry (Mammana & Villani, 1998). Despite the importance of geometry, there is convincing evidence that students do not have a strong understanding of perimeter, area, and volume (Martin & Strutchens, 2000), and past studies give further evidence that many secondary students do not have a firm grasp of length, area or volume concepts (Hart, 1981, 1989).

Geometry, while not synonymous with measurement, “ has everything to do with measuring” (van den Heuvel-Panhuizen, 2005, p. 7), and particularly so in the elementary grades. Students in the elementary grades need ample opportunities to learn how to measure the physical structures in their everyday world.

Geometric measurement is not a simple skill. Rather, it is a combination of concepts and skills that develop slowly over years (Clements & Stephan, 2003; Great Britain. Committee of Inquiry into the Teaching of Mathematics in Schools. & Cockcroft, 1982; Lehrer, 2003; Lehrer, Jaslow, & Curtis, 2003). Among the most important conceptual foundations of spatial measurement are the relationship of the unit to its corresponding attribute and the iteration of that unit to determine the measure (Lehrer, 2003).

While no best sequence of instruction for the teaching of geometric measurement has been proven (Lehrer, 2003), it has been shown that elementary school students should be developing a "theory of measure" in order to gain "flexible adaptability to novel conditions of application and to serve as a foundation to future learning" (Lehrer et al., 2003, p. 100). The sequence of instruction of geometric measurement has been and continues to be length, then area, then volume[1]. It has been shown, though, that the measurement of length need not be a prerequisite for area measurement (Curry & Outhred, 2005). Some researchers have suggested that simultaneous investigations of area and volume may be beneficial (Lehrer et al., 2003); others have suggested that it may also be advisable to delay volume (by packing) until students have mastered area (Curry & Outhred, 2005). Curry, Michelmore, and Outhred (2006) conducted a study, unique in the field, investigating children's concurrent development of understanding of length, area, and volume. They concluded that students, when finding length, area or volume, often do not understand the importance of the use of appropriate identical units that leave no gaps when determining the measure and that students may not be sure what they are measuring. They strongly suggested that students benefit from "tasks where errors can occur if students do not understand the basic principles" (Curry et al., 2006, p. 383). They contended that having such tasks would help focus a discussion on the *reasons* for basic measurement principles, such as the need for appropriate and congruent units, the relationship between the unit and measure, and the structure of unit iteration.

[1] This refers to volume by packing as opposed to volume by filling, which is frequently a Pre-K -2 mathematics experience.

Building on the findings by Curry and Outhred's (2005) and Curry et al. (2006), this exploratory study focused on student's investigating perimeter, area, surface area, and volume concurrently. The hypothesis is that students better recognize and understand the differences, similarities, and relationships among perimeter, area, surface area, and volume when they experience and reflect upon them concurrently. Previous studies addressing cognitive theories of instruction, general pedagogical issues, the classroom learning environment, and techniques for teaching geometry and geometric measurement, also influenced the design of this exploratory study.

Cognitive Theories of Instruction

Piaget's main interest was genetic epistemology; however he argued, and many would agree, that findings from his studies with children could be incorporated into classroom instruction. At the Second International Congress on Mathematical Education, Piaget (1975) stressed the importance of and difference between physical experience (simple or empirical abstraction) and logico-mathematical experience (reflective abstraction). Simple abstraction occurs when a child handles an object. This abstraction would come from the object itself; knowledge would be extracted from the physical properties of the object. For example, a student manipulating square tiles and cubes begins to appreciate the flatness of their surfaces and the perpendicularity of the sides and edges. Reflective abstraction occurs when a child mentally reorganizes and coordinates what he or she has learned from actions carried out on the object and forms mental representations of the results. For example, the student is able to coordinate the movement of square tiles and cubes to form arrays in the absence of the objects themselves.

Piaget (1975) warned educators that it " would be a great mistake, particularly in mathematical education, to neglect the role of actions and always remain on the level of language" (p. 6). Piaget (1975) proposed two principles that influence the role of the teacher:

1) children may give the impression that they understand a concept merely by repeating what the teacher has said or by duplicating what the teacher has done; 2) children may be able to complete tasks, but not be fully aware of and not be able to articulate what he or she is doing. From these principles, it follows that the teacher is someone "who organises situations that will give rise to curiousity and solution-seeking in the child" (Piaget, 1975, p. 9). When students struggle to understand a concept, Piaget (1975) encouraged teachers to provide counterexamples so that students' further explorations would lead them to self-correction.

Piaget (1975) alerted teachers to the fact that students, at all levels even into adolescence, are more capable of *doing* and *understanding in actions* than in expressing themselves verbally; he recommended that teachers have discussions with students and organize work groups "where partners of the same age or similar ages (an older child acting as leader of a small group) discuss between themselves, which in turn favourises verbalisation and 'awareness'" (p. 9).

Piaget's recommendations for group work and discussion are consistent with Vygotsky's theory of *the zone of proximal development*, which is defined as "the distance between the level of actual development and the more advanced level of potential development that comes into existence in interaction between more and less capable participants" (Cole & Wertsch, 1996, p. 5). Thus, students, who might not be able to complete a problem-solving task or mathematical activity when working on their own, are able to participate and perform when interacting with others who are capable of completing the task independently. An essential aspect of this social interaction is that less capable students " must be able to use words and other artifacts in ways that extend beyond their current understanding of them, thereby coordinating with possible future forms of action" (Cole & Wertsch, 1996, p. 5). Both Piaget and Vygotsky maintained that students construct their own knowledge in the context of a social environment, which is often the classroom.

General Pedagogical Issues

Pierre van Hiele and his late wife, Dina van Hiele-Geldof, while echoing some of the educational cautions and principles of Piaget and Vygotsky as stated above, focused their research specifically on geometry instruction. The van Hiele Theory of Geometry Thinking stated that there are sequential *levels of understanding* of geometric concepts. These levels are: visualization; analysis; informal deduction; and deduction (van Hiele, 1984). When there is a disparity between the teaching level of sophistication and the student's level of understanding, learning will be thwarted. While the van Hieles initially conducted their work with high school students, their theory is applicable to younger students as well (Burger & Shaughnessy, 1986; Fuys, Geddes, & Tischler, 1988; Usiskin, 1982; Wirszup, 1976). Gutiérrez, Jaime, and Fortuny (1991, p. 250), who found that students use several levels of thinking at the same time, had reservations about the strict hierarchical structure of van Hiele's levels and instead suggested that the theory be adapted to "the complexity of the human reasoning processes" (1991, p. 250). Similarly, in a longitudinal study of elementary students, Lehrer, Jenkins, and Osana (1998) found that the typical pattern of student responses showed that students may function at multiple levels of geometric thinking.

The five *phases* of the van Hiele Theory of Geometry Thinking are of particular importance because they provide a *framework for teachers* to use as they guide students to learn geometric concepts. These five phases are: information; guided orientation; explication; free orientation; and integration. Van Hiele (1999) stated:

> Instruction intended to foster development from one level to the next should include sequences of activities, beginning with an exploratory phase, gradually building concepts and related language, and culminating in summary activities that help students to integrate what they have learned into what they already know. (p. 311)

Freudenthal, who was a supporter of van Hiele's work, founded the Realistic Mathematics Education (RME) movement; RME viewed mathematics and the teaching of it "as a human activity, the primary educational goal is that the students learn to do mathematics as an activity" (van den Heuvel-Panhuizen, 2005, p. xx). RME continues to stress the importance of connecting everyday knowledge and experience with mathematical understanding. Mathematics that is taught and learned in this manner allows students to connect their everyday experiences and common sense with mathematical understanding rather than viewing mathematics as only a school subject (Elbers, 2003).

In summary, in order for students to understand mathematical concepts, Piaget (1975) recommended structuring situations that give rise to curiosity; Vygotsky focused on social interaction; RME suggested everyday experiences be incorporated into instruction; and van Hiele supported phases of instruction that include exploratory as well as summary activities.

Classroom Learning Environment

In light of the collective body of research outlined thus far, the importance of the classroom teacher's orchestration of tasks and development of social norms that create a robust learning environment cannot be overstated. Pirie and Kieren (1992) used the phrase *creating a constructivist environment* to signify the teacher's ongoing process "of optimizing the opportunities for the construction of mathematical understanding" (p. 526). Classrooms that foster a culture of cooperative responsibility for learning provide students with the optimal opportunity to develop their mathematical thinking. In such a classroom, children are more apt to feel safe to disagree, ask questions for clarification, and work collaboratively rather than competitively. Such a classroom places responsibility on all its members, including the teacher, to contribute ideas and to support all other members of the class (Lo & Wheatley, 1994). Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Oliver, and Wearne (1996) believed that the culture of the classroom supersedes the importance of the tasks that students are asked to complete. Social norms that require students to justify and explain their work are essential to any effective learning environment; social norms are not unique to mathematics classrooms (Yackel, Cobb, & Wood, 1991). In addition, Yackel and Cobb (1996) stressed sociomathematical norms, which they distinguished from social norms as follows:

> The understanding that students are expected to explain their solutions and their ways of thinking is a social norm, whereas the understanding of what counts as an acceptable mathematical explanation is a sociomathematical norm. (p. 461)

In a constructivist learning environment, the following practices have been shown to be effective: discussions, reflections, prediction tasks, constructions, and posters. Productive discussions are focused conversations about a specific mathematical topic; students actively engage in purposeful conversations (Pirie & Schwarzenberger, 1988). In such discussions, the teacher facilitates whole-group dialogue, modeling for students the kinds of exchanges they are expected to have when working in small groups. In such mathematical discussions, terms need to be functional "not only for communication but for reasoning" (Lampert, 1990, p. 47). The teacher is not the dispenser of the mathematics knowledge, but is in fact a "representative of mathematical culture outside of the classroom" (Lampert, 1990, p. 47). As such, a teacher must introduce and model conventional mathematical tools that will enhance students' thinking as they investigate perimeter, area, surface area, and volume. Yackel and Cobb (1996) underscore Lampert's (1990) depiction of what the teacher's role should be.

Lo and Wheatley (1994) stressed that class discussion in mathematics should be viewed as "a setting where students engage in the activities of explaining, clarifying, refuting and revising as an intellectual community" (p. 147). "Participation in reflective discourse therefore, can be seen both to enable and constrain mathematical development, but not determine it.…It is the students who actually do the learning" (Cobb, Boufi, McClain, & Whitenack, 1997, p. 272). McNair (2000) viewed classroom discussion as a form of socially constructed knowledge with two main parts: the text of the discussion, the actual utterances; and its purpose, structure, and coherence. Students contribute to the purpose by making comments or asking questions concerning the assignment or problem being solved.

While Sfard, Nesher, Streefland, Cobb, and Mason (1998) emphasized different aspects of mathematical discussions in the classroom, they all agreed that " mathematical conversation does seem to have great potential as a mode of learning" (p. 30). One must be aware, however, that, as Pirie and Schwarzenberger (1988) cautioned, all students may not benefit from a discussion; they found that students need to demonstrate in some way, by a physical action or verbal exchange, that they have had a change of attitude.

Through the social process of discussion and reflection, children are allowed and encouraged to make their own mathematical constructions (Elbers, 2003). An atmosphere of collaboration and mutual trust permits a community of inquiry to exist in which all students participate in the process of constructing mathematical understanding. The teacher need not direct all classroom activities. It is essential for the teacher to facilitate discussions and shared reflections thus manipulating the *tension* between guiding one's students and at the same time allowing them to construct their mathematical knowledge (Elbers, 2003, p. 78).

The strategy of having students make predictions, prior to determining an answer, has been used by Battista et al. (1998) in a 2-D study involving tiles and by Battista (1999) in a 3-D study involving cubes. Having students make predictions provides them with opportunities to think about outcomes and to reflect on their mental models rather than just moving objects to complete a task (Battista, 1999). Discrepancies that may arise between students' predictions and actual outcomes provide rich opportunities for discussion and reflective thinking.

Constructions, such as making rectangular prisms, allow students to wrestle with key ideas about linear measures of height and width, surface area and units of measure, as well as volume of the completed shape. Such concrete activities help students link their experience to the relevance of the mathematical ideas and skills they already know (Hiebert et al., 1996). Similarly, Dewey (1933) claimed that students learn through *reflective inquiry* and active engagement as problems are fully or partially resolved.

Asking children to produce demonstration materials, such as making a poster, adds interest to learning and forces students to think about what they need to do to convey pertinent information. Berry and Houston (1995) when working with college students warned that results on posters could be reported without students understanding the concepts they had presented. Berry and Houston (1995) also stressed the importance of incorporating a discussion along with a poster construction. When a poster or other such activity is completed with a partner or by a small group, it is not easy to tell "where collective work ends and individual learning begins" (Elbers, 2003, p. 93).

Techniques for Teaching Geometry and Geometric Measurement

Concerning specific proposals for the teaching of basic geometry concepts such as length, area, and volume, it has been proposed that measurement, a real world application of mathematics, connects geometry or spatial relations with real numbers (Clements & Stephan, 2003; Lehrer, 2003; National Council of Teachers of Mathematics., 2000). Understanding the measurement of spatial extent (length, area, and volume) could be, as proposed by (Lehrer et al., 2003), a practical route to grasping spatial structure.

Wheatley and Reynolds (1996) maintained that tiling is a "rich source" for students to use to understand the iteration of a unit to determine an area. However, as Outhred and Mitchelmore (1992) point out, although students could count the tiles to find the area, they do not necessarily "interpret arrays of squares in terms of their rows and columns" (Lynne Outhred & Mitchelmore, 1992). Arrays are not intuitive structures (Battista & Clements, 1996; Battista et al., 1998; Lynne Outhred & Mitchelmore, 2004). Outhred and Mitchelmore (2000) found that having students draw squares to cover a rectangular shape was beneficial. In addition, Outhred and McPhail (2000) stressed the value of drawing arrays as an iteration of rows and columns. Vergnaud's findings (1990) supported the advantage of drawing as one of the ways through which students can identify mathematical relevancy and relationships.

Battista and Clements (1996) proposed that students must be able to structure spatially a 3-D array of cubes in order to enumerate the cubes in a meaningful way beginning with layers, then layering the layers to create the 3-D rectangular array. Battista (1999) concluded that efficiently enumerating the cubes in a 3-D array was complex because of the need to coordinate and integrate the various parts of the array.

Curry's et al. (2006) study of 96 students from grade 1-4 in six public schools in a metropolitan area, with preliminary insights reported in Curry and Outhred (2005), tried to validate the developmental sequence assumed in the research-based program, Count Me in Measurement (Lynne Outhred, Mitchelmore, McPhail, & Gould, 2003). The purpose of the study was to gain additional insight into students' understanding of length, area, and volume measurement and the relationships among them. The researchers attempted to determine if students' understandings of length, area, and volume measurement could be assessed with valid comparisons, and if so, how does the development of students' understandings of these spatial attributes differ.

Curry et al. (2006) sought to uncover possible developmental patterns among five basic principles of measurement: 1. the need for congruent units; 2. the importance of using an appropriate unit; 3. the need to use the same unit when comparing objects (transitivity); 4. the relationship between the unit and the measure; 5. the structure of the unit iteration (two aspects: no gaps or overlaps of units; area and volume units form rectangular arrays). The study included 45-minute interviews consisting of a number of sets of three tasks with each set assessing students' understanding of one of the measurement principles mentioned above.

With difficulties in interpreting students' responses, such as being unable to judge exactly what a student was referring when measuring an object, Curry et al. (2006) determined that the tasks for principles 1 and 2 were not parallel and thus not valid. Valid comparisons were met for principles 3, 4, and 5. The results from the Curry et al. (2006) study confirmed: the *general* order of students developing understanding of spatial measurement was length, area, and then volume; students' understanding of area measurement was not dependent on understanding length measurement because both were affected by precision in recording the unit iteration; students' understanding of volume (packing) measurement was dependent on the *foundation* of their understanding of area measurement. Curry et al. (2006) determined that there was an increase in students' understanding of unit structure of length, area, and volume across grades 1-4 with volume packing being the most difficult. These results "support the wisdom of highlighting the similarities and differences in the unit structures of length, area, and volume measurement in a teaching context" (Curry & Outhred, 2005, p.265).

Aims and Goals of the Study

The aim of this exploratory study was twofold. First, the study explored the effectiveness of the concurrent investigation of perimeter, area, surface area, and volume of rectangular objects among fourth grade students. Second, the study explored the efficacy of the instructive processes of discussions, reflections, prediction tasks, and constructions of 2-D and 3-D rectangular shapes and posters.

The specific questions addressed in this study were:

Question 1: How do tasks that concurrently incorporate the concepts of perimeter, area, surface area, and volume, enhance an elementary school student's capacity to distinguish among the attributes and to see relationships among the attributes of different dimensions?

Question 2: How does the use of certain tools (measuring strings, grid mats, rectangular prisms, pre-cut rectangular shapes, one-inch plastic tiles, and Klick-like puzzles) affect an elementary school student's ability to distinguish among perimeter, area, surface area and volume and promote a student's understanding of the corresponding units of measure?

Question 3: In what ways are the instructional processes effective and how can their effectiveness or lack thereof be documented?

Question 4: In retrospect, what would have made the study stronger?

Methods

Participants

The 11 boys and 8 girls, aged 10 and 11, participating in this study were all the members of the same 4^{th} grade class in a suburban public elementary school. The data from two students (student #4, a boy; student #13, a girl) were not included in the statistical analysis of the pretest and posttest because their tests were incomplete. Other artifacts (e.g. the poster constructions) from these two students are included in the results here reported. Four students (students #2, 13, and 17, all girls and student #12, a boy) were receiving learning center support for both language arts and mathematics and another student (student #1, a boy) was an English-language learner.

The curriculum used in the school (and throughout the school district) is *Everyday Mathematics*. This curriculum has been the core curriculum for grades 1- 4 for over 5 years and the classroom teacher has taught it for 7 years, 3 of these years at the 4th grade level. Mathematics instruction occurred daily for at least one hour. While the researcher was the main teacher/facilitator during the five sessions, the classroom teacher took an active role by stimulating discussions, note-taking on flip charts, clarifying instructions, answering students' questions, and supporting students' engagement in the measurement tasks and poster constructions. The classroom teacher had begun the school year cultivating a classroom culture in which she and her students formed a robust community of learners; her approach to teaching was constructivist. The students viewed themselves as team members in a collaborative environment and took individual responsibility for their own learning. Group work was common practice. Clarity of expression in discussion and written work was a goal for each student; and neatness in presentation of work was also a norm.

Materials

During the hands-on activities, the students used the following materials: 11" x 17" laminated one-inch grid mats, transparent one-inch grid overlays, standard measuring tapes and rulers, measuring strings with alternating intervals of colored beads, solid wood rectangular prisms, one-inch plastic tiles, connected cubes designed from a commercial product called a Klick puzzle, and pre-cut rectangular-shaped cardstock pieces. Photos of these materials can be found in Appendix A. Stationery supplies such as paper, pencils, markers, erasers, scissors, and staplers were also available, particularly for the poster construction. For demonstration purposes and student tasks during the first session, various 2-D polygonal and non-polygonal shapes were also used. Photos of these shapes are found in Appendix A.

Procedures

The study was conducted in June during the regular school day and replaced the students' daily mathematics instruction. The study's sessions lasted between 85 and 135 minutes, which were longer than their regular daily math time of one hour. The basic format of the sessions is shown in Table 1.

Activities/Explorations	Time	
Discussion, comments, questions, feedback from students – recorded on flip chart by researcher or classroom teacher	5 – 10	minutes
Introduction of exploratory activity and/or demonstration by researcher	5 – 15	minutes
Prediction task(s) – made by students individually and independently	5	minutes
Hands-on activity completed by students in partnerships – with recording sheets	60 – 90	minutes
Reflection – written individually and independently, or in collaboration with partner, or orally in whole group discussion	10 – 15	minutes

Table 1. Basic Format of the Sessions

Students were assigned numbers from 1-19 that they used throughout the study for all submitted work. Notes from the whole-group discussions, including questions, responses, and comments, were recorded on a flip chart and color-coded to indicate the session in which they were recorded. These notes were visually accessible to the students throughout the study.

During the first session of the study, students took an untimed pretest; all tests were completed within 30 minutes. After the pretest and during the same session, perimeter and area were explored using one-inch grid paper and 2-D shapes.

For sessions 2-4, students investigated perimeter, area, surface area, and volume using one-inch square tiles, grid mats, pre-cut cardstock pieces (which matched each of the rectangular prisms' faces), rectangular prisms, and one-inch cubes.

During sessions 4 and 5, students were free to design the layout and to incorporate any of the materials used during the study to construct their posters. They discussed, planned, and constructed 11" x 17" posters to convey what they understood about the similarities and differences among perimeter, area, surface area, and volume. They were asked to identify real-world examples of each attribute and to show how these attributes are related and how they are different. The students were told to create a poster that a first grader could understand.

The content focus and agenda of sessions 1, 2, and 3, including prediction tasks, hand-on activities, materials used, and reflection prompts are shown in Table 2; those for sessions 4, 5, and 6 are shown in Table 3.

Content Focus and Agenda for Sessions 1, 2, and 3			
Session	Attribute(s), Prediction Tasks and Activities	Materials Used	Reflection Prompts
1 1	*Pretest* (perimeter, area, surface area, and volume) *Perimeter and Area* Which has the greatest area? Which has the longest perimeter? [Each student was given the same set of four 2-D shapes] Students determined the perimeter and area of each shape and recorded their answers. [Students also identified which materials they used to find their answers.]	• Paper and pencil • One-inch cube (one per student) • Polygonal and non-polygonal 2-D shapes • One-inch grid paper • String • Measuring strings • Rulers • Recording sheet • Reflection sheet	• When an adult talks about the area of something, say a playground, what does that mean to you? • Area and perimeter are different. How would you explain the difference between area and perimeter to a student in the first grade?
2	*Perimeter / Area / Surface Area / Volume* How many different rectangles do you think you can make with 8 square tiles? [Non-congruent rectangles were considered different. Each student was given 8 one-inch tiles.] How many different rectangular prisms do you think you can make with 8 cubes? [Non-congruent prisms were considered different. Each student was given a Klick-like puzzle with 8 connected one-inch cubes.] Students found different rectangles that could be formed with the 8 tiles. For each they determined the length of the rectangle's sides. Students found different rectangular prisms that could be formed with the 8 connected cubes. They then described each prism. For only one prism [of their choice] they traced each of its faces and found its surface area.	• 8 square inch tiles • 8 one-inch cubes	• What connections do you see between area and perimeter of 2-D shapes and volume and surface area of 3-D shapes?
3	*Perimeter / Area / Surface Area / Volume* What do you think the volume of your prism is? [Each student was given one of 6 different rectangular prisms.] If you covered your rectangular prism with paper (no overlaps), how much area would the paper have? The students traced each face of their prism, found the perimeter and area of each face, found the surface area and volume of their prism, and then covered their prism with pre-cut cardstock [The cardstock had been cut to the sizes of the rectangular faces of the prisms.]	• Solid rectangular prisms of various sizes • Pre-cut rectangular cardstock	• How are area and volume alike and how are they different? Give an example of each to help explain.

