# Memory and Probability

A thesis

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# ABSTRACT

This thesis examines how we store probabilities, how remembered probabilities affect decisions, and how memory for probability judgments determined by a single event interacts with implicit probability judgments formed by exposure to repeated events. The first experiment deployed a novel paradigm in the form of a card game to examine how memory for stochastic events influences choice following intervening decision tasks. The second experiment investigated memory for the context of game trials. The third experiment modified an existing memory research paradigm in order to examine the ability to remember probabilistic information following a single presentation of an event with a visible sample space. Increasing retention interval has a significant, systematic, and degrading effect on optimal choices based on judgments of relative probability, but reinforcement is somewhat more robust. However, memory for simple probabilities derived from events with clearly presented sample spaces is more accurate. Implications and future research are discussed.

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# INTRODUCTION

The present experiments explore the nature of memory storage and retrieval for probabilities. The probability of an event is an important consideration in higher-order reasoning. For example, theories of decision-making have assumed for centuries that the relative probabilities - perceived or actual - of two or more choices are an independent factor in how people make decisions under conditions of risk and uncertainty (Bernoulli, 1738; Luce, 1991; Pascal, 1670; Tversky & Kahneman, 1992). Thus, for an individual to make a choice, he or she must have some knowledge of those relative probabilities. Memory for a probability judgment itself, then, is essential: if such a memory degrades over time, it may lead to different choices than those which would have been made had probabilistic information been readily available. How, then, do we store and retrieve information related to probability? Specifically, how do we remember probabilistic information related to a single event with stochastic properties?

There have been numerous investigations on the role of memory in creating probability judgments based on memories not of individual events, but of series of events and their outcomes. For example, stochastic reinforcement schedules (e.g., Ferster & Skinner, 1957; Rescorla & Wagner, 1972) may be considered an example of this type of learning; operant conditioning impels an organism to perform behaviors that provide a sufficiently high probability of reinforcement. However, line of research does not asses memory for the specific probability value being reinforced. The Weather Prediction Task has examined choice as a habit learning process mediated by probabilistic reinforcement (e.g., Knowlton, Mangels, & Squire, 1996), and from the Iowa Gambling Task has shown that people can develop preferences for choices that yield relatively higher probabilities of reinforcement long before those preferences can be explicitly expressed or explained (e.g., Bechara, et al., 1997). Still other research has examined the role of working memory in making explicit judgments of success probabilities based on declarative calculation of event outcomes (e.g., Brainerd, 1981, Jensen & Neuringer, 2008). In these paradigms, participants decide which options bear relatively higher probabilities of reinforcement based on a series of learning opportunities. Missing from this literature, though, is how this probabilistic updating process - the development of habit - might compete with previously held knowledge about optimal choice - hewing to principles. Do we remember our instruction and take the best option, even if it has failed us before? Or, do we simply repeat what has worked in the past?

Previous research on numeracy and decision-making suggests a general tendency for intuition to drive decisions at the expense of the application of rules and calculation, particularly when tasks become difficult to manage (e.g., Pacini & Epstein, 1999; Reyna & Brainerd, 2008). However, it is still unclear how the two processes compete. To address these questions, Experiment 1 provided all of the information necessary to make judgments of relative reinforcement probabilities at the first learning opportunity, and studied how people use that information after a delay. In doing so, we developed a novel paradigm in which participants both learned rules to help them make optimal choices and received immediate outcome feedback. The experimental paradigm took the form of a card game in which participants learned rules on how best to succeed on a given trial. When the trial repeated, following those rules yielded optimal responses. However, participants were not guaranteed victory with optimal responses, and it was quite possible to win with a suboptimal choice. Thus, we pitted rule-learning directly against reinforcement learning in order to study patterns of each.

Experiment 2 examined memory for the stimuli themselves, an ability necessary to drive performance on the card game task. This experiment presented the same stimuli in the same order as in Experiment 1, but replaced the gameplay aspect with a yes/no recognition task. Thus, participants did not need to choose a card, but rather to simply state whether or not they had seen a given configuration of cards. In this way, we wanted to highlight, if not isolate, the effect of remembering probability judgments apart from remembering the stages of the game. A fourth condition in Experiment 2 took this investigation a step further by using line-drawings from the Snodgrass-Vanderwart (1980) set in the place of cards in order to directly test memory for combinations of items on a set of stimuli that had already been normed for similarity. In short, Experiment 2 aimed to demonstrate that neither an inherent difficulty in remembering combinations of items or hands of cards could account for whatever particular difficulty arose in making optimal decisions in the game scenario.

