- e. But the latus rectum is just 2/A
- f. Therefore, for any conic section trajectory governed by centripetal forces aimed at the focus, the force along the trajectory varies as  $1/r^2$
- 4. The Scholium immediately following Problem 3 has mystified people ever since -- vide Wilson "The major planets orbit, therefore [sic!], in ellipses having a focus at the centre of the Sun, and with their radii drawn to the Sun describe areas proportional to the times, exactly as Kepler supposed"
  - a. To infer that the area rule holds exactly, need to establish that the only forces acting on the planets are aimed to the center of the sun
  - b. To infer that the orbits are exactly elliptical, then need the converse of Problem 3 -- i.e. the problem Hooke originally wanted solved, and Newton told Halley he had solved: if a body is governed by inverse-square centripetal forces, then its only closed circuit trajectory is an ellipse
  - c. And even then need the claim that the centripetal force is inverse-square, something that has so far been shown for the planets only under the assumption of uniform motion in perfectly circular orbits
  - d. The seeming logical lacuna persists right through the first edition of the *Principia*, to be filled in the second edition following some pointed comments by Johann Bernoulli
- 5. My suspicion is that Newton was here engaging in a type of evidential reasoning (hence appropriate to a Scholium) that he employs widely (provoking much controversy) in Book III of the *Principia* 
  - a. From the rough phenomena alone -- e.g. circular orbit -- can conclude from Theorem 1 and 2 that, at least to a first approximation, an inverse-square centripetal force aimed at the sun is the dominant factor keeping the planets in their orbits
  - b. Problem 3 shows that the next level of refinement of the phenomena, to a roughly elliptical orbit sweeping equal areas vis-a-vis the focus, requires no revision of that conclusion
  - c. Licenses the inference that, at the second level of approximation, the planets obey Kepler's first two rules exactly, for nothing beyond forces inferred from the first approximation is required for them to do so
- 6. Whether such evidential reasoning is legitimate -- or more to the point, exactly what conclusion is to be drawn from it -- I leave as an open question for now
  - a. But the interpretation I offer at least has the virtue of not saddling Newton with a glaring blunder in logic
  - b. I find the idea that Newton would fall trap to such a blunder in simple 'if-then' reasoning beyond belief

## B. Theorem 4: The Keplerian 3/2 Power Rule

1. Theorem 4 provides a generalization of one half of Corollary 5 of Theorem 2: Kepler's 3/2 power rule holds for bodies in elliptical orbits governed by centripetal forces aimed at (a common) focus when those forces vary as  $1/r^2$  -- i.e.  $P^2$  varies as  $a^3$