Table 2. Content Focus and Agenda for Sessions 1, 2, and 3

Content Focus and Agenda for Sessions 4, 5, and 6			
Session	Attribute(s), Prediction Tasks and Activities	Materials Used	Reflection Prompts
4	*Perimeter / Area / Surface Area / Volume* If you have 36 square tiles, how many different rectangles do you think you can make using all 36 tiles? [Non-congruent rectangles were considered different.] If you have 36 cubes, how many rectangular prisms do you think you can make with 36 cubes? [Non-congruent prism were considered different.] Students discussed in whole group, and then drafted with a partner, a poster 11" x 17" that would: • Display perimeter, area, surface area, and volume; • Demonstrate how these attributes are related and how they are different; • Show a real-world example that would help to convey their thinking; and • Be understood by a first grader. The students had to decide what and how to display this information.	• Paper and pencil • One-inch cube (one per student) • Polygonal and non-polygonal 2-D shapes • One-inch grid paper • String • Measuring strings • Rulers • Scissors, tape, paper clips, and other classroom construction supplies	• What would be important to include on the poster? • Draw a sketch indicating the overall layout of your poster.
5	*Perimeter / Area / Surface Area / Volume* There was no prediction task. Students in partnerships (of 2 or 3 students) constructed their posters. (posters were later displayed in classroom)	• Any material that had been used in previous sessions	• Which activities and materials were most helpful in understanding the concepts? How? Whole-group discussion
6	*Posttest* (same as pretest)	• Paper and pencil • One-inch cube (one per student)	----

Table 3. Content Focus and Agenda for Sessions 4, 5, and 6.

Whole-group Discussions.

After completion of the pretest, the researcher of the study facilitated the first of the whole-group discussions that were part of each session. The first discussion solicited student responses on 2-D and 3-D shapes. For each of the four remaining sessions, discussions were held at the start of the session as well as at the end of sessions 4 and 5. The researcher related previous work and experiences to the content focus of the day's session. Discussions held at the end of sessions were driven by students' questions and reflections on their engagement with the tasks. The prompts used to elicit students' oral reflections are given in Figure 1. The classroom teacher assisted with the scribing of the notes as well as leading discussions at the end of sessions 4 and 5.

Oral Reflection Prompts

Session 4: How would you explain to a student in the first grade what perimeter, area, surface area, and volume mean?

Session 5: What activity helped you to make sense of perimeter, area, surface area, and/or volume?

What materials helped you to make sense of perimeter, area, surface area, and/or volume?

Figure 1. Oral Reflection Prompts

The whole-group discussion notes, which were recorded on flip-chart paper, are in Appendix B. Different color inks were used at each session. Session 1's notes start with "2-D" and 3-D" written in blue with the rest of the notes from that session written in purple. Session 2's notes start with "8 cubic inches—Volume"; all notes taken in that session were written in green ink even when recorded with the notes from the first session.

Session 3's notes (which appear on two sheets) start with a point (•) and "0 D" on one sheet and "Real-Life examples of 2D and 3D" on the other sheet; these notes were written in red ink. Session 4's notes, which were written in pink, start with the frequency of the students' predictions on the number of possible rectangles made with 36 square tiles as well as their predictions on the number of possible rectangular prisms made with 36 cubes. During session 4, the classroom teacher facilitated a whole-group discussion as students made a list of what would be important to incorporate in their posters and what materials they might use to present the geometric measurement concepts of perimeter, area, surface area, and volume. These notes were written on the classroom whiteboard, and since they did not photograph well, they have been rewritten and also appear in Appendix B. Session five's notes were written in black ink.

Written Reflections.

While students ended each session with a reflection exercise, the *written* reflections were only a part of sessions 1, 2, and 3. The prompts for these written reflections appear in Figure 2, and the students' responses to these reflection prompts are in Appendix C. Students' *oral* reflections shared during whole-group discussions were part of sessions 4 and 5 and are part of the whole-group discussion notes found in Appendix B as mentioned earlier in this paper.

Written Reflection Prompts	
Session 1:	When an adult talks about the area of something, say a playground, what does that mean to you? Area and perimeter are different. How would you explain the difference between area and perimeter to a student in the first grade?
Session 2:	What connections do you see between area and perimeter of 2-D shapes and volume and surface area of 3-D shapes?
Session 3:	How are area and volume alike and how are they different? Give examples of each to help explain.

Figure 2. Written Reflection Prompts

Prediction Tasks.

After the opening discussion and introduction of the day's activity in sessions 1, 2, 3, and 4, the researcher presented a prediction task to the students. The students wrote their predictions on their recording sheet for that session. The prediction tasks completed in sessions 1, 2, 3, and 4 are found in Figures 3 through 6, and a copy of the student-recording sheets for sessions 1, 2, 3, and 4 are found in Appendix D.

Prediction Tasks - Session 1

Which of the 2-D shapes on the board do you think has the **largest area**? A, B, or C Why?

Which of the 2-D shapes do you think has the **longest perimeter**? A, B, or C Why?

Which of the 2-D shapes in front of you do you think has the **largest area**? A, B, C, or D Why?

Which of the 2-D shapes in front of you do you think has the **longest perimeter**? A, B, C, or D Why?

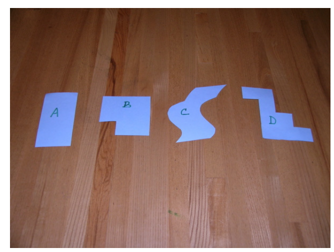


Figure 3. Prediction Tasks - Session 1

Prediction Tasks - Session 2

How many different rectangles do you think you can make with 8 square tiles?

How many different rectangular prisms do you think you can make with 8 cubes?

Figure 4. Prediction Tasks - Session 2

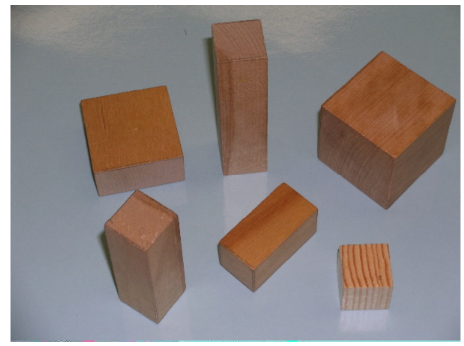
Prediction Tasks - Session 3

What do you think is the volume (in cubic inches) of your rectangular prism?

If you cover your rectangular prism with paper (no overlaps) how much area would the paper have?
(What is the surface area of your rectangular prism?)

Put a √ beside the interval in which you think the surface area would be.

- ☐ 4 and 7 square inches
- ☐ 7 and 11 square inches
- ☐ 11 and 15 square inches
- ☐ 15 and 29 square inches
- ☐ 29 and 45 square inches



(These are representative of the various prisms used.)

Figure 5. Prediction Tasks - Session 3

Prediction Tasks - Session 4

If you have 36 square tiles, how many different rectangles do you think you can make using all 36 tiles?

- ☐ Between 1 and 3 rectangles
- ☐ Between 4 and 6 rectangles
- ☐ Between 7 and 9 rectangles
- ☐ More than 9 rectangles

If you have 36 cubes, how many different rectangular prisms do you think you can make using all 36 cubes?

- ☐ Between 1 and 3 rectangular prisms
- ☐ Between 4 and 6 rectangular prisms
- ☐ Between 7 and 9 rectangular prisms
- ☐ More than 9 rectangular prisms

Figure 6. Prediction Tasks - Session 4

Partnerships and Poster Constructions.

After session 4's opening discussion and prediction tasks, students discussed with their partner (one partnership had three students) the activities that they had worked on during the previous sessions, what they had learned, and how they could present information about perimeter, area, surface area, and volume to a first grader on an 11" x 17" poster. The classroom teacher facilitated a discussion during which she used language, such as *essential knowledge* that she had used with her students throughout the year to indicate *the must know* knowledge as opposed to the *nice to know* knowledge. She stressed that the focus should be on the information that would be shared through the poster, rather than on the *prettiness* of the poster. She also reminded students that there was a time factor and that the poster had to be completed by the end of the next day's session. Session 4 was devoted to the planning of the poster so that during session 5 the plan would be executed and the poster would be completed. The students were given prompts, as seen in Figure 7, and asked to write down some of their thoughts.

Prompts for Poster Constructions – Session 4

With your partner discuss:

- What are differences among area, perimeter, surface area, and volume?
- How will you explain to a student in the first grade what area, perimeter, surface area, and volume mean?
- What will you write?
- What examples will you give?
- What will you make?
- How will you display all this information on a sheet of paper 11" x 17" so that it helps a first grader to understand?

Figure 7. Prompts for Poster Constructions

The classroom teacher thoughtfully formed partnerships consisting of two students (one had three students) for the poster construction. The teacher's rationale for each partnership is given in Table 4. All 19 students in the class worked on a poster construction.

Partnerships	Rationale
Student #1 (boy) Student #6 (girl)	Student #1 was an ELL student who spoke little English when he arrived in November of this school year. He was an average math student and hard worker when supervised, but could get off-task easily. Student #6 was a capable math student who could be bossy but at the same time compassionate when working with others whom she felt were trying their best.
Student # 2 (girl) Student # 9 (boy)	Student #2 was an inconsistent math performer, received learning center support, and had worked with student # 9 before. Student #9, who could be very impatient, was supportive of student #2, and listened to her input.
Student # 2 (girl) Student # 9 (boy)	Student #2 was an inconsistent math performer, received learning center support, and had worked with student # 9 before. Student #9, who could be very impatient, was supportive of student #2, and listened to her input.
Student #3 (boy) Student #4[2] (boy)	Student #3 was easily distracted, but as the year had progressed had begun to take his work more seriously. Student #4 was a strong overall student.
Student # 5 (girl) Student #13[3] (girl)	Student #5 could be overbearing at times, but her family and student #13's family were friends and so there was a certain acceptance by both students. The classroom teacher hoped to encourage this working relationship as the parents were attempting to foster a social relationship between the two. Student #13 had been reluctant to share her thinking in small groups, whole-group and at times, one-on-one with her classroom teacher.
Student #7 (girl) Student #10 (boy) Student #17 (girl)	Student #7 was a capable student, but was more willing to take direction than assume a leadership role. Student #10 was very capable academically, but often fatigued and would peter out. Student #17 was a natural leader, who always gave 100%. She struggled with math and reading. Although she enjoyed "thinking" about math, she often needed assistance with planning and organizing the academic component of a project.
Student #8 (girl) Student #14 (boy)	Student #8 was very creative, academically strong, but had not always made the expected progress. She liked to organize others. Student #14 had difficulty focusing his energy when working independently.
Student #11 (girl) Student #19 (girl)	Student #11 had strong number sense and worked at a very slow pace. Student #19 had developed a variety of strategies to break problems down into manageable parts, and had intense drive to complete her work.
Student #18 (boy) Student #12 (boy)	Student #18 was popular among classmates. An average math student, who never got discouraged, he was patient with others. Student #12 had difficulty with math and with reading and frequently gave up. If he felt that someone was critical of him, he would shut down. Student #12 was very artistic and did better when actively involved with materials.
Student #15 (boy) Student #16 (boy)	Student #15 loved math, and although he was capable of solving more difficult problems, he often did so in an inefficient way. He was very competitive, rushed to complete his work, and tended to point out to others what he knew. Student #16 was very quiet and an average math student. Both students were sports-minded and student #15 respected the athletic ability of student #16.

Table 4. Partnerships for Poster Constructions

[2] Student #4's pretest and posttest data were incomplete and were not included in the statistical analysis.

[3] Student #13's pretest and posttest data were incomplete and were not included in the statistical analysis.

Results/Discussion

Pretest/Posttest

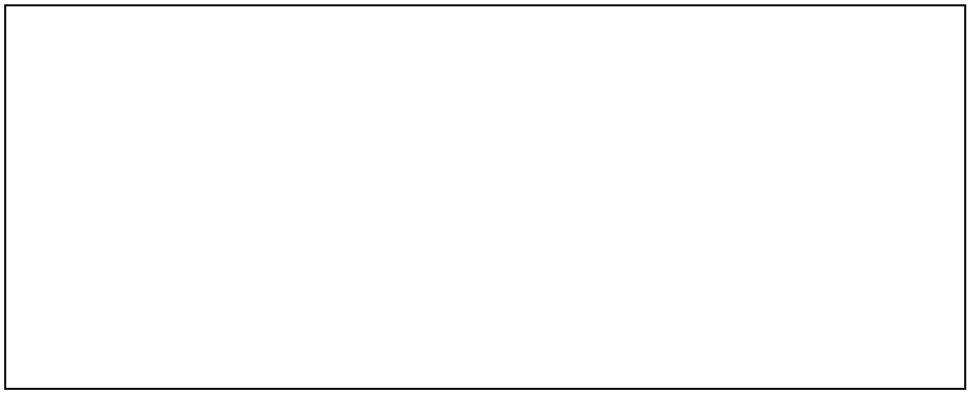
The pretest and identical posttest consisted of 4 multi-step questions and is found in Appendix E. For analytical purposes, the questions were given a score of one point for the numerical part of the answer, and, if appropriate, a score of one point for the unit of measurement.[4] For example, for question 1, as shown in Figure 8, the students were asked to find the perimeter of the drawn rectangle with the information that the rectangle had two sides with a length of five inches and two sides with a length of two inches. They were then asked to draw a different rectangle with the same perimeter and to label the length of the sides of their drawing. Numerical answers varied. Since the given rectangle had a perimeter of 14 inches, a 1” x 6” and a 3” x 4” rectangle were possible answers. Thus, for analytical purposes, question 1 has four parts to its answer: the first part is “14,” which is the numerical part of the answer; the second part is the indication of the unit of measurement; the third part is the drawing of a different rectangle whose lengths of sides add up to 14; and the fourth part is the indication of the unit of measurement. There were no restrictions as to using inches or centimeters or of using fractional amounts; no student used centimeters or fractional amounts.

---

[4] This accentuates the important structure of (magnitude, unit), a “mass noun,” a phrase coined by Schwartz (1996).

Question 1

This rectangle has two sides that are five inches long and two sides that are two inches long. What is *the perimeter* of the rectangle?



Can you draw a *different* rectangle that has the *same perimeter*? If so, draw it below and label the length of all the sides.

Figure 8. Question 1 on Pretest and Posttest

The full breakdown of the four Pretest/Posttest questions with reassigned numbers (1-27) and listed categories (perimeter, area, surface area, volume, and units of measurement) is shown in Table 5. Question 1 on the student test was scored as question numbers 1 - 4; question 2 as numbers 5 - 12; question 3 as numbers 13 - 19; and question 4 as numbers 20 - 27. In this way, scores could be aggregated and data examined according to the categories of attributes (i.e., perimeter, area, surface area, and volume), and units of measurement.

		Specific Category					
Question Numbers (as they appeared on the student's test)	Reassigned Numbers (used for analytical purposes)	Perimeter	Area	Surface Area	Volume	Units of Measurement	General Context of Question
1	1	X					Involves drawing a rectangle
	2					X linear	
	3	X					
	4					X linear	
2	5		X				Involves answering questions about a 3-D object for which only the rectangular footprint is given
	6					X square	
	7	X					
	8					X linear	
	9				X		
	10					X cubic	
	11			X			
	12					X square	
3	13				X		Involves answering questions about a 3-D object which is depicted in a 2-D drawing
	14	X					
	15					X linear	
	16		X				
	17					X square	
	18			X			
	19					X square	
4	20			X			Involves writing out one's thinking
	21				X		
	22	X					Involves answering questions using a 3-D object made up of rectangular prisms
	23	X					
	24					X linear	
	25			X			
	26			X			
	27					X square	
4 multi-step questions	Totals	6	2	5	3	11	27 questions

Table 5. Breakdown of Pretest/Posttest Questions by Category

Each student was given a pencil, ruler, a one-inch cube, and a 3-D shape composed of rectangular prisms glued together, which was used in question 4. No more than two students received the same 3-D shape. And each student received the same 3-D shape for the pretest and the posttest with the exception of student #16. Inadvertently, student #16 received a different shape on the posttest, but the researcher determined that the level of difficulty in answering the questions related to the shapes was equivalent and data for this student was included in the analysis.

Each student's pretest and posttest scores are displayed in the double bar graph in Figure 9. The total possible score was 27 points. The minimum and maximum values for the pretest were 6 and 23 respectively, and for the posttest they were 5 and 26, respectively. The mean for the pretest was 12; the mean for the posttest was 16.5. The median for the pretest was 12; the median for the posttest was 14. The mode for the pretest was 6; the modes for the posttest were 10, 13, 14, and 25, with two students attaining each of these scores. The range for the pretest was 17; the range for the posttest was 21.

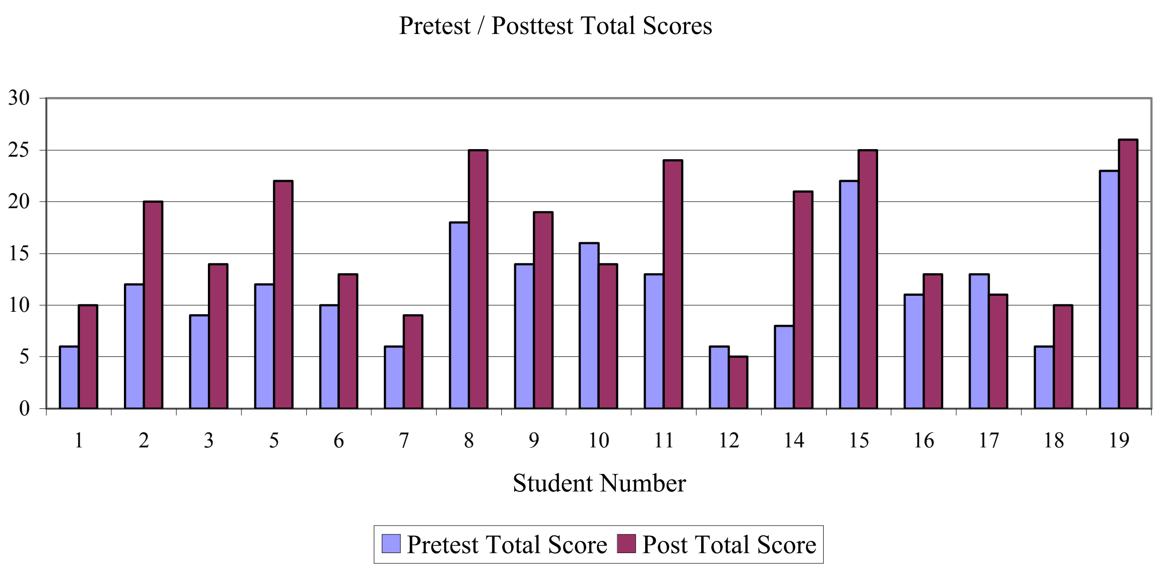


Figure 9. Students' Scores on Pretest and Posttest

Fourteen of the 17 students[5] improved from the pretest to the posttest, but student #10, student #12, and student #17 did not. Student #12 and student #17 were students

[5] Students #4 and #13 had incomplete data and are not included in the statistical analysis of pretest and posttest scores.

receiving special education services in mathematics; student #10 was, as noted in Table 4, “often fatigued and petered out.”

Due to the small size of the sample (N =17), non-parametric tests were conducted. The Wilcoxon Signed Ranks Test was applied to the matched pairs of *total scores* for each student, and their totals on the aggregate *just area* questions, *just volume* questions, *just surface area* questions, and *just units of measurement* questions. When the Wilcoxon Signed Ranks Test was applied to students’ matched aggregate scores on *just area* questions, *just surface area* questions, *just volume* questions, and *just units of measurement* questions, it was shown that at $\alpha = .05$, N = 17, there was a significant difference in each of these categories. There was no significant difference for *just perimeter* questions. The topic category and the significance value for each of these aggregates are shown in Table 6.

Wilcoxon Signed Ranks Test on Pretest and Posttest Matched Pairs	
Aggregate Scores	Significance Value
Total Score	.001
Just Area Question	.013
Just Perimeter Questions	.118
Just Surface Area Questions	.002
Just Volume Questions	.015
Units of Measurement Questions	.002

Table 6. Results of Wilcoxon Signed Ranks Test on the Matched Pairs of Aggregate Scores on Pretest and Posttest

The Spearman Rho test was conducted on students’ posttest scores for perimeter, area, surface area, and volume to determine if there were any correlations between students’ performances on questions dealing with these attributes. Correlation at $\alpha = .05$ between the number of correct responses on perimeter questions and the number of correct responses on surface area questions was statistically significant and strong, $\rho(15) = .606$, $p < .05$. At the same alpha level, there was a statistically significant and strong correlation between correct area and volume responses, $\rho(15) = .685$, $p < .01$, and there was a statistically significant and moderate correlation between correct perimeter and volume responses, $\rho(15) = .523$, $p < .05$.

The intervals of percent change from pretest to posttest are shown in Figure 10. Of the 14 students, who improved, more than half of them made a gain of between 7% and 20%. Among them are student #15 and student #19 each of whom scored the highest on the pretest and the posttest and therefore were limited in the percent gains they could achieve between tests. Student #5, student #11, and student #14 are in the 35% to 48% interval of percent change. The work of these students will be highlighted in the remaining discussion of this paper in order to gain insight into their substantial improvement.

Percent Change in Student Scores

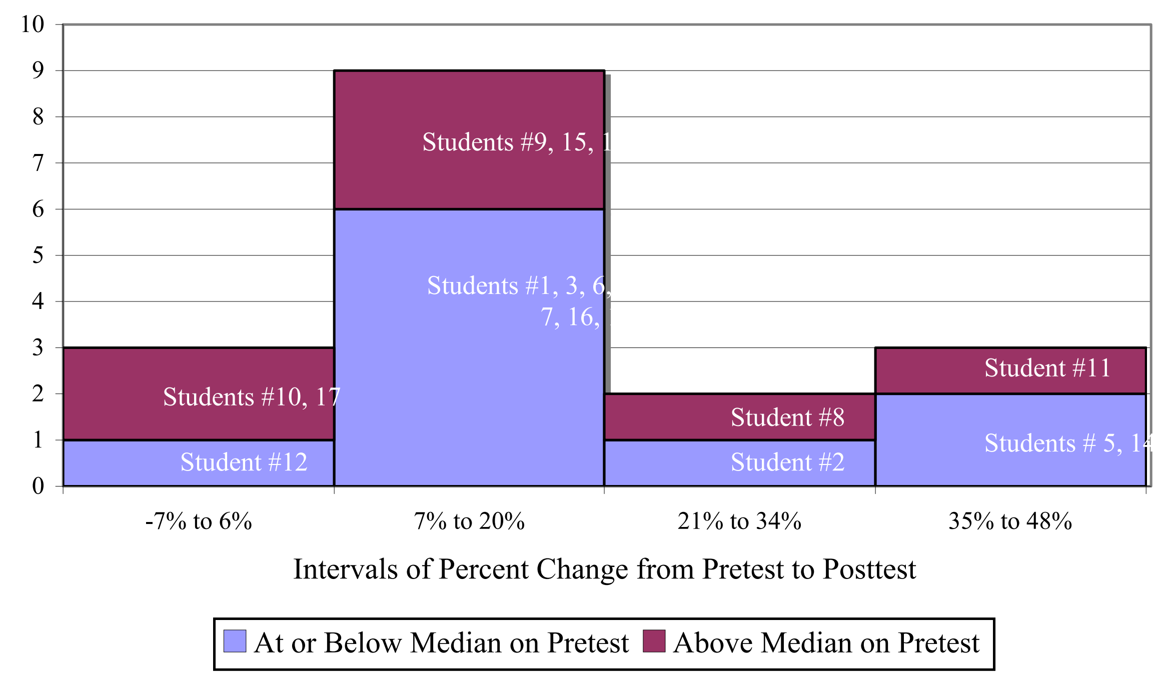


Figure 10. Intervals of Percent Change from Pretest to Posttest

In order to determine more precisely where students' gains and losses were made, the students' scores on questions that dealt only with units of measurement were separated from all other questions. The percent change for each student's scores in these two categories (numeric answers and unit of measurement), as well as in total score, is shown in Figure 11. Note that student #12, who showed an overall loss between pretest and posttest, actually improved on units of measurement.

