a. As we noted earlier in the course, thoroughly anti-Aristotelian to think of two motions compounding to form a third motion: one motion always dominant

b. But can be seen in some classical geometry, especially post-Euclidean geometry -- e.g. Archimedes' spiral -- as well as in Ptolemy (compound circles) and Kepler (ellipse out of circle from radial motion)

4. Theorem 1: the resulting trajectory compounded from uniform horizontal and uniformly accelerated vertical motions is a semiparabolic line

   a. Galileo's proof employs the converse of lemma 1, which amounts to claim that y proportional to x^2 in parabolas

   b. Basic idea of proof straightforward: x proportional to t, y proportional to t^2, resulting in y proportional to x^2

5. Theorem 1 more limited than it may at first appear to be, for it says nothing about which parabola is described by the motion -- i.e. the dimensions of the parabola

   a. Theorem 1 the counterpart of Kepler's "law" stating that orbits are elliptical (conforming to area rule), without specifying what factors might determine which parameters

   b. Kepler never offers factors suitable for solving the "initial value" problem, but Galileo does

B. A Problem: Local Versus Circular Motion

1. Much as in the "Third Day" the basic claim immediately elicits a series of challenges to the very idea of conceptualizing projected motion in this way [273-274]

   a. Sagredo: surely not a parabola all the way to the center of the earth, and hence why a parabola even over first segment

   b. Simplicio: motion in a straight line along a horizontal is not uniformly removed from the surface of the earth, and hence will experience a deceleration as if going up an inclined plane!

   c. Simplicio: resistance effects cannot be removed, and they will "destroy" the two separate patterns of motion being compounded

      "All these difficulties make it highly improbable that anything demonstrated from such fickle assumptions can ever be verified in actual experiments" -- [274]

2. Galileo's response to the challenge on horizontal motion grants the point -- i.e. true horizontal motion not eternal -- but then invokes the authority of Archimedes to treat the earth's radius as infinite

   a. Idealization that is approximately correct for short horizontal distances, as evidenced by such practical procedures as parallel plumb lines in architecture

   b. Concedes that some correction may be needed when applying to various real motions -- thereby opening problems for the future

3. Galileo's response to the challenge about the path to the center of the earth: while reaching surface of earth, parabolic shape altered "only insensibly, whereas that shape is conceded to be enormously transformed in going on to end at the center" [275]

a. A famous problem to which Galileo had offered a solution that he had already learned to be mistaken (from Mersenne and Fermat) -- see Koyré's monograph
b. Concession drops claim that parabola even an idealization, for no longer said to hold exactly in absence of resistance
c. Raises a question for others: what trajectory does a projectile truly follow in the absence of resistance, with the earth's curvature taken into consideration

4. Galileo has a real problem here that he does not resolve, namely how to reconcile local motion near the surface of the earth with circular motion
   a. Response here simply withdraws claim that results would hold exactly without resistance -- hold only approximately, but still of practical value
   b. In process opens up a number of problems for others to address in future -- exact motions, relation between local motion and (celestial) circular motion, etc.
   c. These questions will receive increasing attention in subsequent years
5. Galileo's response to question about resistance is maybe his clearest statement that no science of resistance possible at all [275ff]

   "No firm science can be given of such events [*accidenti*] of heaviness, speed, and shape, which are variable in infinitely many ways. Hence to deal with such matters scientifically, it is necessary to abstract from them. We must find and demonstrate conclusions abstracted from the impediments, in order to make use of them in practice under those limitations that experience will teach us. And it will be of no little utility that materials and their shapes shall be selected which are least subject to impediments from the medium, as are things that are very heavy, and rounded."

   a. Practical defense: use scientific result as basic approximation and then fudge it as needed -- engineering defense
   b. Further argument: fudge not that large in cases of interest -- in particular, small enough that need not worry about how resistance separately undermines the individual patterns of the two compounded motions; so, combining separate components okay
   c. {Note reference to "supernatural" motion in cannon -- speeds greater than generable by nature, [278]}

C. The Problem of Compound Impetus: Which Parabola
1. Galileo's next concern is with the impetus and hence speed that occurs in compound motion
   a. Remember again that impetus relates to percussive effects on impact, which Galileo took to be proportional to weight and (mistakenly) speed
   b. Speed is the scalar magnitude of the velocity vector, always taken in the direction of the motion
2. Impetus and hence speed of two compounded uniform motions is simply the vector sum of the respective uniform impetuses and speeds
   a. For, spaces traversed in equal times can be used as a measure of speed and hence of impetus in the case of uniform motion