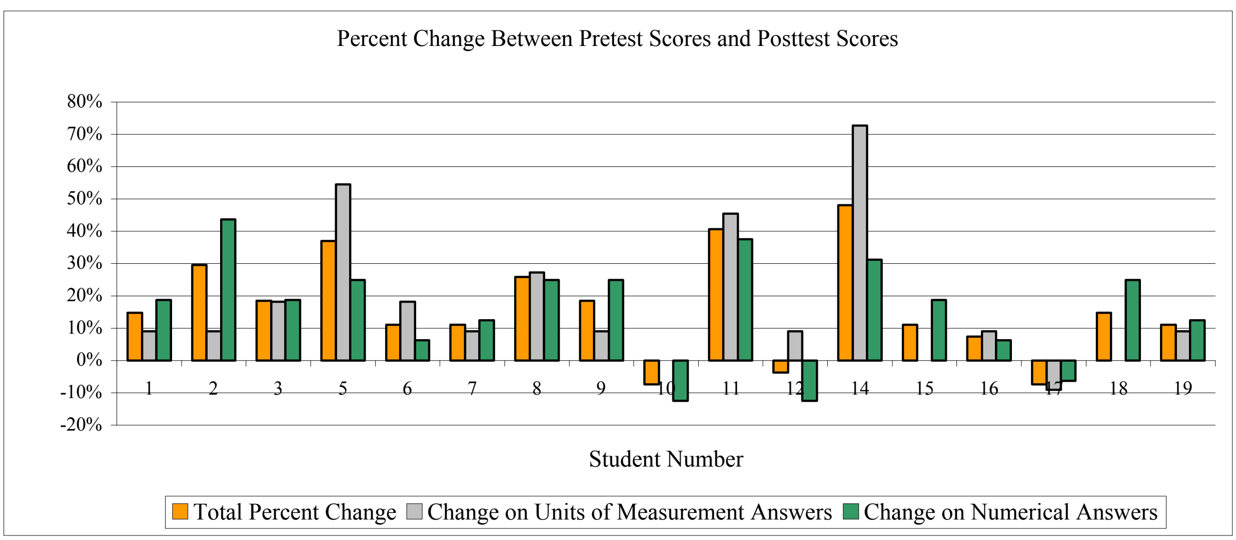


Figure 11. Percent Change of Each Student's Gains or Losses from Pretest to Posttest

Student #14 made a dramatic improvement on questions involving units with over a 70% gain; student #5 made a substantial gain on units of measurement with over a 50%

change. Student #14 had a slightly greater than 30% change on numeric answers, and student #5 had approximately a 25% gain on numeric answers. Student #11's substantial gains in both categories of questions were less disparate with an approximate 45% change on units of measurement and an approximate 38 % change on numeric answers.

The scores of all the students were disaggregated into categories of perimeter, area, surface area, volume, and units of measurement; the percentage of the possible scores in each of these categories is shown in Figure 12. Note the descending order of the percentages of maximum scores of perimeter, area, and surface area. This would reflect the order in which these topics are taught and the relative length of school exposure the students have had in solving problems related to these attributes.

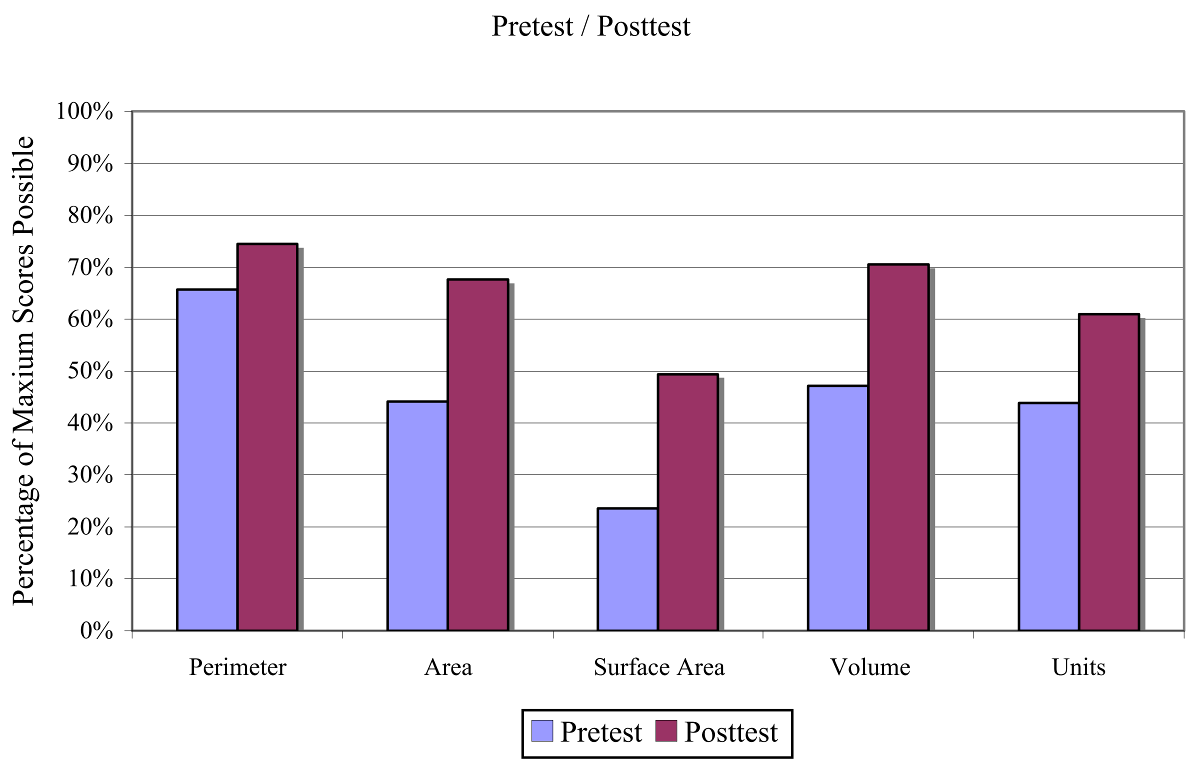


Figure 12. Percentage of Maximum Points Scored on Pretest and Posttest

Since volume is a topic that has not yet been formally taught, it may be surprising that the volume percentage is actually higher than that for area questions. This may be due in part to the disparity among the number of questions asked in each category as shown in Table 5. There were only two questions on area and three questions on volume, but six questions were asked on perimeter, five questions on surface area and eleven questions on units. While this disparity does not diminish the positive outcome, it makes the comparisons of the relative gains among the categories more difficult. Also, the level of difficulty of questions and the impact on results should be considered. Some of the questions, such as 20 and 21, were open response, while other questions, such as 23 and 26, required students to estimate a measurement.

In order to get an alternative perspective on the differences in scores between pretest and posttest, the frequency of correct responses for each of the 27 test questions is shown in Figure 13.

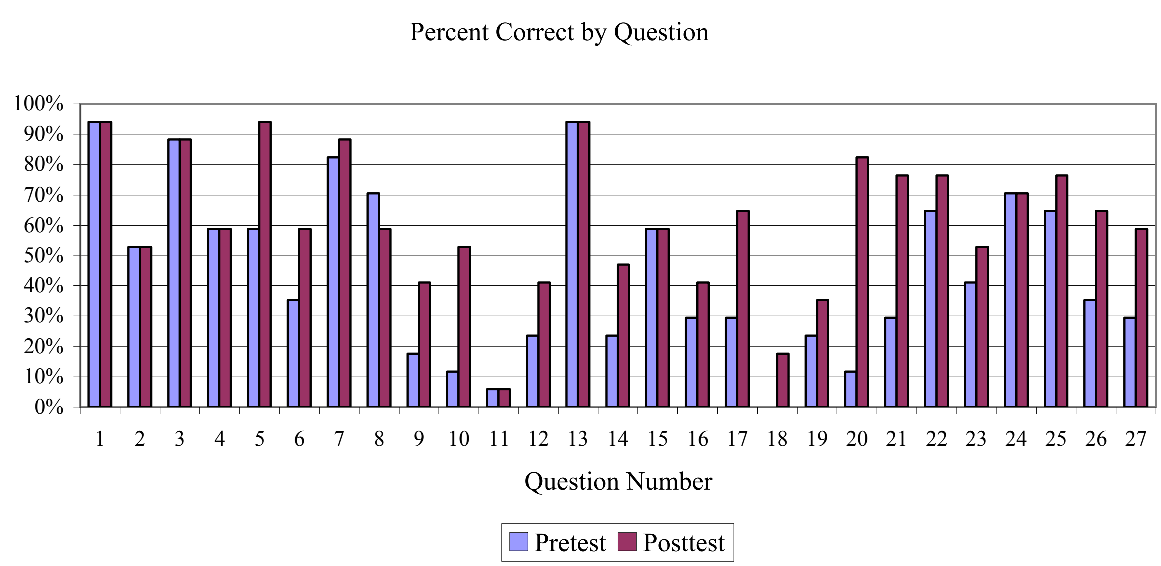


Figure 13. Frequency of Correct Responses for Each Test Question on Pretest and Posttest

Note that for question 13, over 90% of the students answered it correctly on the pretest. This should seem surprising because it was considered a volume question. On closer examination of the question, which asked students to give the number of cubes in a drawn 3-D shape, it is clear that students needed merely to count pictured cubes to arrive at the correct answer. This could be done without any consideration of the concept of volume. This being the case, it gives some explanation as to why so many students answered question 13 correctly without the benefit of volume instruction; it sheds more light on the unexpected high percentage of volume points scored in the pretest as shown in Figure 12.

No student correctly answered pretest question 18, which was a surface area question. This is not surprising because students had not been exposed formally to the topic of surface area and most were not familiar with that concept or expression. For these same reasons, it is not surprising that on pretest question 20 (also a surface area question) only 12% of students could articulate how to go about finding the surface area of a concrete 3-D shape. On the posttest, 82% of students were able to answer question 20 correctly; this question showed the greatest increase from pretest to posttest in the number of students answering it correctly.

Written Responses from Session 1 and the Use of a Rhyme

The students' responses to session 1's reflection prompts, found in Appendix C, revealed some interesting information, particularly when reviewed along with the students' perimeter and area scores on the pretest. The students earlier in the school year had learned the rhyme, *"Perimeter goes around the rim, area fills it in"* in order to help them distinguish the concepts of area and perimeter. Many of the students used the rhyme throughout the sessions, and this rhyme was used by the majority of students when responding to the written reflection prompt in session 1. Because of its extensive use it seemed important to investigate its effectiveness more closely. Each of the students' written responses from the first session were categorized and organized into a matrix as shown in Table 7. The matrix registers those students who used the rhyme and those who did not and identifies which students answered with correct or incorrect answers or ones with partial understanding of interior, or left the answer blank, or indicated that they did not understand.

Students Who	Used Rhyme		Did Not Use Rhyme		% of Total Number of Students (17 students)
	Student #	% on Pretest*	Student #	% on Pretest*	
Gave correct response related to a real-world use of area	3 6	47% 40%	----	----	
Gave a correct mathematical response related to area	10 14 15 16	73% 47% 87% 67%	2	60%	41%
Gave a response that indicates a partial understanding of interior	18 7	27% 33%	12	27%	
Indicated that they did not understand	5 9 11 19	67% 60% 60% 100%	----	----	
Left answer blank	8 17	87% 67%	1	27%	59%
% of Total Number of Students	82%		18%		100%

*This reflects the percent score on pretest questions that involved perimeter, area and their corresponding units of measurement (15 out of the 27 questions on the pretest).

Table 7. Students' Use of the Rhyme,
"*Perimeter goes around the rim, area fills it in,*"
Related to Pretest Scores on Perimeter and Area

Eighty-two percent of the students included the rhyme, *"Perimeter goes around the rim, area fills it in"* in their response to distinguish area from perimeter, and yet only 41% of the students were able to explain what "area of a playground" meant to them with an answer that could be considered a real-world use of area or a correct mathematical response. One student gave the practical example of mulch for area, while student #14 referenced the number of square units in the playground. Student #5 also used the rhyme, but indicated confusion with the response, "It makes me remember what area is. [a space of a few line] What?" Student #11 also indicated a lack of understanding and wrote a question mark. The full complement of the students' written responses to the reflection prompts in sessions 1, 2, and 3 is in Appendix C.

For some students, the use of the rhyme appears to have masked their inability to explain what area meant to them. This supports Piaget's (1975) contention that children may give the impression that they understand a concept by repeating something that a teacher has said. For some students, the use of the rhyme may actually provide a hook that can be useful to attach meaning to experiences dealing with area and perimeter; the interplay of remembering the rhyme will strengthen the distinction of the vocabulary words when the concepts of area and perimeter have been firmly constructed. Student # 19, who scored the highest in both the pretest and the posttest, answered in the same fashion as student # 5 and student #11, and merely wrote "What?" Clearly, with such high scores one would have said that student #19 understood the concepts of area and perimeter even before the exploratory study started. This discrepancy of performance, at the start of the study, may actually indicate that this student disconnected her schoolwork with area and perimeter from everyday real world uses of area and perimeter.

A Closer Look at the Prediction Tasks and Activities in Sessions 1, 2, and 3 Through the Work of Students #5, #11, and #14

Since Students #5, #11, and #14 made the largest improvements between the pretest and the posttest, their work will be looked at more closely to gain insight into the development of their understanding of perimeter, area, surface area, and volume. In session 1, students were shown a set of three closed 2-D shapes[6], two were polygons, one of which was a rectangle; these shapes were taped to the whiteboard at the front of the room. Students were asked to predict which of the three had the largest area and which one had the longest perimeter. Student #5 predicted shape B for having the largest area and said it was “a guess”; student #11 said, “I don’t know,” followed with “they’re probably almost the same measurement.” Student #14 predicted shape C and said, “because it has a lot of space on the inside.”

All three students predicted that shape B had the longest perimeter. Student #5 commented that “It is all curved up so it [is] longer.” Student #11 said, “Because it’s thinner and it might take longer to get around.” Student #14 said, “because it is really long.” The students’ use of everyday language is somewhat imprecise and definitely relative, but their responses make sense. For example, attributing thickness to a 2-D shape, and using the expressions, “curved up” and “really long,” do not hinder one from understanding the students’ meanings, nor does student #11’s use of time for designating the perimeter of shape B as the longest. Student #11 seems to be visualizing a walk around the shape; this possibly relates to the “rim” in the rhyme, “Perimeter goes around the rim,

[6] These are pictured in Appendix A. Note that all three shapes have the same area; shape B has the longest perimeter.

area fills it in." Student #11 did use the rhyme repeatedly in her recorded work and it seems to have helped her distinguish attributes.

In session 2, the students predicted the number of different (non-congruent) rectangles they thought could be made with 8 square tiles and the number of different rectangular prisms that could be made with 8 cubes. The students' predictions are listed in Table 8.

Predictions Made in Session 2		
Student #	Number of Different Rectangles with 8 Tiles	Number of Different Prisms with 8 Cubes
5	1 - 20	1 - 20
11	1 - 8	1 - 8
14	1	1

Table 8. Students' Predictions Made in Session 2

Students #5 and #11, who gave intervals rather than a single value for their prediction, were both girls; the only other student in the class to predict an interval was also a girl. The researcher suspects that these students made their estimates on the conservative, safe side. Student #14, on the other hand, either did not understand the problem posed or was too rigid to consider the possibility of different shapes, either 2-D or 3-D, being made using 8 tiles or 8 cubes respectively.

After the students made their predictions, they manipulated the tiles to determine exactly how many different rectangles were possible; they recorded their findings by tracing each rectangle they made with the 8 tiles and by finding the lengths of each side of the rectangles. Student #5's, student #11's, and student #14's recorded work on Session 2, Part 1 are shown in Figures 14 through Figure 16.

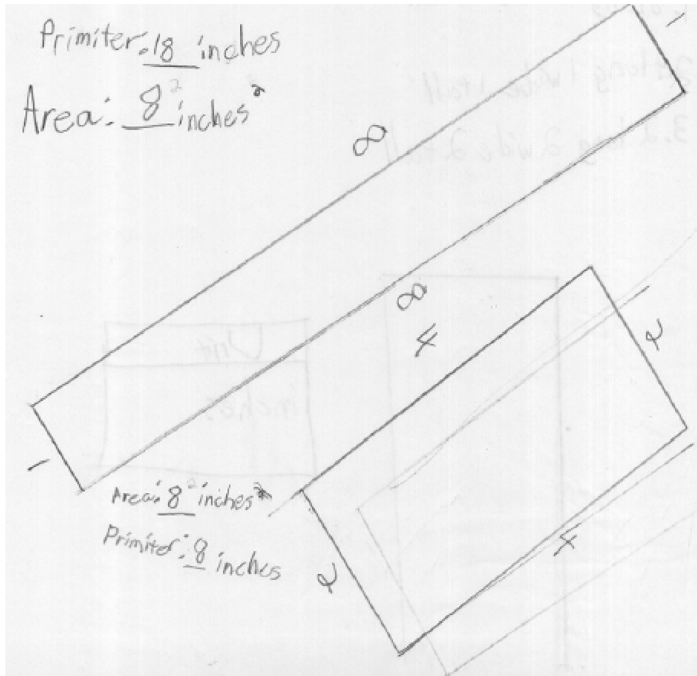


Figure 14. Student #5's Work from Session 2, Part 1

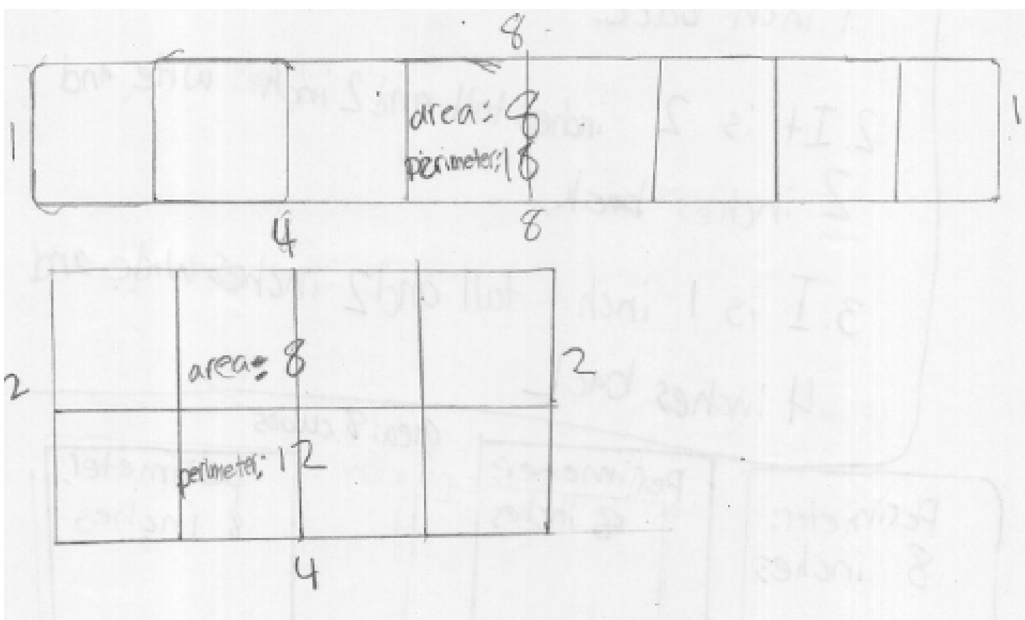


Figure 15. Student #11's Work from Session 2, Part 1

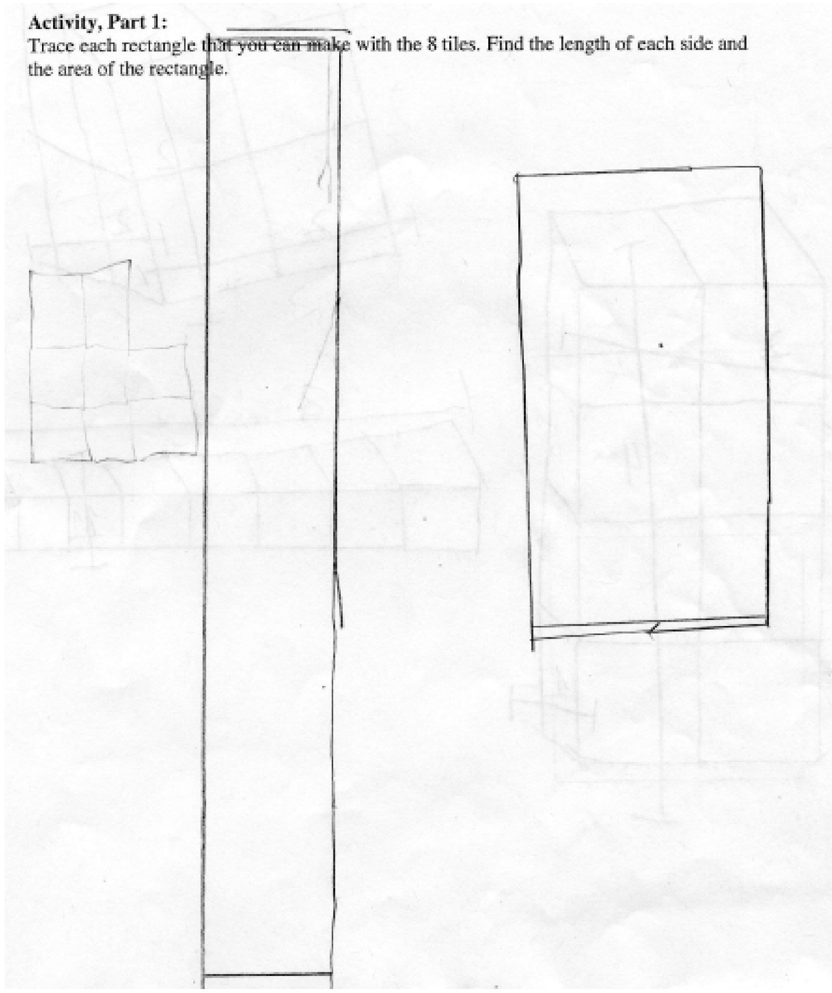


Figure 16. Student #14's Work from Session 2, Part 1

Note that student #11 marked off the square units, but they are not uniform. Students #5 and #11 correctly labeled the side lengths, but failed to include the units of measurement. Student #14 drew correctly sized rectangles, but did not include lengths or units of measurement. Student #14 may have overlooked the details of the directions. Only student #5 correctly included the units of measurement for the perimeter and area of each rectangle, but she wrote "8 inches" for the perimeter of the 4" x 2" rectangle (even though perimeter was not requested). This may have been a careless addition error, because student #5 did find the correct perimeter for the 8" x 1" rectangle.

In session 2, part 2, the students manipulated 8 cubes to determine exactly how many different rectangular prisms they could form with them. The students recorded how long, how tall, and how deep each prism was. The students were asked to choose one of the prisms and then trace each of its faces, find the perimeter and area of each face, and then find the total sum of all the areas. Student #5's, student #11's, and student #14's recorded work is shown in Figure 17 through Figure 19.

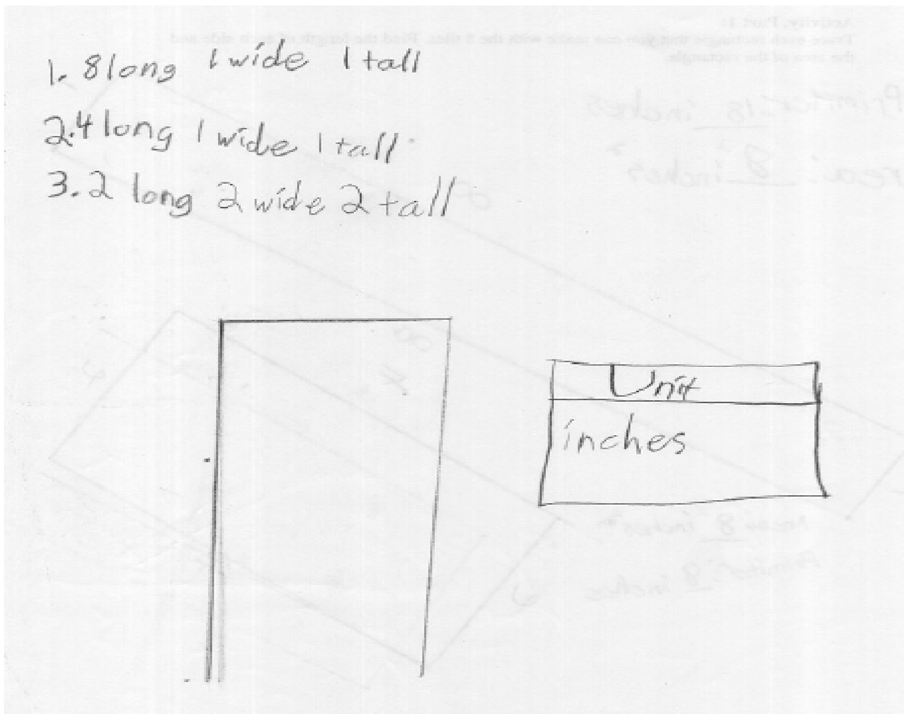


Figure 17. Student #5's Work from Session 2, Part 2

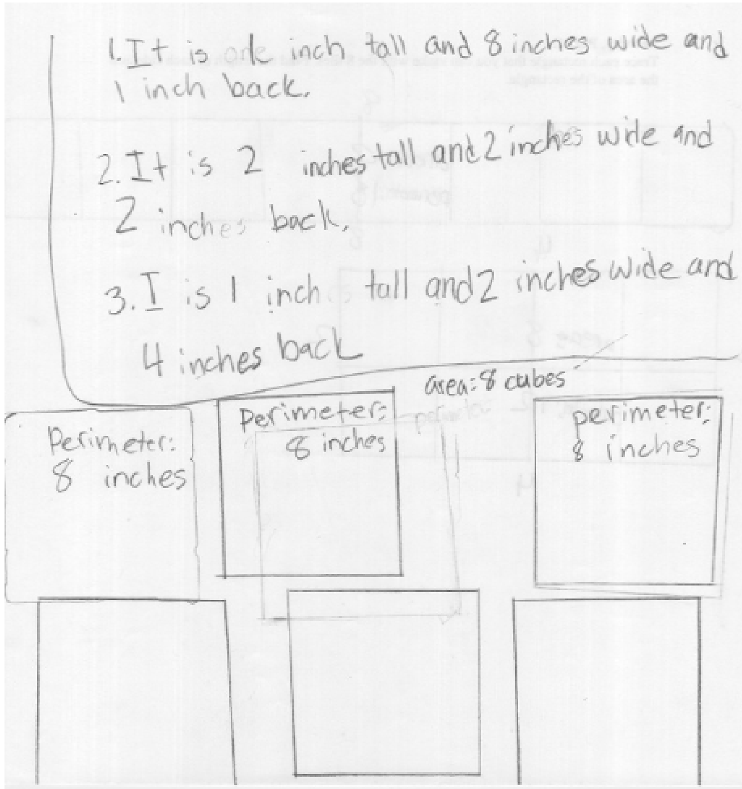


Figure 18. Student #11's Work from Session 2, Part 2

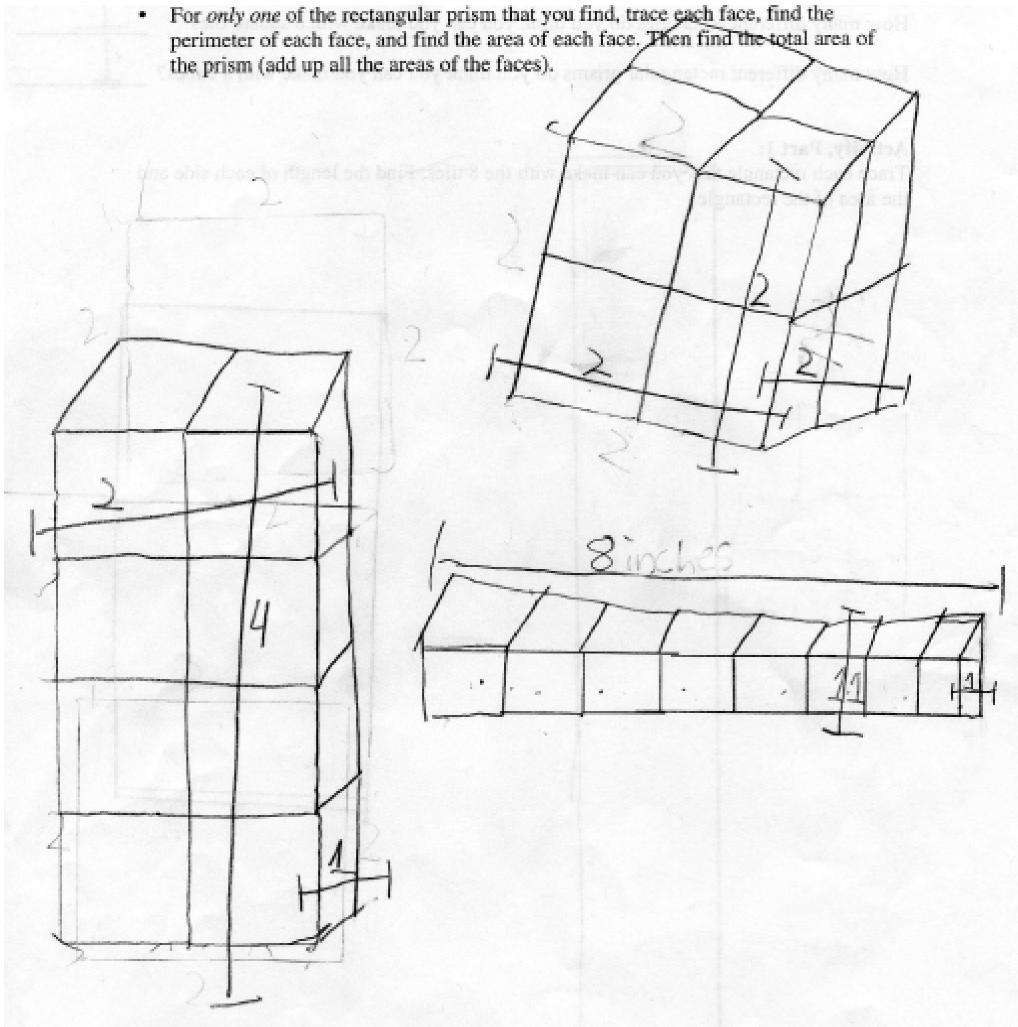


Figure 19. Student #14's Work from Session 2, Part 2

Student #5's work was inaccurate and incomplete; she identified a "4 long 1 wide 1 tall" as one of the three rectangular prisms with a volume of 8 cubes. On the right-hand side of the paper, she included a unit reference and indicated inches on it. This is a format that is used in *Everyday Mathematics* to reinforce the importance that units play in giving meaning to numbers in context. This is in agreement with Schwartz's (1996) idea of *mass noun.*

Student #11 correctly identified the three rectangular prisms with a volume of 8 cubes. She chose the 2" x 2" x 2" prism to complete the activity; she showed all six 2" x

2” faces and correctly marked the perimeter of 8 inches for these congruent faces. She incorrectly stated that “area = 8 cubes,” but it is eight cubes that make up the volume. This could indicate that student #11 did not pay attention to details; or she had used the face of the cube 8 times to verify that the area was 8 of these; or she may have been confused by all the vocabulary used in the directions. Since student #11 wrote only a question mark as her response to the first session’s reflection prompt[7], it is possible there is a disconnect in mathematical communication. This relates to van Hiele’s (1984) warning that if the levels of understanding differ between the teaching and the student, learning can be thwarted.

Student #14 drew three 3-D drawings of the three prisms and correctly gave the numerical values of their dimensions, but he failed to include the units of measurement. It is quite possible that the student identified the number of cubes and therefore saw no need to have written “inches” along with the value. Merely having the count of the cubes on each of the dimensions, without requiring the unit of measurement, may contribute to the dilemma of students not being sure what part of the cube is being considered when asked to measure using it. In this instance, the prism on the left is 4 cubes high; it is the sum of the one-inch edges of each of the four cubes that results in the prism being 4 inches high. This supports Outhred’s and McPhail’s (2000) and Bragg’s and Outhred’s (2001) findings that elementary students may not know whether to focus on an edge, face, or the cube itself when using it to measure.

Student #14 also drew the 2” x 4” x 1” and the 2” x 2” x 2” prisms with accurate measurements of their front faces, but he did not do the same with the 8” x 1” x1” prism. It appears that the length marked “8,” which is actually 4” long, has been divided into 8

[7] The reflection prompt was: When an adult talks about the area of something, say a playground, what does that mean to you?

intervals, but these intervals are not uniform. This work indicates a concern for the external structure of the prism and is reminiscent of student work from Outhred and Mitchelmore (1992). Student #14 used the edges of the front face and then divided it into eight unequal intervals. This relates to Piaget's findings that children place a great deal of attention on the boundaries of objects and attention to the details comes later (Piaget, Inhelder, & Szeminska, 1960).

The reflection prompt at the end of session 2 was "What connections do you see between area and perimeter of 2-D shapes and volume and surface area of 3-D shapes?" At this midway mark in the exploratory study, it is interesting to read some of the responses from these three students. Student #5 wrote, " Area and surface area …both have the word area in them so that means that they both mesure [*sic*] the area of surface. Primeter [*sic*] and volume don't really have much in common." This revealed that she merely recognized the word area in the names of the two attributes. Although she acknowledged that perimeter and volume are quite different, she did not give reasons why. Student #11 wrote, "I think area is the same as volume in 3-D shapes because it's how much is in the 3-D shape." This student created an equality where an analogy, such as, area in 2-D is like volume in 3-D, would have been more precise. Student #14 wrote, " I think a connection between 2-D shapes and 3-D shapes is 3-D shapes are made of layers of 2-D shapes." This last response indicates the student's concept of layering to form a rectangular prism; this layering procedure was considered by Battista and Clements (1996) as the most effective strategy for enumerating cubes in a 3-D array. Each of these students is building a network of understanding and forming a theory of measurement that makes sense to them.

The prediction tasks in session 3, which involved surface area and volume, were dependent on the particular rectangular prism the student received; therefore, the students' prediction tasks cannot really be compared. Session 3's activity required the students to trace each face of their prism; to find the perimeter and area of each face; and to find the surface area and volume of their prism. The students then made a covering[8] for their prisms using the pre-cut rectangular cardstock pieces. Student #5's, student #11's, and student #14's recorded work is shown in Figures 20 through Figure 22 and a tabulation of the work from session 3 completed by these three students is shown in Table 9.

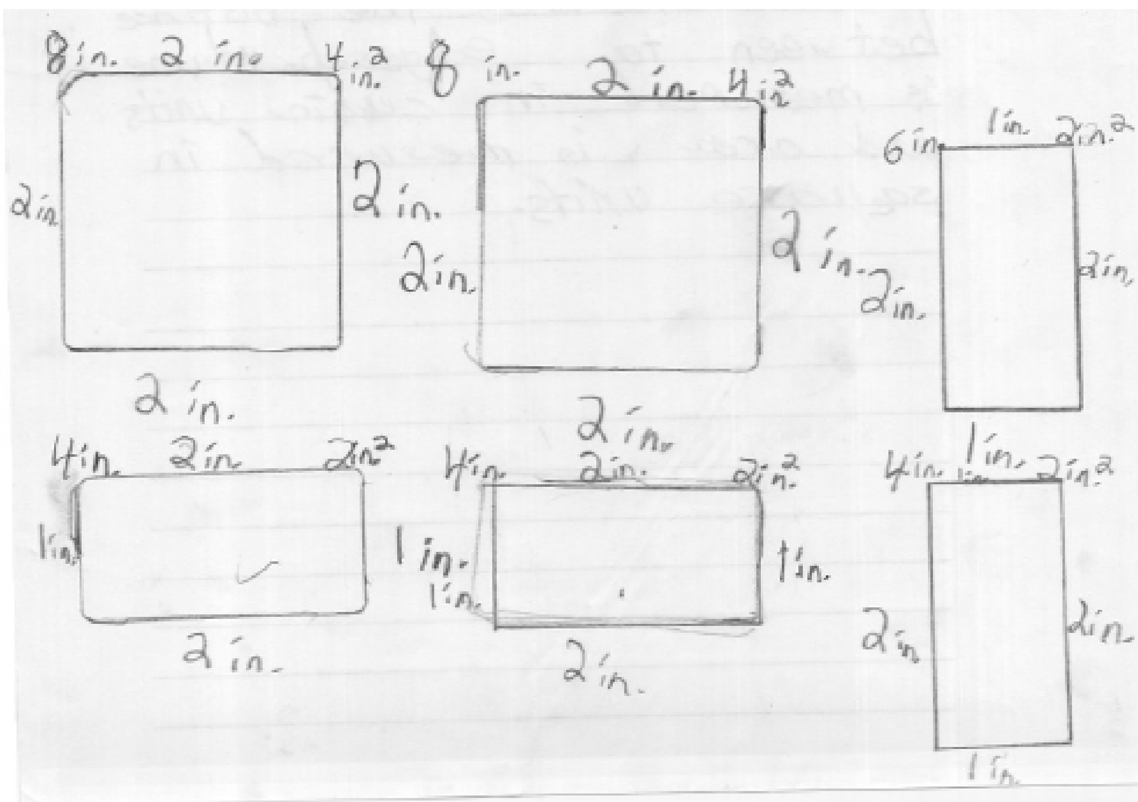


Figure 20. Student #5's Work from Session 3

[8] The students later in the study referred to these as "jackets."

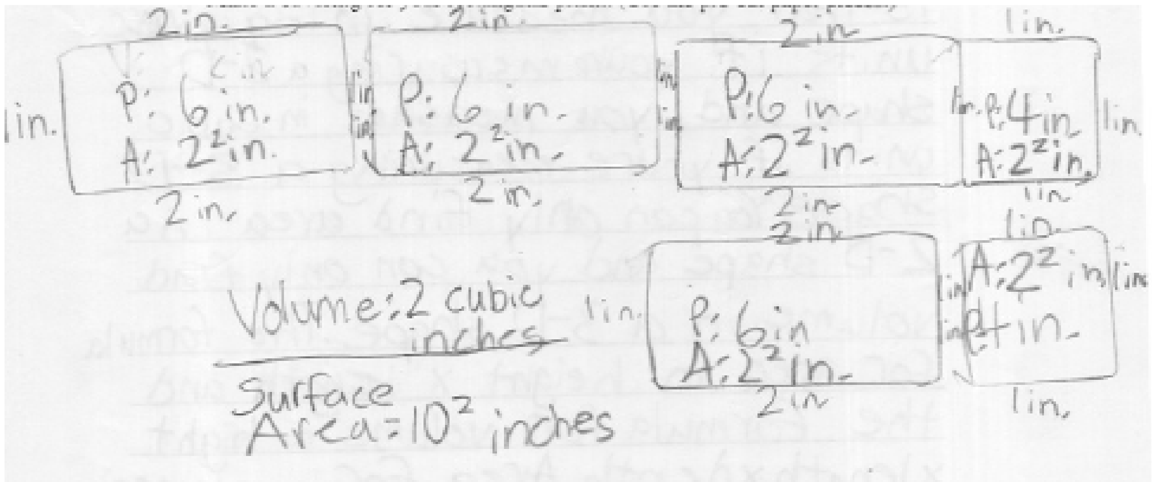


Figure 21. Student #11's Work from Session 3

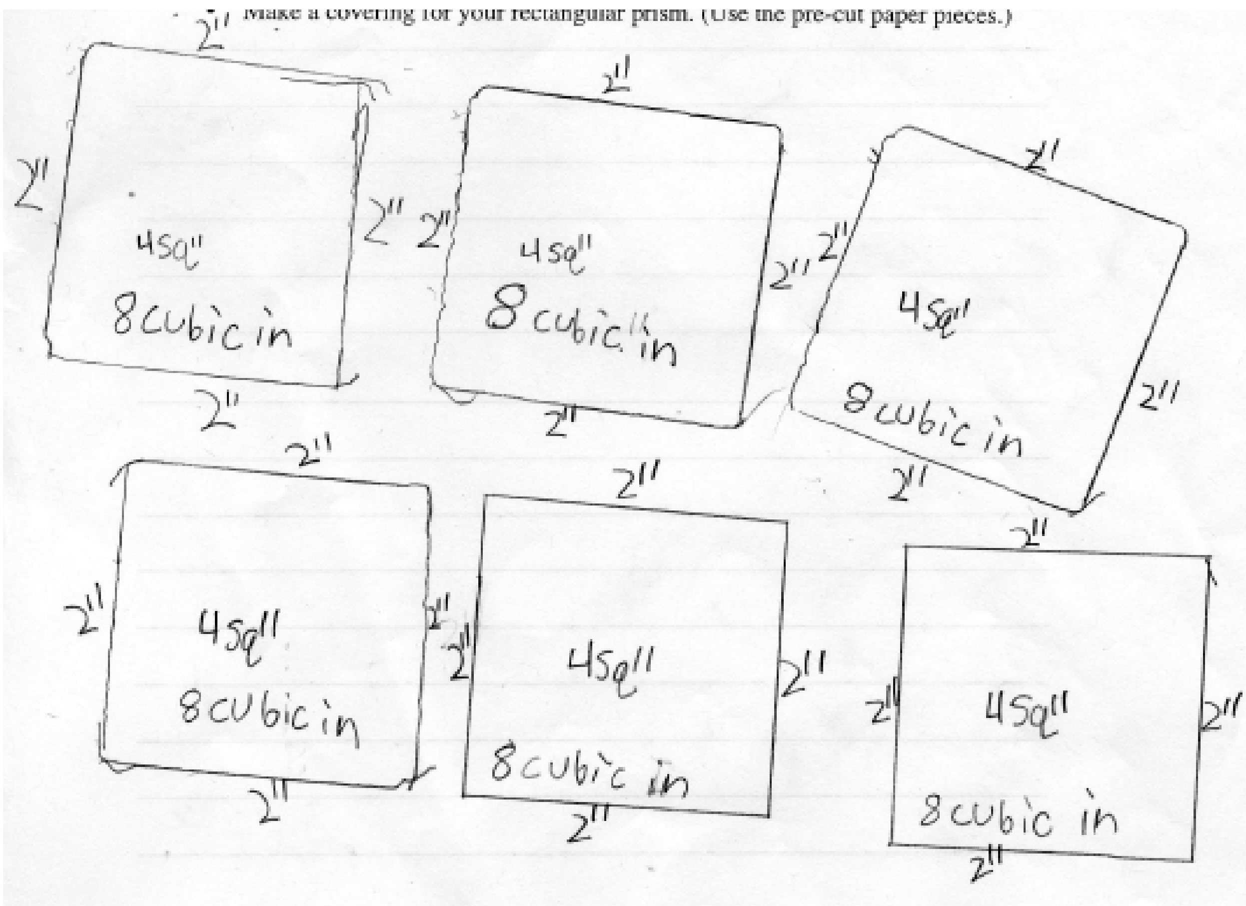


Figure 22. Student #14's Work from Session 3

Session 3 Activities									
Student #	Dimensions of Their Rectangular Prism	Perimeter of Each Face		Area of Each Face		Surface Area of Rectangular Prism		Volume of Rectangular Prism	
		Numerical Value	Unit	Numerical Value	Unit	Numerical Value	Unit	Numerical Value	Unit
5	2” x 2” x 1”	error	√	√	√	√	√	√	√
11	2” x 1” x 1”	√	√	error	√	√	√	√	√
14	2” x 2” x 2”	√	error	√	√	missing	missing	√	√

Table 9. Tabulation of Session 3 Work by Students #5, #11, and #14

Student #5 had a single error. She wrote “4 in.” in the upper left-hand corner of three of the four 2” x 1” faces drawn. This may have been careless, because she had the correct perimeter of “6 in.” on the fourth 2” x 1” face. Although these values were not labeled as perimeters, after considering all parts of the student’s work, it seems to be the most logical conclusion. There is the possibility, though, that the student understood the concepts (the correct units of measurement are noted), but confused the words of perimeter and area, and therefore omitted their labels. The remainder of her work contains correct numerical values and units of measurement for the areas of each face, but the label, area, was similarly absent as was the label, perimeter.

Student #11 gave an incorrect area of “2^2in.,” rather than the correct 1in^2, for each of the two 1” x 1” faces. While 2^2in. actually equals 4 inches, it appears that the student’s intention was actually 2in^2, indicating that the student was aware that the appropriate unit for area was a square unit. Student #11 chose to use the exponent 2 (to represent squaring as opposed to using the abbreviation sq.) and made a literal translation from English to mathematical symbols for square inches. She therefore failed to use the customary abbreviation of in^2 for square inches, yet her error is interesting and quite logical.

Student #14 correctly shows each of the six congruent faces of his 2-inch cube with each side correctly labeled "2″." The student wrote "4sq ″" in the interior of each face; the researcher assumes this amount represents the area, even though the student did not label it as such. Also, in the interior of each face the student wrote, "8 cubic in." It is not clear whether this is an error in the unit of measurement for perimeter of the face, or if "8 cubic in." designates that each face belongs to the prism that has a volume of 8 cubic inches. This seems to support Outhred's and McPhail's (2000) study that uncovered the dilemma students have when distracted by details, when it is not clear to them to focus on the block's edges, faces, or the block itself. It appears that student #14 has made progress in distinguishing the abstract ideas of area and volume. In responding to session 3's reflection prompt, "How are area and volume alike and how are they different?", student #14 wrote:

> Volume and area are different because volume has depth and area 2-D [*sic*]. Area and volume are alike because volume must have surface area. Volume and area are different because volume measures the compacity [*sic*] and area measures something flat.

Student #14 included capacity in his explanation of differences between area and volume. This concept was brought up in whole-group discussion during the second session and the notes from the discussions were accessible to all students for reference. While Student #14's writing was awkward, he included sophisticated ideas.

Student #5 wrote a lengthy explanation for this same prompt:

> Area and volume don't really have many things in comen [*sic*]. One thing that is the same is that they are both a kind of measurement. A few things that are different are a. volume is the area that fills in something and area measures the distance between the edges b.volume is measured in cubic units and area is measured in square units

Student #5 had no difficulty in identifying that volume and area are measurements with different units. Her statement that "volume is area" was not mathematically correct, but when used as an analogy it makes sense. Volume fills in a 3-D shape, in a similar fashion that area fills in a 2-D shape. Saying that "area measures the distance between the edges" is also incorrect, but again an underlying analogy appears to be the intension of the statement. It is interesting that student #5 made dimensional connections between the attributes of different dimensions—volume was associated with area and area was associated with distance. It is quite possible that student #5 was incorporating ideas from the whole-group discussion from the third session. Those ideas included: a 1-D shape has distance and is bounded by points that secure a location and have no dimension; a 2-D shape has area and is bounded by 1-D shapes; 3-D shapes have volume and are bounded by 2-D shapes. It appears that student #5 may understand the concepts better than her writing expresses. This would support Piaget's (1975) contention that student often understand more than what they are able to express.

Student #11, who wrote a lengthy, but somewhat redundant response, ended it with a powerful analogy:

> They both fill in shapes whether it's 2-D or 3-D. To find area in a square you have to count how many square units and in a cube to find the volume you have to count the cubic inches. So the only difference between measuring a 2-D shape and a 3-D shape is that you measure in square units if you're measuring a 2-D shape and you measure in cubic units if you're measuring a 3-d [*sic*] shape. You can only find area in a 2-D shape and you can only find volume in a 3-D shape. The formula for area in [*sic*] height x length and the formula for volume is height x length x depth. Area for a square is like volume for a cube

Student #11's thinking is easy to follow. She shows understanding of attributes, units and strategies to find area for squares and volume for cubes. It is disturbing, though, that she has generalized the area formula for a rectangle to be *the* area formula and the volume formula for a rectangular prism to be *the* volume formula. Student #15 also made the same generalization about formulas. Student #19 used the volume formula accurately as part of an example of finding the volume of a box. Formulas were not a focus of this study, but because only rectangular prisms were used, students may have been given too narrow an experience. The study did not, as suggested by Pirie and Kieren (1992), optimize "the opportunities for the construction of mathematical understanding" (p. 526). This concern will be noted further in the conclusion of this paper.

Posters

The established social and socio-mathematical norms of this class allowed the students to be focused and productive in the planning and constructing of the posters[9]. Since the students worked in partnerships, though, each poster can only be assessed as a joint effort. Even though the students were enthusiastic and the assignment had specific requirements, it is difficult, if not impossible, to determine the impact this activity had on furthering students' understanding of the concepts of perimeter, area, surface area, and volume. It is possible, though, to cite some commonalities that the students displayed on their posters and thus considered important in understanding of perimeter, area, surface area, and volume. All posters had attached models of rectangular prisms made from the pre-cut rectangular shapes, and these prisms varied in their linear dimensions.

The rhyme, "*Perimeter goes around the rim, area fills it in*," appears in its entirety or partially on six of the nine posters. Student #5 (with partner student #13) and student #14 (with partner student #8) both used the rhyme, but student #11(with partner student #19) did not use the rhyme. It is interesting to note that student #11 (and #19) had real-world examples of perimeter, area, surface area, and volume, but students #5 (and #13) and #14 (and #8) gave strictly mathematical representations. The poster created by student #11 (and #19) was exceptional in that the examples that they gave for the attributes were not ones that had been shared in whole-group discussions. For instance, for the volume representation the students attached a 2" x 2" x 2" prism made from the pre-cut rectangular shapes with one face unattached on one edge. They referred to the prism as "a toy box" and in the prism they included the "toys," which were 8 one-inch cubes made

[9] Photos of all posters can be found in Appendix F.

from the pre-cut squares. For perimeter they wrote about a running race and related the track to a perimeter. They demonstrated surface area with 1-inch squares, which they referred to as tiles for covering a backyard. Student #19 scored the highest on both the pretest and the posttest. The researcher contends that this student, who was the more capable mathematics student, was able to articulate well the concepts of perimeter, area, surface area, and volume and impacted student #11's understanding of these attributes. Their interaction on the poster construction had a positive outcome. Their finished poster was exemplary and student #11 made a substantial improvement in her posttest. These findings support both Piaget's recommendations to teachers for group work and discussion and Vygotsky's theory of the zone of proximal development (Cole & Wertsch, 1996; Piaget, 1975).

Issues of Concern

While the direction of the study's data is consistent and positive indicating a successful intervention with gains attained by the 4th grade students, there are a few issues that merit concern. Firstly, the size of the sample and the pretest/posttest assessments pose some concern. With both the small sample size and the relatively limited number of pretest/posttest questions asked, particularly on area (2 questions) and volume (3 questions), the statistical results should be considered with some reservation.

Secondly, the qualitative findings support that the interaction of the instructional processes used in this study did enhance its effect. This anecdotal evidence gives credence to the fruitfulness of the processes used: discussing, reflecting, predicting, and constructing; working independently and in partnerships; and using particular tools and manipulatives. But, there is a lack of statistical evidence that would indicate which one of these processes (or combination thereof) and in what particular ways these processes improved students' understanding of perimeter, area, surface area, and volume.

Conclusions

This study explored the effectiveness of the concurrent investigation of perimeter, area, surface area, and volume. Curry et al. (2006) used interviews and parallel tasks to investigate students' development of basic principles of spatial measurement. Using different methods, the present study incorporated findings from the Curry et al. (2006) study that addressed the teaching of these attributes. This incorporation was in keeping with Hiebert's and Carpenter's (1992) strong appeal for research "designed to provide fine-grained analysis of how the elements of the teaching-learning process interact" (p. 92).

While Curry and Outhred (2005) suggested that volume be delayed until mastery of area, this study included both volume and area in order to highlight the differences and similarities concurrently between the two unit structures involved. Also, surface area was not addressed in the Curry et al. (2006) study, but was included in this study in order to provide additional tasks that forced students to consider the relationships among the spatial attributes. Curry and Outhred (2005) in their preliminary findings of the Curry et al. (2006) study reported that more experience with packing of rectangular boxes with unit cubes of a variety of sizes would be beneficial for students' understanding of the 3-D array structure. Besides packing boxes, this study had students manipulating the connected cubes (forming 3-D array structures); both tasks provided students with valuable and different experiences with volume. When students covered the 3-D arrays with pre-cut rectangular shapes, they explored the relationship of volume and surface area.

This study's findings support the implications of the Curry et al. (2006) study as related to teaching and learning of spatial attributes. Tasks and activities where errors could occur when students do not fully understand spatial measurement, and which could help focus discussions on *reasons* behind the basic principles of spatial measurement, are vital to the teaching and learning of spatial attributes.

The four specific questions, which this study attempted to answer, will now be addressed.

Question 1: How do tasks that concurrently incorporate the concepts of perimeter, area, surface area, and volume, enhance a student's capacity to distinguish among the attributes and to see relationships among the attributes of different dimensions?

Having students work with 3-D objects to find perimeter, area, surface area, and volume, encouraged students to attend to details, which fostered the occurrence of more cognitive conflict for some students than for others. It did not cause any apparent frustration. Students were engaged both physically and mentally with what they were doing when handling the materials and making rectangular prisms. The concrete experience allowed students to relate surface area with volume. Besides making a prism, the students constructed with pre-cut square units multiple unit cubes and used these cubes to fill the prism. They could make the layers and construct 3-D arrays. Students could count the square units used to determine the surface area and the cubic units used to determine the volume. As each 2-D rectangular shape was manipulated to make a prism or to cover an existing prism, perimeters and areas of the rectangular shapes were considered. Coordination and integration of the rectangular shapes were needed to complete the activities. The activities in each session were designed to help the students focus on each attribute but also to see similarities and differences and analogies between the geometric features of 2-D and 3-D shapes. Also, the activities allowed all the students access to the experience at their level of understanding. Students could build their mathematical thinking on what made sense to them.

While the sample size of this study was small and there was no comparison group, the evidence from the study shows that fourth grade students are capable of concurrently investigating perimeter, area, surface area, and volume of rectangular objects. The results of this study suggest that in designing instruction, particular attention should be paid to manipulative tasks that incorporate both 2-D and 3-D concepts. The study indicates that the materials used did enhance the students' understanding, which will be addressed in the response to the next question.

Question 2: How does the use of certain tools (measuring strings, grid mats, rectangular prisms, pre-cut rectangular shapes, one-inch plastic tiles, and Klick-like puzzles) affect an elementary school student's ability to distinguish among perimeter, area, surface area and volume and promote a student's understanding of the corresponding units of measure?

During the fourth session of the study, the students as a whole group discussed what would be important to include in their posters, which they would be making the following day. These posters were to convey what the students understood about the similarities and differences of the attributes of perimeter, area, surface area, and volume; examples of each attribute were to be given that would show how the attributes are related and how they are different. Students were free to incorporate any of the materials used during the study. The students overwhelmingly chose to include the “blue covers” (the pre-cut rectangular shapes). These shapes lend themselves to easy formation of rectangular 2-D and 3-D shapes. It may be contested that students were developing a narrow interpretation of area, surface area, and volume restricted only to 2-D and 3-D rectangular shapes, and not a broader view to include non-rectangular shapes. One could argue that the pre-cut rectangular shapes provided students with a versatile familiar shape that students could use to understand perimeter, area, surface area, and volume in a variety of ways. The use of the pre-cut rectangular shapes allowed the students to concentrate with greater focus on their explanation of the abstract concepts of the attributes. Students, during session 5 also identified the pre-cut rectangular shapes, which they called “jackets,” as being helpful in understanding surface area. Even the use of the word “jacket” reflects an intuitive understanding of the surface area concept. The use of this concrete material seemed to accentuate what it intended to illustrate, and, in fact, allowed students to consider the relationships that perimeter, area, surface area, and volume have among each other.

During the fifth session of the study, the students in a whole-group discussion overwhelmingly said that the connected cubes (Klick-like puzzles) helped them to understand volume and how the same number of cubes could form different prisms that had different surface areas. Using the connected cubes provided a different challenge than that of enumerating cubes in an existing 3-D array, which is a more complex task (Battista, 1999). The use of the connected cubes facilitated the manipulation of a constant number of cubes to form different 3-D shapes, which may or may not have the same surface area. A student had the opportunity for simple abstraction by handling the cubes and for reflective abstraction by taking deliberate actions such as twisting and turning the cubes to form different shapes (Piaget, 1975). Piaget asserted that these two avenues of abstraction together contribute to understanding abstract concepts (Gruber & Voneche, 1995; Piaget, 1975). Having connected cubes allowed the students to create arrays and to examine the arrays by turning them upside down and over without the threat of having the shape fall apart. Students could observe the orthogonal views of the shape; the connected cubes provided valuable experiences to help students learn to visualize, coordinate, and integrate such views in order to enumerate cubes in other 3-D arrays whether with concrete objects or pictorial representations. The researcher maintains that these experiences did help students to develop a firmer understanding of the spatial structuring of layering arrays of cubes, which Battista and Clements (1996) showed is not intuitive but must be learned.

Few students used the measuring strings. Some students seemed interested only in the novelty of them. During the first session there was a need to measure the perimeter of a non-polygonal 2-D shape where the solitary use of a ruler would not serve well. Most students chose using a string and then measuring the string using a ruler. While the measuring strings appropriately highlight the units of measure as linear intervals, they did not prove of any particular value in this study.

The grid mats were used by a number of students at the start of the study, but were used less as the pre-cut rectangular shapes were used more. Some students continued to use them almost as an organizing placemat rather than a measuring device, which was actually the intent in the study.

The tiles were familiar manipulatives for the students, who had used them in earlier grades for a variety of reasons. Students did not use them to any great extent other than to complete the tasks of finding the number of rectangles possible with 36 tiles in session 2. When students were finding the area of a prism face or the surface area of the prism, they were more apt to use the cubes. The students would use a number of one-inch cubes or pre-cut rectangular shapes to cover the face(s) or a single cube to *stamp out* the area on each face.

The evidence from this study strongly suggests that with the use of carefully planned manipulative models, students can have success in learning about perimeter, area, surface area, and volume concurrently. It also suggests that further research in classrooms on the use of the connected cubes and "jackets" would be useful. Additional work could be done in identifying the pitfalls and advantages associated with the use of these tools and other 3-D tools.

Question 3: In what ways are the instructional processes effective and how can their effectiveness or lack thereof be documented?

The social and socio-mathematical norms had already been established by this class of 4th graders and facilitated by a very competent teacher. Students took responsibility for their own learning in that they were expected to ask questions for clarification, to participate actively, and to be respectful and supportive of all other members in the class. Carrying out the study toward the end of the school year in such a classroom, the researcher reaped the benefits of working with a cooperative group of students, who were accustomed to high expectations for a concerted effort in learning. While some students participated more comfortably and frequently in whole-group discussions, all were active participants when working independently or with a partner with the manipulative materials.

The students commented during session 5 that the recording sheets were a help to them. The students found that having simple, direct instructions and sufficient space to record their drawings, explanations, and reflections allowed them to concentrate on the hands-on activities. This may have been the case, but there were in fact a fair number of blank responses. The absence of responses (and certainly the unclear ones) supports Piaget's (1975) contention that students are often more capable of doing and understanding in actions than in expressing themselves verbally.

The prediction tasks proved worthwhile, but risky for some students. The students were invested in thinking about what would be an appropriate answer; and requesting an interval rather than a single amount, the idea of a reasonable estimate was reinforced. By the number of erasures and crossing-outs on the recording sheets, though, it appears that it was difficult for some students to commit to a prediction even though it was stressed that predicting was a learning experience, not a test, and that they would be checking out their predictions with hands-on activities. One implication is that frequent experiences of making predictions would be beneficial and transferable to many problem-solving situations.

The poster construction was in fact a culminating project that incorporated most of the materials used in the study, and more importantly required the students to present their understanding of perimeter, area, surface area, and volume. The well-planned student partnerships of this study proved essential to productivity. But, it was not evident by observing the posters to know which students were responsible for which components, or if all partners understood all that was included in their poster. Maybe it is not necessary to have such information; the purpose of the poster construction was to provide yet another avenue for students to think, share ideas, and then come to agreement on displaying their ideas about perimeter, area, surface area, and volume using pictures, models, and explanations. The true effectiveness of this working partnership, besides resulting in a physical product of a poster, hopefully strengthened each student's network of understanding of perimeter, area, surface area, and volume. The researcher thinks that the posters were in fact effective, even though there is no way to determine to what extent these constructions contributed to the overall improvement of the class as shown by the pretest/posttest data.

Question 4: In retrospect, what would have made the study stronger?

There are two major considerations that would have made the study stronger and would have provided more insight into students' thinking about perimeter, area, surface area, and volume. The first involves additional observation time and interviews with the students and the second involves the materials used in the study.

The first consideration could have been accomplished at three different junctures in the study. The first juncture could have occurred if the students actually did present their posters to first graders. This would have raised the poster construction activity up another notch. It would have given students another opportunity for what van Hiele (1999) endorsed: using related language and integrating what they have learned into what they already know. This could have shown what actions the 4th graders took and what vocabulary and explanations the 4th graders used to convey the meaning of the geometric and measurement concepts. Listening to the 1st graders' questions and the 4th graders' responses would have provided additional evidence of the students' grasp of understanding of perimeter, area, surface area, and volume.

The second opportunity to gather direct data from students would have been follow-up interviews about a week after the posttest. The researcher would have had sufficient time to review the data so that students, who left answers unfinished or blank, or who gave ambiguous or incorrect responses, or who gave insightful or sophisticated answers, could be interviewed and more information could have been obtained about their thinking. For example, it would have been enlightening to learn from Student #14 exactly what "8 cubic in." referred to in his neatly laid out work (see p. 48).

The third opportunity to gather direct data would have been interviews conducted about four to five months after the posttest. In this way, retention or lack thereof could be determined. Any new approaches or insights that the student had acquired could be identified. It would be interesting to see if the rhyme, *Perimeter goes around the rim, area fills it in,* was still remembered and if it was used to help distinguish or mask the ability to distinguish between area and perimeter.

The second consideration that would have made the study stronger would be changes in the materials used in the study. While the Klick-like puzzles were successful, there are now manipulatives, such as magnetized cubes (Magna-cubes) that allow greater flexibility in constructing 3-D rectangular shapes. The connected cubes of the Klick-like puzzles were restrictive in that each puzzle had a specific number of cubes, and the study only included a few different Klick-like puzzles (e.g., 8 cubes, 12 cubes, 64 cubes). The loose cubes of the Magna-cubes would provide opportunities for students to manipulate the cubes very quickly into a variety of configurations without limitation to the number of cubes that could be joined magnetically.

Another change in materials that would have made the study stronger would be the inclusion of non-rectangular shapes. In session 1, a few non-rectangular 2-D shapes were investigated, but all the 3-D shapes used in the study consisted of rectangular prisms. While using rectangular prisms seemed to help lay the foundation for understanding volume, two students (#11 and #15 in session 3's written reflection, pp. 53-54) generalized the rectangular area formula to be the area formula for all 2-D shapes and similarly considered the rectangular prism formula to be the formula for all 3-D shape. Without examples of non-rectangular prisms, students were deprived of the opportunity to gain what Lehrer et al. (2003) referred to as "flexible adaptability to novel conditions of application" (p.100). It is in keeping with Piaget's recommendation that for students struggling to understand a concept, their exploration of counterexamples may lead them to self-correction (1975).

Hence, even though these two major considerations would have improved the study, the exploration as conducted did increase students' performance and understanding of perimeter, area, surface area, and volume of rectangular shapes. Findings did suggest that the interaction of planned student partnerships, hands-on activities with manipulative 2-D and 3-D materials, predictions, reflections, and discussions enhanced students' thinking and understanding of the attributes, their units of measurement, and the relationships among them.

References

Battista, M. (1999). Fifth graders' enumeration of cubes in 3D arrays: conceptual progress in an inquiry classroom. *Journal for Research in Mathematics Education, 30*, 417-448.

Battista, M., & Clements, D. H. (1996). Students' understanding of three-dimensional rectangular arrays of cubes. *Journal for Research in Mathematics Education, 27*(3), 258-292.

Battista, M., Clements, D. H., Arnoff, J., Battista, K., & Van Auken Borrow, C. (1998). Students' spatial structuring of 2D arrays of squares. *Journal for Research in Mathematics Education, 29*, 503-532.

Berry, J., & Houston, K. (1995). Students using posters as a means of communication and assessment. *Educational Studies in Mathematics, 29*(1), 21-27.

Bragg, P., & Outhred, L. (2001). So that's what a centimetre looks like: Students' understandings of linear units. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th International Conference on the Psychology of Mathematics Education* (Vol. 2, pp. 209-216). Utrecht, Netherlands: Program Committee.

Burger, W., & Shaughnessy, M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics education, 17*, 31-48.

Clements, D. H., & Stephan, M. (2003). Measurement in Pre-K to grade 2 mathematics. In D. H. Clements, J. Sarama & A.-M. DiBiase (Eds.), *Engaging young children in* *mathematics : standards for pre-school and kindergarten mathematics education* (pp. 299-317). Mahwah, N.J.: Lawrence Erlbaum Associates.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education, 28*(3), 258-277.

Cole, M., & Wertsch, J. V. (1996). Beyond the individual-social antimony in discussions of Piaget and Vygotsky. *Human Development, 39*(5), 250-256.

Curry, M., Mitchelmore, M., & Outhred, L. (2006). Development of children's understanding of length, area, and volume measurement principles. In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 377-384). Prague.

Curry, M., & Outhred, L. (Eds.). (2005). *Conceptual understanding of spatial measurement* (Vol. 1). Sydney: MERGA.

Dewey, J. (1933). *How we think, a restatement of the relation of reflective thinking to the educative process*. Boston, New York [etc.]: D.C. Heath and company.

Elbers, E. (2003). Classroom interaction as reflection: Learning and teaching mathematics in a community of learners. *Educational Studies in Mathematics, 54*(1), 177-199.

Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht,: Reidel.

Freudenthal, H. (1981). Major problems of mathematics education. *Educational Studies in Mathematics, 12*(2), 133-150.

Fuys, D. J., Geddes, D., & Tischler, R. (1988). *The van Hiele model of thinking in geometry among adolescents*. Reston, VA: NCTM.

Great Britain. Committee of Inquiry into the Teaching of Mathematics in Schools., & Cockcroft, W. H. (1982). *Mathematics counts : report of the Committee of Inquiry into the Teaching of Mathematics in Schools under the chairmanship of W.H. Cockcroft*. London: H.M.S.O.

Gruber, H. E., & Voneche, J. J. (Eds.). (1995). *The essential Piaget*. Northvale, NJ: Jason Aronson.

Gutiérrez, A., Jaime, A., & Fortuny, J. M. (1991). An alternative paradigm to evaluate the acquisition of the van Hiele levels. *Journal for Research in Mathematics Education, 22*(3), 237-251.

Hart, K. (1981). Measurement. In D. E. Kuchemann & M. McCartney (Eds.), *Children's understanding of mathematics: 11 - 16* (pp. 9-22). London: John Murray.

Hart, K. (1989). Volume of a cuboid. In K. Hart, D. Johnson, M. Brown, L. Dickson & R. Carkson (Eds.), *Children's mathematical frameworks 8-13: A study of classroom teaching* (pp. 126-150). London: NFER-Nelson.

Hiebert, J., Carpenter, E., Fuson, K., Human, P., Murray, H., Olivier, A., et al. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher, 25*(4), 12-21.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 771). New York: MacMillan Publishing Company.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical Knowing and Teaching. *American Educational Research Journal, 27*(1), 29-63.

Lehrer, R. (2003). Developing understanding of measurement. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 179-192). Reston, VA: NCTM.

Lehrer, R., Jaslow, L., & Curtis, C. L. (2003). Developing and understanding of measurement in the elementary grades. In D. H. Clements (Ed.), *Learning and teaching measurement* (pp. 100 - 121). Reston: NCTM.

Lehrer, R., Jenkins, M., & Osana, H. (1998). Longitudinal study of children's reasoning about space and geometry. In R. Lehrer & D. Chazan (Eds.), *Designing learning environments for developing understanding of geometry and space* (pp. 137-167). Mahwah, NJ: Lawrence Erlbaum Associates.

Lo, J.-J., & Wheatley, G. H. (1994). learning opportunities and negotiating social norms in mathematics class discussion. *Educational Studies in Mathematics, 27*(2), 145-164.

Mammana, C., & Villani, V. (Eds.). (1998). *Perspectives on the teaching of geometry for the 21st century*. Dordrecht [Netherlands]; Boston: Kluwer Academic Publishers.

Martin, W. G., & Strutchens, M. E. (2000). Geometry and measurement. In E. A. Silver & P. A. Kenney (Eds.), *Results from the seventh mathematics assessment of the national of educational progress* (pp. 193-234). Reston, VA: NCTM.

McNair, R. E. (2000). Working in the mathematics frame: Maximizing the potential to learn from students' mathematics classroom discussions. *Educational Studies in Mathematics, 42*(2), 197-209.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Outhred, L., & McPhail, D. (2000). A framework for teaching early measurement. *MERGA23-July 2000*, 487-494.

Outhred, L., & Mitchelmore, M. (1992). Representation of area : A pictorial perspective. In W. Geeslin & K. Graham (Eds.), *Proceedings of the 16th annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 194-201). Durham, NH: Program Committee.

Outhred, L., & Mitchelmore, M. (2000). Young children's intuitive understanding of rectangular area measurement. *Journal for Research in Mathematics Education, 31*(2), 144-167.

Outhred, L., & Mitchelmore, M. (2004). Students' structuring of rectangular arrays. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 3*, 465-472.

Outhred, L., Mitchelmore, M., McPhail, D., & Gould, P. (2003). Count me into measurement. In D. H. Clements (Ed.), *Learning and Teaching Measurement* (pp. 81 - 99). Reston, VA: NCTM.

Piaget, J. (1975). Comments on mathematical education. *Contemporary Education, 47*(1), 5-10.

Piaget, J., Inhelder, B., & Szeminska, A. (1960). *The child's conception of geometry* (E. A. Lunzer, Trans.). New York: Harper and Row, Publishers.

Pirie, S., & Kieren, T. (1992). Creating constructivist environments and constructing creative mathematics. *Educational Studies in Mathematics, 23*(5), 505-528.

Pirie, S., & Schwarzenberger, R. (1988). Mathematical discussion and mathematical understanding. *Educational Studies in Mathematics, 19*(4), 459-470.

Schwartz, J. (1996). *Semantic aspects of quantity*.Unpublished manuscript, Cambridge, MA.

Sfard, A., Neshler, P., Streefland, L., Cobb, P., & Mason, J. (1998). Learning mathematics through conversation: is it as good as they say? *for the learning of mathematics, 18*(1).

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry*. Chicago: University of Chicago.

van den Heuvel-Panhuizen, M. (2005, Nov. 14-15). *More space for geometry in primary school - A learning-teaching trajectory as a new impulse.* Paper presented at the Realistic Mathematics Education Conference, Madison, WI.

van Hiele, P. M. (1984). A child's thought and geometry. In D. Fuys, Geddes, D., & Tischler, R. (Ed.), *English translation of selected writings of Dina van Hiele-Geldof and P.M.van Hiele* (pp. 243-252). Brooklyn: Brooklyn College.

van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. *Teaching Children Mathematics, 5 no. 6*(February 1999).

Vergnaud, G. (1990). Epistemology and psychology of mathematics education. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and cognition: A research synthesis by the international group for the psychology of mathematics education* (pp. 14-30).

Wheatley, G. H., & Reynolds, A. (1996). The construction of abstract units in geometric and numeric settings. *Educational Studies in Mathematics, 30*, 67-83.

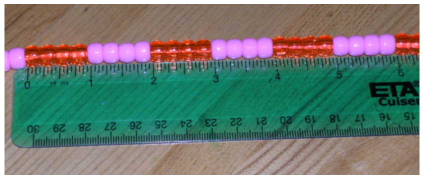
Wirszup, I. (1976). Breakthroughs in the psychology of learning and teaching geometry. In J. L. Martin & D. A. Bradbard (Eds.), *Space and geometry: Papers from a research workshop* (pp. 75-97). Athens, GA: University of Georgia, Georgia

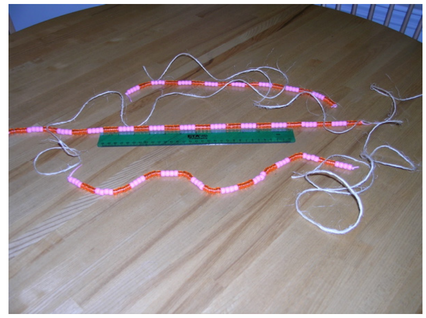
Center for the Study of Learning and Teaching Mathematics. (Eric Document Reproduction Service No.ED132 033.).

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

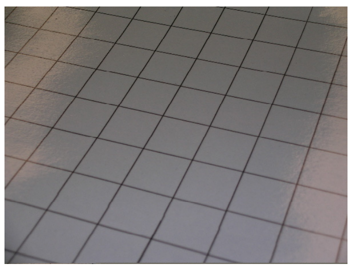
Yackel, E., Cobb, P., & Wood, T. (1991). Small-group interactions as a source of learning opportunities in second-grade mathematics. *Journal for Research in Mathematics Education, 22*(5), 390-408.

Appendix A - Materials

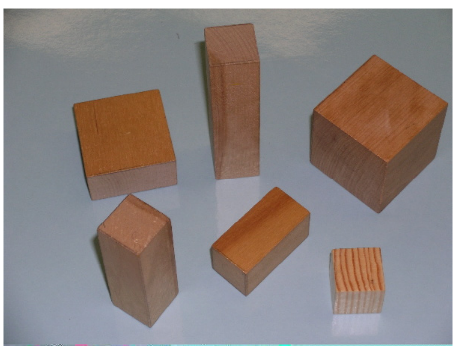




Measuring Strings
and Rulers

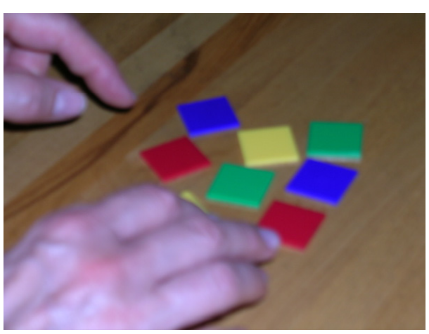


Grid Mat and Overlay



Rectangular Prisms

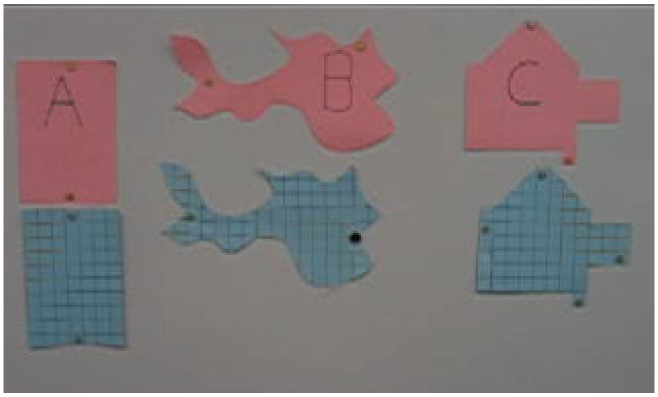
Pre-cut Rectangular Shapes
(In session 4, students referred to
these shapes as the "blue covers.")



One-inch Plastic Tiles

Connected Cubes
Adapted from commercial product Klick-puzzle

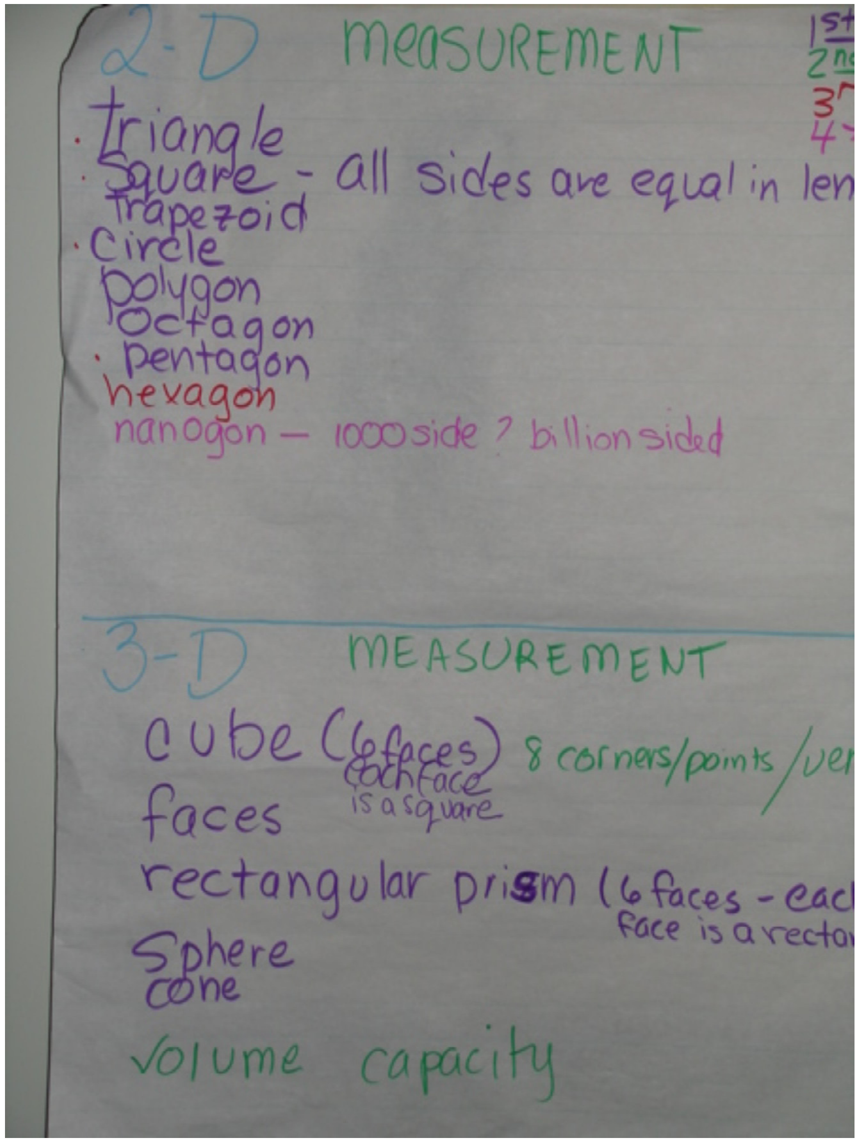
Session 1 Materials Used in Prediction Tasks2-D



The top three shapes were on the whiteboard. The students did not see the bottom row until after they made their predictions.

Each student was then given a set of the above four shapes. The students predicted which shape had the largest area and which one had the longest perimeter. After all made their predictions, the students verified them.

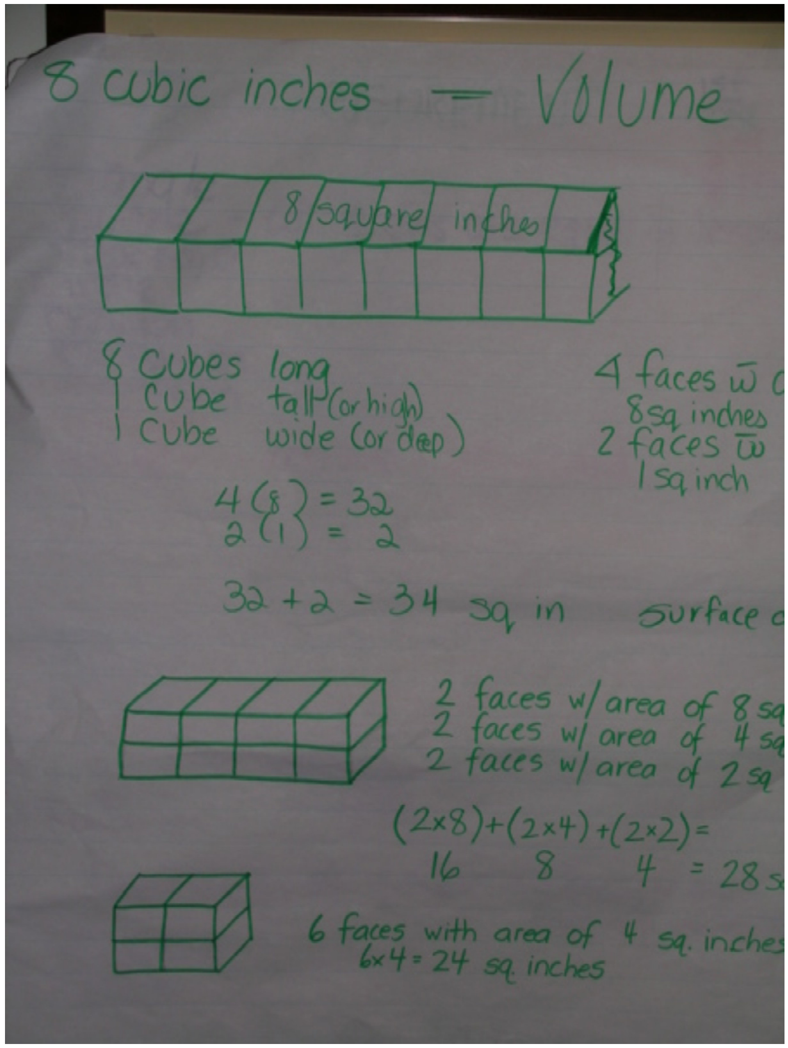
Appendix B - Whole-group Discussion Notes



This sheet of notes was started in Session 1

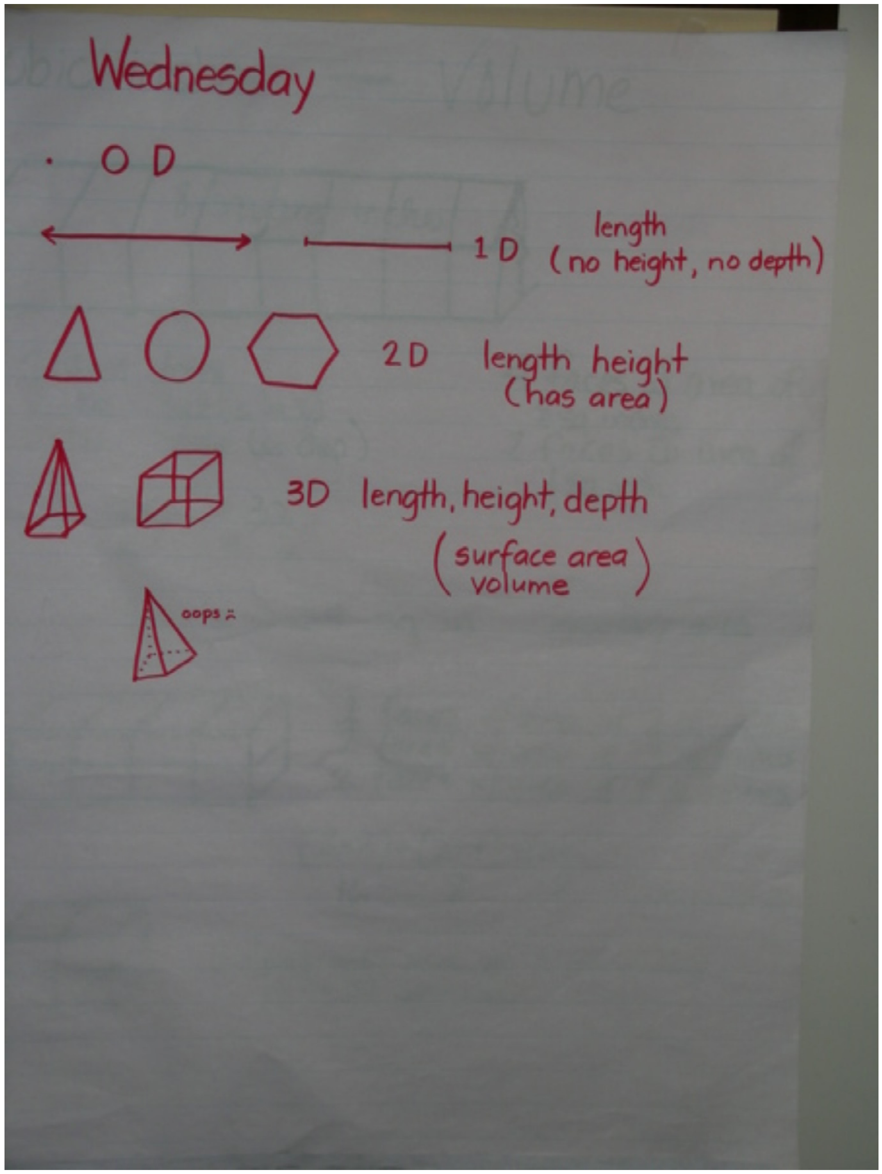
Session 1 notes were written in blue and purple.
Session 2 notes were written in green.
Session 3 notes were written in red.
Session 4 notes were written in pink.

Session 2



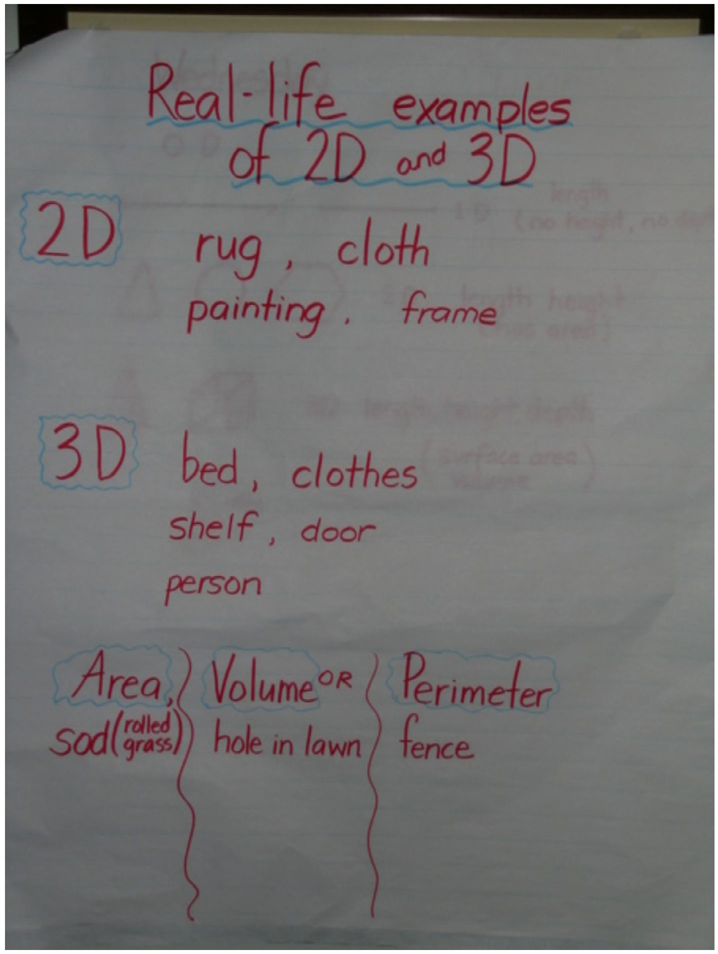
This sheet of notes was started in Session 2.
Session 2 notes are written in green.

Session 3



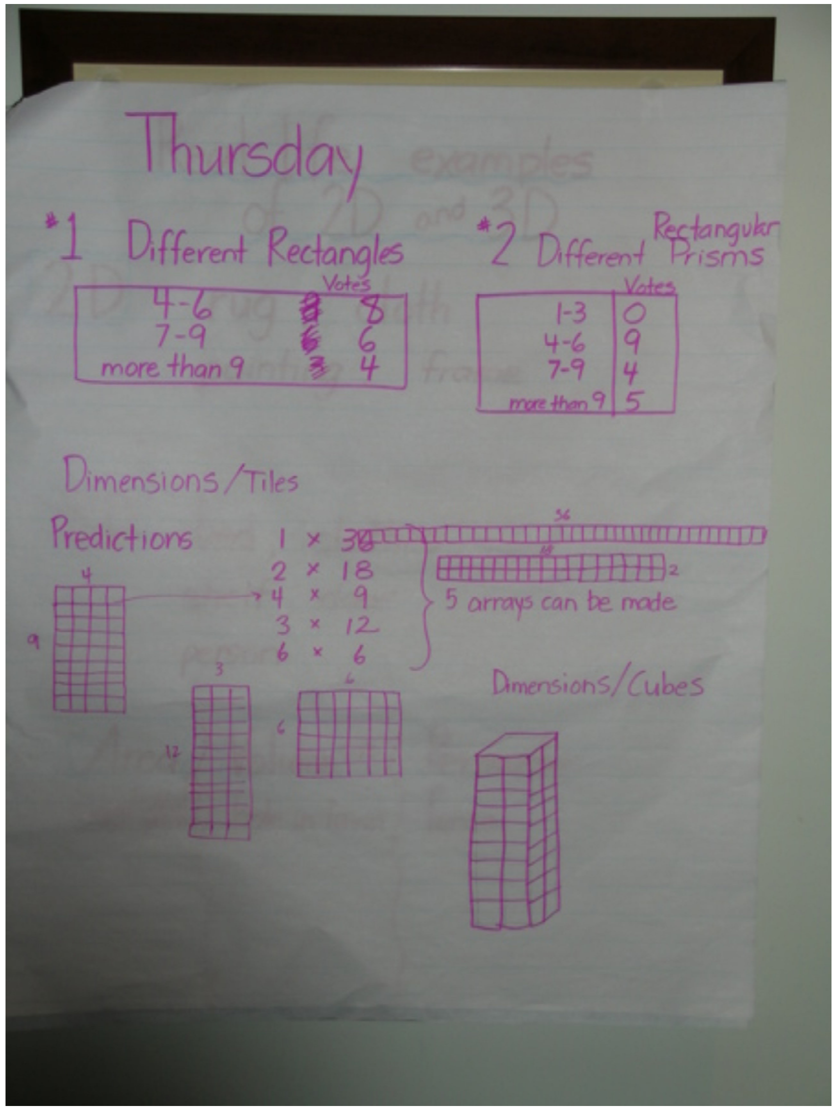
This sheet of notes was started in Session 3.
Session 3 notes were written in red.

Session 3 continued



This sheet of notes was started in Session 3.
Session 3 notes were written in red.
(The underlining and boxing-in in blue ink was done for emphasis during Session 3's discussion.)

Session 4



This sheet of notes was started in Session 4.
Session 4 notes were written in pink.

Session 4 continued

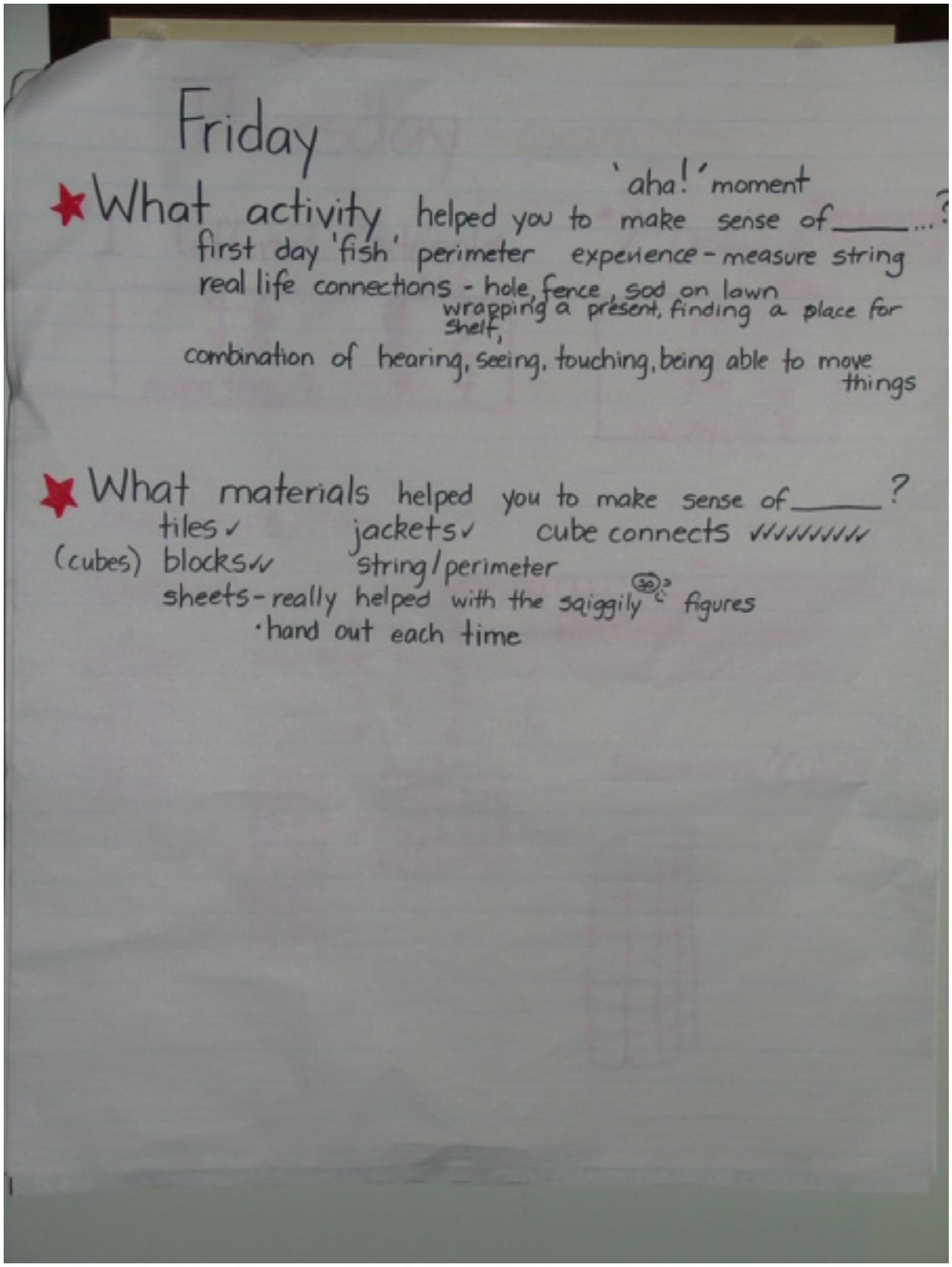
Classroom Teacher

- Pictures
- Examples
- Explanations
- Definitions
- The “blue covers”

Notes Scribed on Whiteboard by Classroom Teacher
The “blue covers” were the pre-cut rectangular shapes students used to construct prisms. A photo of them is shown in Appendix A- Materials.

The classroom teacher facilitated a whole-group discussion on what would be important to include on the posters that the students would be making in Session 5.

Session 5


Friday
'aha!' moment
What activity helped you to make sense of ___...?
first day 'fish' perimeter experience - measure string
real life connections - hole, fence, sod on lawn
wrapping a present, finding a place for shelf,
combination of hearing, seeing, touching, being able to move things
What materials helped you to make sense of ___?
tiles ✓
jackets ✓
cube connects ✓✓✓✓✓✓✓✓✓✓
(cubes) blocks ✓✓
string / perimeter
sheets - really helped with the sqiggily figures
· hand out each time

Appendix C - Students' Written Responses to Reflection Prompts

Session 1 Reflection Prompts		
Student #	*When an adult talks about the area of something, say a playground, what does that mean to you?*	*Area and perimeter are different. How would you explain the difference between area and perimeter to a student in the first grade?*
1	-----	-----
2	It means the emount [*sic*] that fils [*sic*] the shape in. And to find the eara [*sic*] I would multiply1 roy [*sic*] and 1 column.	I would saw [*sic*]. Well, this is a toon [*sic*] that should alway [*sic*] remember at math time.
3	It means like putting mulch on the ground of the play-ground [*sic*].	I would say area fills it in and perimeter goes around the line.
5	It makes me remember what area is. What?	Perimeter goes around the rim and area fills it in.
6	The perimeter to play in or how much space do I have or what the playground has in it.	How I would explain perimeter to a 1st grader is perimeter has rime in it because perimeter goes around the rime. How I would explain to a 1st grader is I would help them memorize perimeter goes around the rime [*sic*] and area fill it in!
7	The number of cubes you can find inside the shape, square units.	Well, lik [*sic*] our rime [*sic*], the perimeter goes around the rim, and area fills it in, so, I imagine a square, for the area, brake the square into little squares, and count them. For perimeter, take another square, measure the top, then bottom, then sides, then add it up.
8	-----	I would say that area is the inside of a shape, and perimeter is the rim on the outside.
9	To me it means almost nothing because math is not my favorite subject but after the past months I've learned to like area and perimeter so there both fine [*sic*].	If I told it to a 1st grader I would tell them that well first of all I would tell them that perimeter goes around and area fills it in. And then I'll say that perimeter goes outside and area goes inside.
10	it fills in a 2D shape.	Area fills in the shape and perimeter

		goes around the shape.
11	?	Perimeter goes around the shape and area goes in the shape
12	Area is in the mitell [*sic*].	-----
14	how many square units are in the playground	perimeter goes around the rim and area fills it in.
15	What it means to me, "how many square units to fill it in?"	Perimeter goes around the rim, area fills it in.
16	Area fills in 2D shapes in sqare [*sic*] units.	'Perimeter goes round the rim, area fills it in.' Area is measured is [*sic*] square units. Perimeter is measured in inches.
17	-----	I would say that perimeter is the outside of a lets say a boy and the outside is the rim. And area is the inside.
18	To me it means how much can you fit inside the shape.	Perimeter goes around the rim, area fills it in.
19	What?	Okay Kevin today I'm going to teach you what perimeter is and what area is. Perimeter goes around the rim, area fills it in. What that means is perimeter is for example the number inch around a shape. Area is the number of square units insides a shape.

Session 2 and Session 3 Reflection Prompts			
Session 2		Session 3	
Student #	*What connections do you see between area and perimeter of 2-D shapes and volume and surface area of 3-D shapes?*	*How are area and volume alike and how are they different?*	*Give examples of each to help explain.*
1 Res-ponses scribed for ELL student	• area is the inside of a square or a rectangle • surface area is about 3D • surface area is like a 3D shape • you can look at it and see the back of it if you draw it	Same because they are the inside [A square is sketched with an arrow pointing to it from the following text] – area is 2D [A cube is sketched with an arrow pointing to it from the following text] – Volume is inside. This is a 3D shape How are they different One is 2D (area) One is 3D (volume)	2D Examples No faces - a piece of paper - paper towel when you take it out - your money when - you go to pay (pantomimed paying with a dollar bill) 3D Examples a movie box a pencil a box a candy paper bag
2	I see between area and perimeter of 2-D shape is, area fills in the shape, and perimeter goes around the shape. If you're trying to find the area of a big box or something then you could count how tall it is and how much the length is the [*sic*]	Area and volume are alike because, bothe [*sic*] area and volume meger [*sic*] the inside of the shape, but not the outside. But area and volume are all so not alike because, volume meger [*sic*] the inside of a 3-D shape, but area meger [*sic*] the inside of a 2-D shape.	In real life for a example, you would have to meger [*sic*] the volume of a house if you're building a house for a family. And, for the area you would have to meger [*sic*] the area of a poster, if you're making a poster for the school.
3	The difference I see	Area and volume are alike	Here is a chart for

	between 2 dimensional shapes and 3 dimensional shapes are when you are looking at a 3-D shape it looks 2-d and when you look all around a 3-D shape it looks 3-D. When you look all around a 2-D shape there's no extra cubes to. see so it's 2-D	because they both fill in a shape. Area is different because area fills in a 2-D shape and volume fills in 3-D shapes.	area/volume. Area rugs floors walls paint Volume hole cup shelf water
5	The connections I see between area and perimeter of 2-D shapes and volume and surface area of 3-D shapes. Area and surface area is that a. they are both measurements b. they both have the word area in them so that means that they both measure the area of surface. Primiter [*sic*] and volume don't really have much in common.	Area and volume don't really have many things in comen [*sic*]. One thing that is the same is that they are both a kind of measurement. A few things that are different are a. volume is the area that fills in something and area measures the distance between the edges b. volume is measured in cubic units and area is measured in square units.	----
6	I recanized [*sic*] between 3-D and 2-D shapes that they have different perimeter and area because [*sic*]	How area and volume are different is area is for 2-D shapes and volume is for 3-D shapes. How area and volume are alike is area and volume both fill shapes in and they both can use inches.	Example: Say you had a hole and you had to fill it in with dirt you would use volume because it is a 3-D shape.
7	Well, the area of a 2-d shape is like a footprint of a 3-D shape. The perimeter is always the rim. Here is the square, [a square is sketched]. The area is going to be 4 and the perimeter is going to be 12. Now imagine a cube, [a cube is sketched]	Alike • they are both about the inside • they are both counted in units • they are both on 2-D shapes, but volume has layers of 2-D shapes Different	Examples if you are measuring a rug, you probably wouldn't use volume, you would use area, because it can be measured in square units, and your floor is not a ditch !! If you were measuring

	it has the same thing on one of the sides, but it is multiplying it by four. The area is turning into volume, because it now has height. So area is volume after adding height, and instead of one surface (side) like a square, it has six, because it's a cube.	• volume is 3-D • area is 2-D • area is always measured in square units	how low the hole is that a mole had dug, you would use volume, because it has depth and height.
8	I know that surface area is counting all the faces on the outside, and area is counting all the faces in the inside. Volume is the inside of a 3-D shape that makes up the shape.	Area is the amount of square units inside a 2-D shape. Volume is the depth that fills up a 3-D shape. Area and volume both represent the inside of a shape, but Area represents the inside of a 2-D shape, and volume a 3-D shape.	----
9	I first of all that the area of the 2-D shape has to fill it in and the surface are [*sic*] has to see for example how much rapping [*sic*] paper you will need to rap [*sic*] a present so if youre [*sic*] rapping [*sic*] a gift the rapping [*sic*] paper is surface area and the area is the present. Surface area is mainly well if your rapping [*sic*] a present and you need to figure out how much wrapping paper you'll need measure the area of the surface of the present and that what surface area is!!	Area and volume are alike just tiny bit. One reason is that area is if your [*sic*] buying a rug and you need to know how much space it will take up and volume is like if you have the GathPool for instince [*sic*] how mutch [*sic*] water do you need to fill it. Area and volume are alike because if you were planning to dig a hole you would need to figure out how much dirt will it take. A difference between them is area is 2-D and volume is 3-D.	
10	I learned that surface area is like a jacket that covers a rectangular prizem's [*sic*] faces. Area is for a	Area and volume are both measuring the space inside a shape. Only volume is measuring a 3D	Volume would be measuring the hollow part of a box, and area would be measuring a

	2D shape and surface area is for a 3D shape.	shape and area is for a 2D shape.	rug.
11	Area in one 2-D shape is like surface area on one of the faces on a 3-D shape. I think area is the same as volume in 3-D shapes because it's how much is in the 3-D shape. I learned today that perimeter cannot be measure [*sic*] on a 3-D shape unless you are measuring one of the faces. That's how perimeter and surface area are alike because you can't measure the inside of either the 2-D shape or the 3-D shape.	They both fill in shapes whether it's 2-D or 3-D. To find area in a square you have to count how many square units and in a cube to find the volume you have to count the cubic inches. So the only difference between measuring a 2-D shape and a 3-D shape is that you measure in square units if you're measuring a 2-D shape and you measure in cubic units if you're measuring a 3-d [*sic*] shape. You can only find area in [*sic*] a 2-D shape and you can only find volume in a 3-D shape. The formula for area in [*sic*] height x length and the formula for volume is height x length x depth.	Area for a square is like volume for a cube.
12	2-D is when it is on flat paper 3-D is when it is 3 dmeccinal [*sic*] like a bear	Area and volume are different and the same. (Different)3-D is volume (Same) area is 2-D	An example for area in 2-D is a circle. 2-d is when you need to do a farme [*sic*]. An example for volume in 3-D is rectangular prism. 3-D is when you need to do volume is a chere [*sic*].
14	1st I think a connection between 2-D shapes and 3-D shapes is 3-D shapes are made of layers of 2-D shapes. 2nd the surface of 3-D shapes still has area and perimeter.	Volume and area are different because volume has depth and area 2-D [*sic*]. Area and volume are alike because volume must have surface area. Volume and area are different because volume	-----

		measures the copacity [*sic*] and area measure something flat.	
15	The connections I see between area and perimeter of 2-D shapes and volume and surface area are the following. First, 3-D shapes have 2-D shapes in them. 3-D shape also have volume, which, the formula for cubes is area of 1 side times depth or, a.k.a. height times width times depth. Those are two of many connections between the area and perimeter of 2-D shapes and volume and surface area of 3-D shapes.	Area and volume are alike for these reasons. First, area is part of a formula to find volume, the formula is (base x height [*sic*]) = area area x depth = volume. Volume and area are different for the following. Volume is base x hight [*sic*] x depth = volume, for 3-D objects. Area is base x height = area for 2-D objects. Those are the differences and similarities of volume and area.	-------
16	Surface area is the same as area except 3D. Area fills in a 2D shapes, surface area fills in 3D shapes. Perimeter goes around the shape. To figure out the volume, do base x height x weight = volume	Area fills in 2D shapes, volume fills in 3D things. •Area is 2D, volume is 3D.	Area would fill in a circle, volume would fill in a hole. •Area would fill in a rectangle, volume would fill in a rectangular prism. Volume would fill in a hexagonal prism, area would fill in a hexagon.
17	Area fills the inside of the squar [*sic*]. perimeter is the outside of a squar [*sic*]. volume is filling in an area.	They both meger [*sic*] the inside. But area measures the inside of 2-D shaps [*sic*] and Volume meger [*sic*] the inside of 3-D shapes.	You would need to use area for a rug in the room. You would need to use volume for the grass in the yard.
18	I learned that surface area is basically the outside layer of a shape and that volume is what you put in a hole like dirt. I also learned that the there is more than 2 ways to make a rectangular prism out of	Area and volume are alike because they both have to do with basically the same thing like area is how much you can put in it and volume is what you put in it. Area is also like volume because they can	------

	8 cubes.	both be used on 3 demeniniol [*sic*] shapes.	
19	I see a couple of connections. In 2-D shapes to find the perimeter of a rectangle you do (base x 2) + (height x 2) = ____. In 3-D shapes, it's called a surface area. What you do to find the surface area is you find the area of each face then you do face + face + face and so on until you reach the number of faces there are. In 2-D you also have area. How you find the area of a rectangle is you do height x base = area. In 3-D you also have volume. How you find the volume of a rectangular prism is you do height x length x depth = volume. I think that area and volume are alike. Area is finding the inside of a shape with flat units. Volume is when you find the inside of a 3-D shape with 3-D units.	Area and volume are alike because in area you find how much a 2-D shape cover. How area and volume are different is, you measure area with 2-D units and volume with 3-D units. Also when you measure area your measuring 2-D shapes and when your measuring volume your measuring 3-D shapes.	For example, a rug, you need to know if you have room in your house. To find the are of the rug you measure each side and do side + side + side etc. until you added the measurements of every side once. The total is the area. In volume you are talking about a 3-D shape, for example a box. Pretend the box is a cube and your putting your little brother 1-inch cube blocks. To find the volume of the box you do (height x length) x depth = volume.

Appendix D – Recording Sheets for Sessions 1, 2, 3 and 4

Session 1 – Recording Sheets for Predictions, Activity Results, and Written Reflection (p. 1 of 2)

Name ________________________________ **Date** ____________________

Predictions – For each question, circle one of the letters.

1. Which of the 2-D shapes on the board do you think has the **largest area**? A, B, or C
   Why? ____
   ____

2. Which of the 2-D shapes do you think has the **longest perimeter**? A, B, or C
   Why? ____
   ____

3. Which of the 2-D shapes in front of you do you think has the **largest area**? A, B, C, or D
   Why? ____
   ____

4. Which of the 2-D shapes in front of you do you think has the **longest perimeter**? A, B, C, or D Why? ____
   ____

Activity

Determine the area and perimeter of each shape. Show work and be sure to include the unit of measurement.

Shape	Area	Perimeter
A		
B		
C		
D		

What materials did you use and how did you find the areas and perimeters?

____
____
____
____
____
____

Session 1 – Recording Sheets for Predictions, Activity Results, and Written Reflection (p. 2 of 2)

Reflection

When an adult talks about the area of something, say a playground, what does that mean to you?

Area and perimeter are different. How would you explain the difference between area and perimeter to a student in the first grade?

Session 2 – Recording Sheet for Predictions, Activity Results, and Written Reflection

Name ________________________________ **Date** ____________________

Prediction – How many different rectangles do you think you can make with 8 square tiles? ________

Activity, Part 1:
Trace each rectangle that you can make with the 8 tiles. Find the length of each side and the area of the rectangle.

Prediction – How many different rectangular prisms do you think you can make with 8 cubes? ________

Activity, Part 2:
- Find as many different rectangular prisms as you can with 8 cubes.
- Describe each rectangular prism that you find. (Tell how long, how tall, and how deep it is.)

For *only one* of the rectangular prisms that you find, trace each face, find the perimeter of each face, and find the area of each face. Then find the surface area of the prism (add up all the areas of the faces).

Reflection:
What connections do you see between area and perimeter of 2-D shapes and volume and surface area of 3-D shapes?

____

____

____

____

____

____

____

____

____

Session 3 – Recording Sheet for Predictions, Activity Results, and Written Reflection

Name ________________________________ **Date** ____________________

Prediction
1. What do you think the volume (in cubic inches) of your rectangular prism is? ________________

2. If you cover your rectangular prism with paper (no overlaps) how much area would the paper have? (What is the surface area of your prism?)
Put a √ beside the interval in which you think the surface area would be.

☐ 4 and 7 square inches
☐ 7 and 11 square inches
☐ 11 and 15 square inches
☐ 15 and 29 square inches
☐ 29 and 45 square inches

Activity
- Trace each face.
- Find the perimeter of each face.
- Find the area of each face.
- Find the surface area of your rectangular prism.
- Find the volume of your rectangular prism.
- Make a covering for your rectangular prism. (Use the pre-cut paper pieces.)

Reflection:
How are area and volume alike and how are they different? Give examples of each to help explain.

____

____

____

____

____

____

____

____

____

Session 4 – Recording Sheet for Predictions, Activity Results, and Written Reflection

Name ________________________________ **Date** ____________________

Prediction

1. If you have 36 square tiles, how many different rectangles do you think you can make using all 36 tiles?

- ☐ Between 1 and 3 rectangles
- ☐ Between 4 and 6 rectangles
- ☐ Between 7 and 9 rectangles
- ☐ More than 9 rectangles

2. If you have 36 cubes, how many different rectangular prisms do you think you can make using all 36 cubes?

- ☐ Between 1 and 3 rectangular prisms
- ☐ Between 4 and 6 rectangular prisms
- ☐ Between 7 and 9 rectangular prisms
- ☐ More than 9 rectangular prisms

Activity

With your partner discuss -

- What are differences among perimeter, area, surface area, and volume?
- How will you explain to a student in the first grade what perimeter, area, surface area, and volume mean?
- What will you write?
- What examples will you give?
- What will you draw?
- What will you make?
- How will you display all this information on a sheet of paper 11" x 14" so that it helps a first grader to understand?

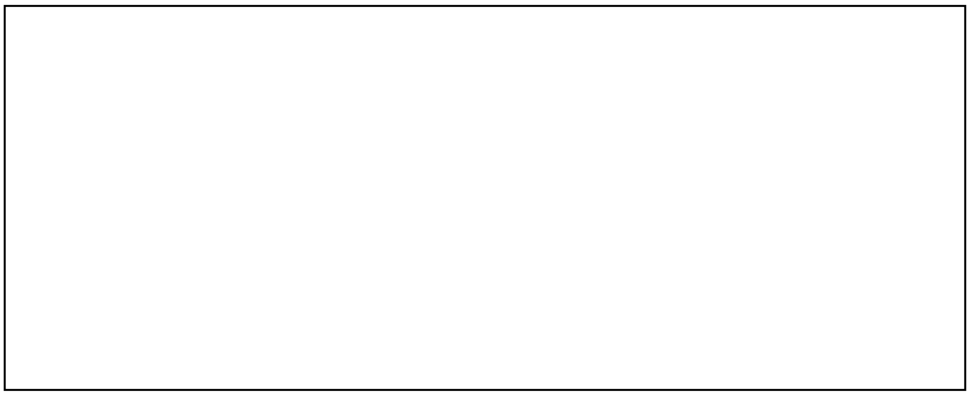
Appendix E - Pretest/Posttest

Name ______________________________________ Date _________________________
Teacher ____________________________________

Question # and answers in red

1. This rectangle has two sides that are five inches long and two sides that are two inches long. What is *the perimeter* of the rectangle?

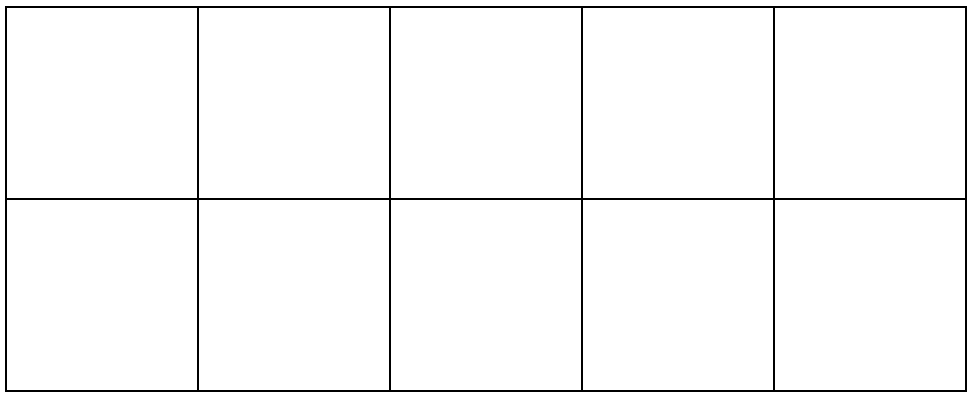
#1 - 14
#2 - inches



Can you draw a *different* rectangle that has the *same perimeter*? If so, draw it below and label the length of all the sides.

#3 – drawn rectangle
#4 – inches

2. This is the “footprint” of a rectangular prism that is three inches tall.



Show your work.

What is the area of this footprint?

#5 – 10
#6 – square inches

What is the perimeter of the footprint?

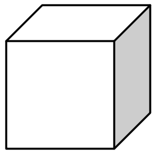
#7 – 14
#8 - inches

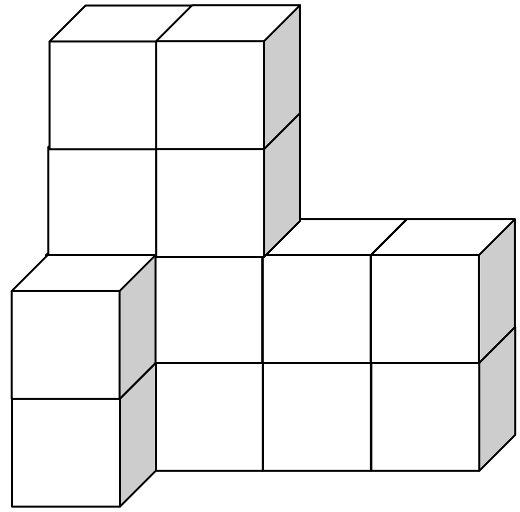
What is the volume of the rectangular prism?

#9 – 30
#10 – cubic inches

What is the surface area of the rectangular prism?

#11 – 62
#12 – square inches

3.  = one-inch cube



#13 - 14

How many one-inch cubes make up this 3-dimensional shape?

What is the perimeter of the face of this 3-dimensional shape that would be resting on the table if the shape was really 3D?

#14 - 12 #15 - inches

What is the area of the face of this 3-dimensional shape that would be resting on the table if the shape was really 3D?

#16 - 5 #17 - square inches

What is the surface area of this 3-dimensional shape?

#18 - 46 #19 - square inches

4. What is the letter and number of the 3-dimensional shape you have been given? ----

How would you go about finding the surface area of your 3-dimensional shape?

#20 - answers may vary

How would you go about finding the volume of your 3 dimensional shape?

#21 - answers may vary

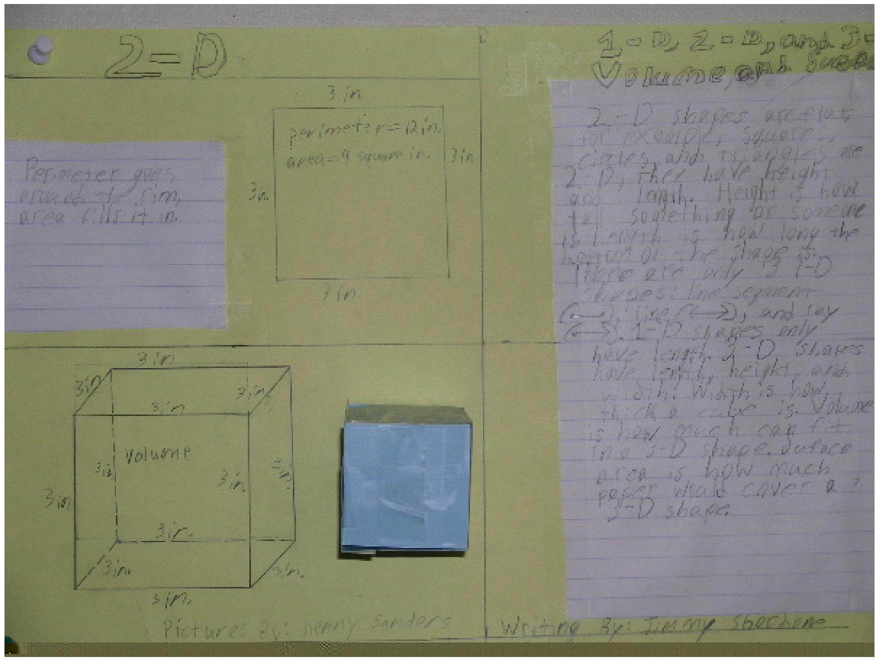
Put a post-it sticker (labeled "GP") on the face of your 3-dimensional shape that has the greatest perimeter. Give an approximate measurement for the perimeter and be sure to include the unit of measurement.

#22 - identify the face #23 - approximate # #24 - inches

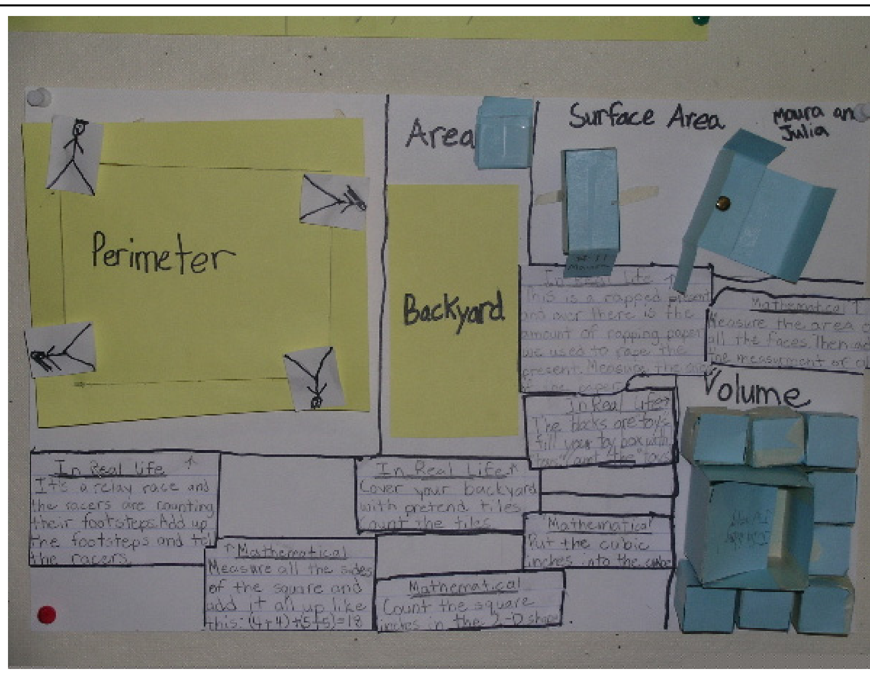
Put a post-it sticker (labeled "SA") on the face of your 3-dimensional shape that has the smallest area. Give an approximate measurement for the area and be sure to include the unit of measurement.

#25 - identify the face #26 - approximate # #27 - square inches

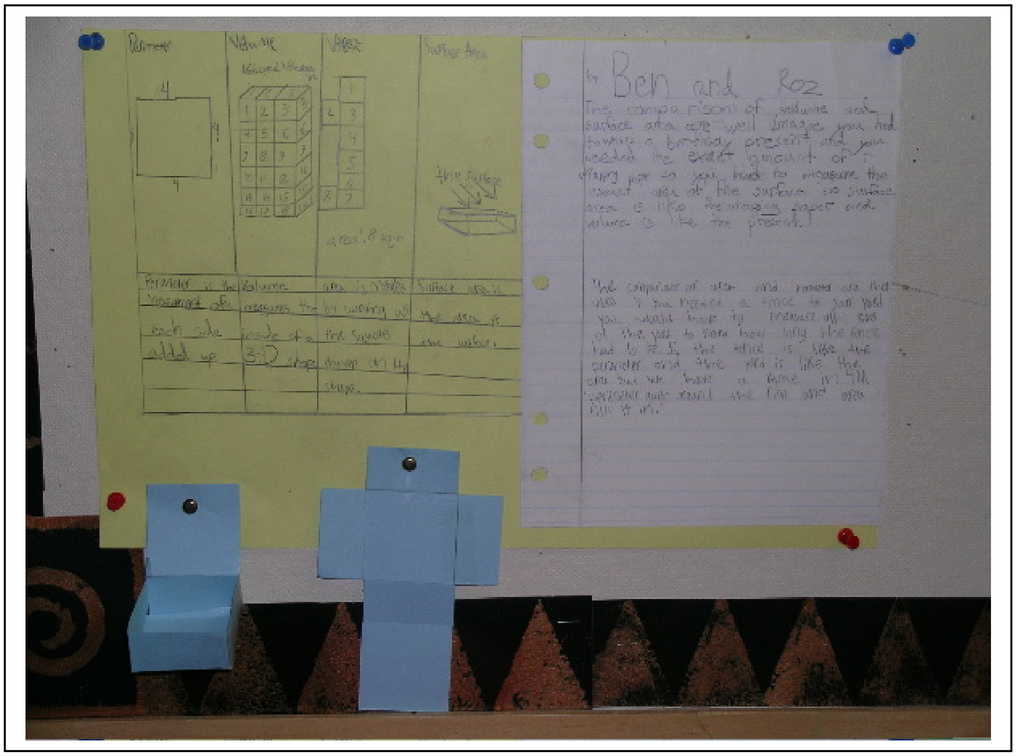
Appendix F - Students' Posters



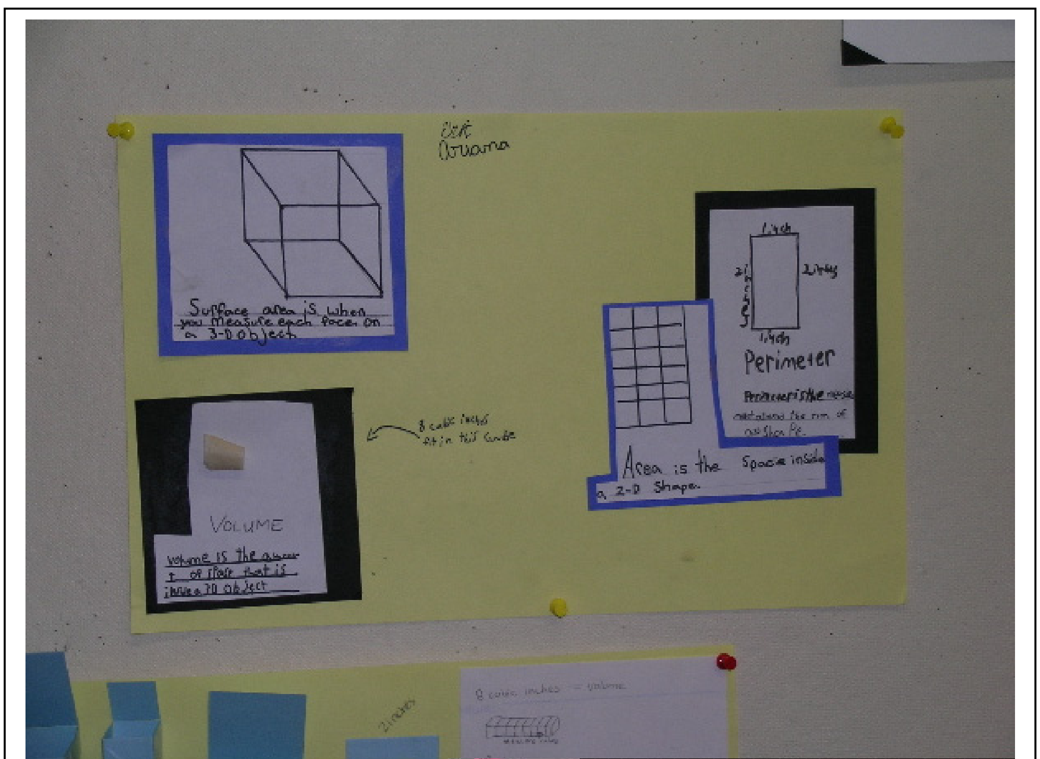
Students #15 and #16



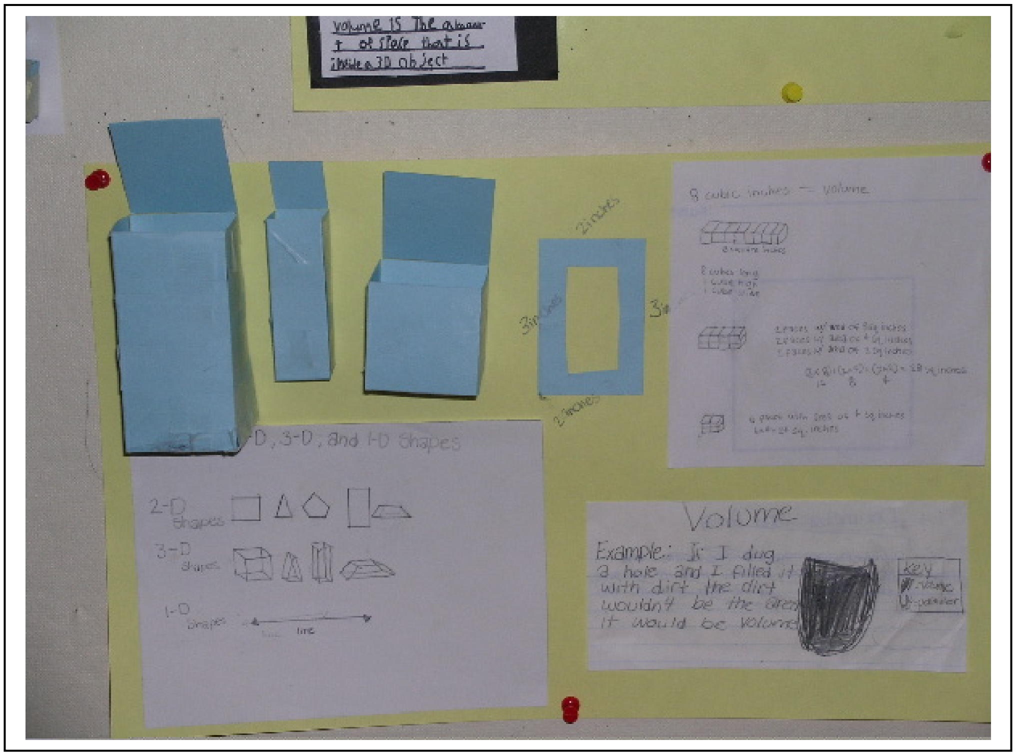
Students #11 and #19



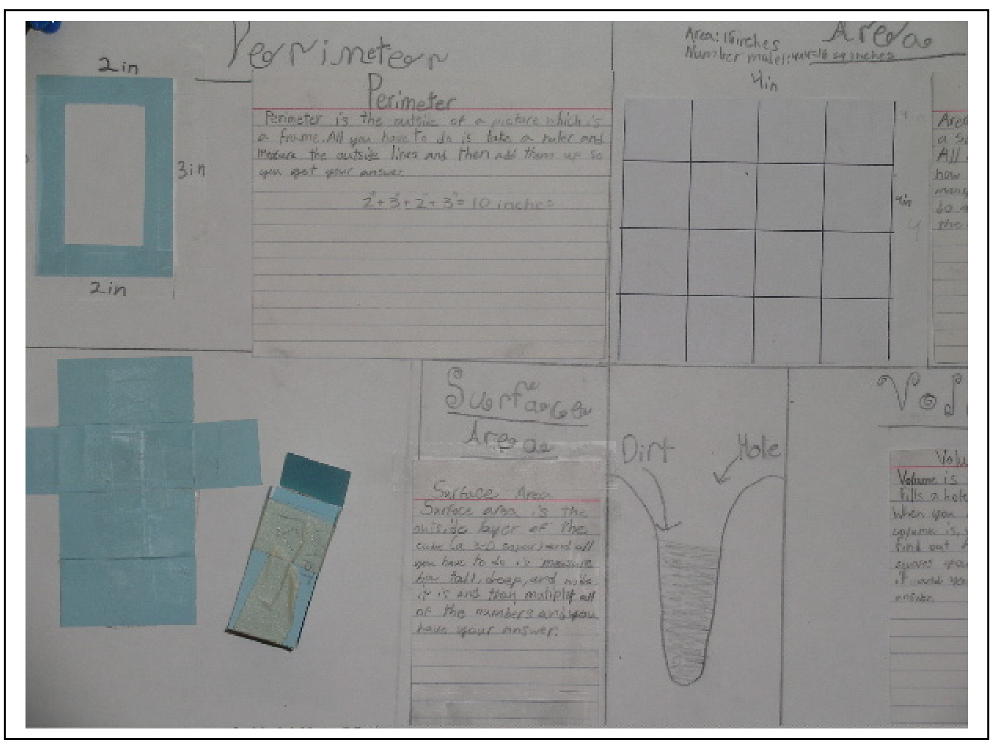
Students #2 and #9



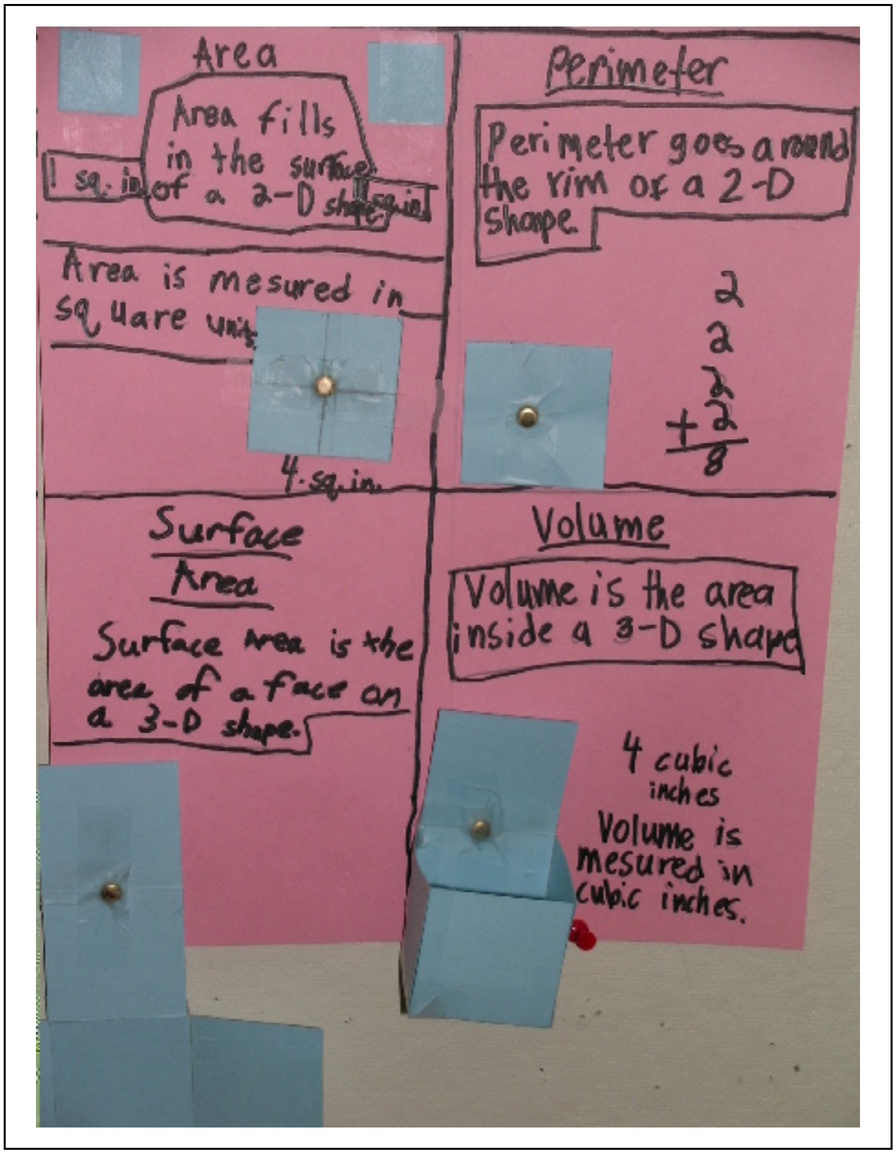
Students #7, #10, and #17



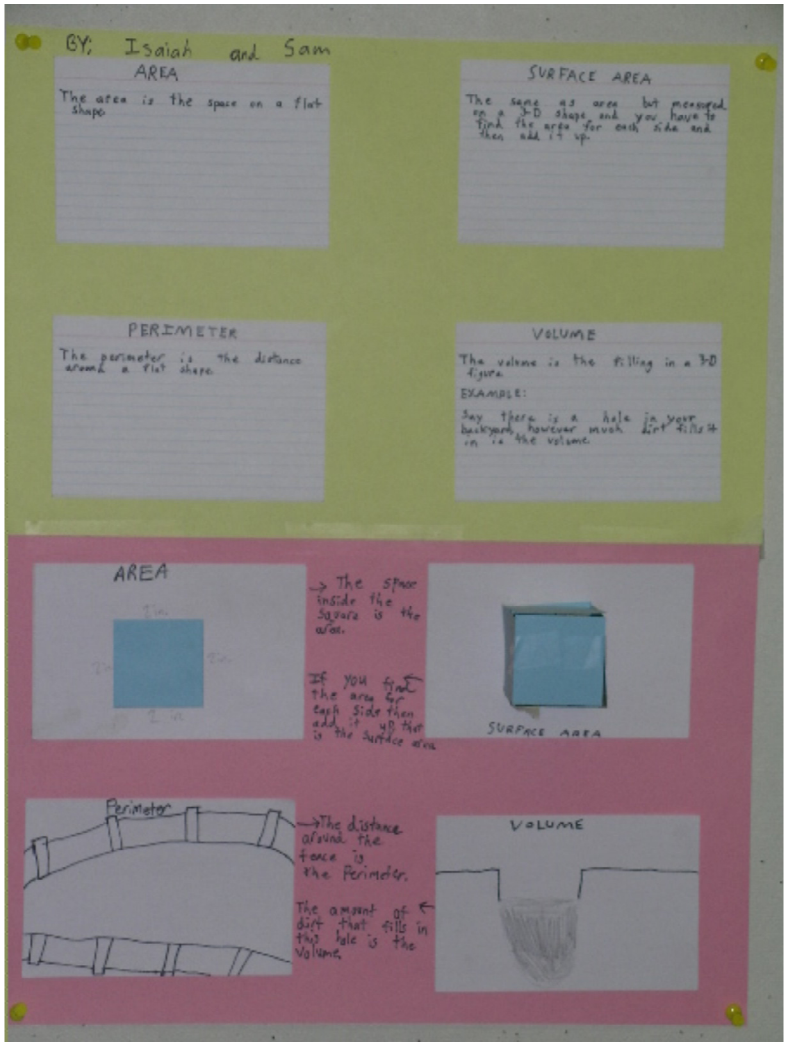
Students #1 and #6



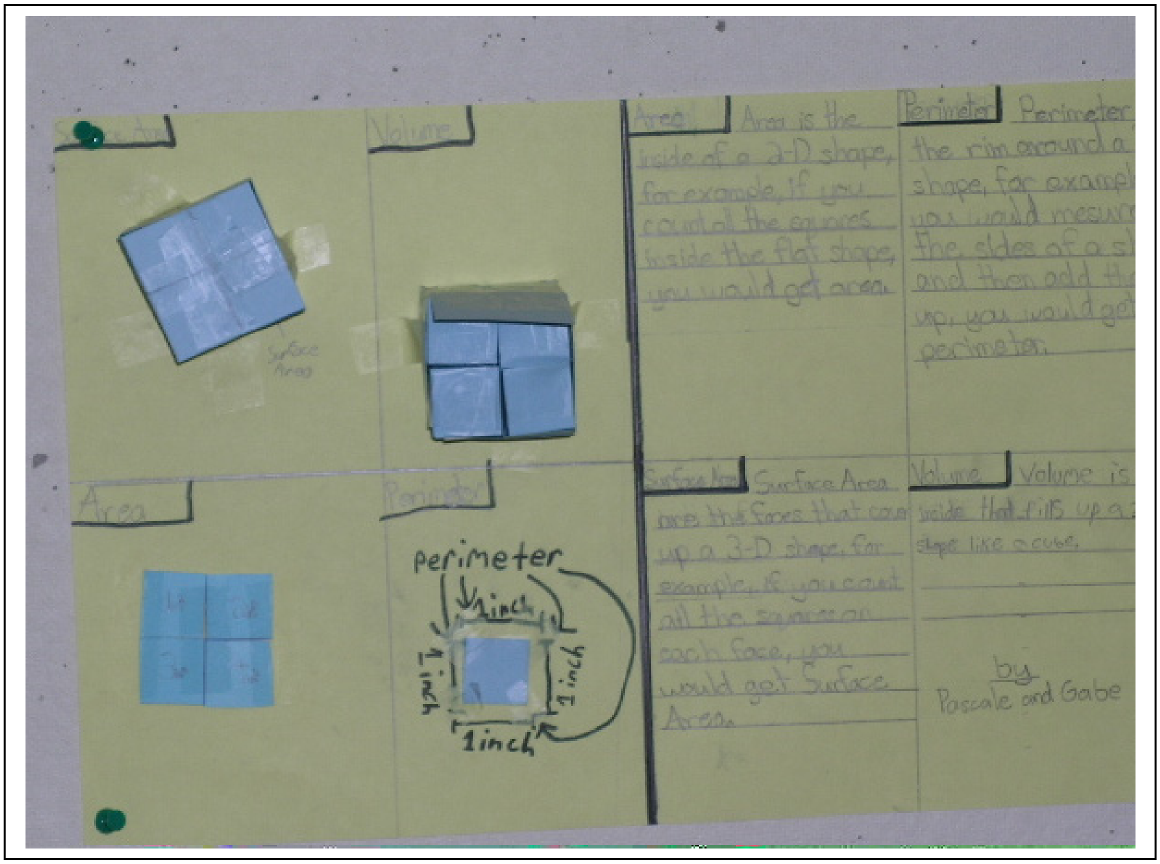
Students #12 and #18



Students #5 and #13



Students #3 and #4



Students #8 and #14