Experiment 3 looked more directly at memory for probability by testing memory for one event at a time. Instead of using the continuous recognition approach of Experiments 1 and 2, we used a Brown-Peterson paradigm (Brown, 1958; Peterson & Peterson, 1959). Where individuals viewed consonant triads in the classic Peterson & Peterson (1959) experiment, individuals in Experiment 3 viewed events bearing probabilistic information instead. Participants viewed a brief (less than 2 seconds in duration) event and then, following an intervening distractor task of varying length (Brown, 1958; Peterson & Peterson, 1959). Thus, Experiment 3 examined memory for probability separately from the effects rule learning and reinforcement that were explored in Experiment 1.

What happens to probability judgments if and when they are degraded by memory was the central issue that we sought to address with Experiment 3. Would individuals be forced to guess blindly following longer intervals, or would they produce accurate probability judgments? We predicted that probability judgments would become increasingly inaccurate with time, and that judgment errors would follow a predictable pattern. Previous research on probability estimation has shown a natural tendency to systematically overweight small probabilities and to underweight large probabilities (e.g., Prelec, 1998). We predicted to see this effect at very short intervals and become exaggerated as retention interval increases and details of the event were lost to memory.

Taken together, these experiments explore how individuals remember the stochastic nature of events. We examined how people remember probability judgments after a period of time. We looked at how people remembered rules that identified choices that offered the highest probability of reinforcement (and distinguished this mnemonic ability from memory from features of the task itself). We examined the extent to which choice behavior could be explained by reinforcement of winning (though possibly suboptimal) choices or by properly recalled rules for optimal decision-making. This investigation represents the beginning of a program to understand memory for probability – not how we remember series of events and how we infer probability from remembering their outcomes, but rather, how we remember and use probability when given a single event that carries with it all of the information needed to make a judgment.

## EXPERIMENT 1

The experimental paradigm for Experiment 1 took the form of a card game in which no move was perfect, that is, optimal choices often but not always resulted in winning. In this game, each choice was followed by instruction that provided information about what the best option was for that choice. In the tradition of Bartlett (1932), we designed the paradigm to mirror real-world scenarios in which both rules for optimal behavior and reinforcement are available as feedback. We hypothesized that people would construct representations of stochastic events that drew on features of repeated events that were present at time of test. Naturally, we expected an effect of retention interval, that is, that increasing the number of intervening trials between presentations of identical choices would lead to increased forgetting and therefore reduced optimal choice behavior. More importantly, we planned to investigate the shape and magnitude of that retention effect. We also developed analyses of response patterns to compare rule-learning and reinforcement learning. By examining the effect of trial outcomes on choices made on trial repetitions, we sought to disentangle which choices were motivated by the principles suggested by rules and which were driven by the habits encouraged by outcomes.

The three conditions in this experiment were designed to examine how participants could remember and apply judgments of relative reinforcement probabilities after varying delays. The conditions differed in the size of the array of choices participants were asked to make. The varied number of choices between the three experiments allowed us to study the hypothesized effects over different cognitive load conditions. The game created for this paradigm pitted participants against a computerized opponent. In no instance could the participant make a choice that was guaranteed to beat the computer. However, the same rule applied on each occasion that a given hand appeared, so with perfect memory, choices could be made that would consistently win. The object of the game was to choose cards that were closest to a value - e.g., closest to seven or closest to king - that was associated with a given hand.

## METHODS

### *Participants*

Undergraduates at Tufts University participated to complete part of the course requirements for either Introductory Psychology or Statistics for the Behavior Sciences, and registered for the studies via an online interface. Forty participants were tested in each of three conditions that differed in terms of the number of cards in a hand (3,4, or 5 cards).

### *Design and Methods*

All experimental software was created and presented run using the Quickbasic programming language and implemented on a Dell microcomputer. The computer display was a 14" LCD Dell monitor. In each experimental condition, data were automatically outputted by the software developed for the task.

The following design procedure applies for each of the first three conditions. For Experiment 1, the design elements refer to the creation of three-card hands, for Experiment 2, four-card hands, and for Experiment 3, five-card hands. Hands of cards were also created using QuickBasic. A random number generator algorithm was used to create 1,000 unique hands of cards as if dealt from a 52-card deck. Hands were selected from the set that would be most suitable to the game. Since the game requires one card to be clearly optimal, hands with cards clustered around a single value were considered inappropriate. In the most extreme example, if there were four cards of the same rank – four kings, for example – in the four-card experiment, then we could not have designated an optimal choice. We chose 90 hands and applied a rule to each one. The rule associated with each hand indicated a target card. For example, "closest to 7" indicated 7 as the target. Rules were assigned to hands such that the target card was never presented in the hand, and that choosing the card that gave the best chance of beating the randomly choosing computer - the optimal choice - carried a relatively high chance of winning. Given the target card for each hand, we calculated the probability of each card in each hand beating a randomly-choosing computer. Take, for example, a hand made up of the King of Hearts, the Ten of Clubs, and the Five of Spades, and assume that the rule for this hand is "Closest to Ace wins." Here, the King of Hearts is clearly the optimal choice. If a player chose the King of Hearts, then there would be four cards remaining in the deck for the computer to choose that would result in a loss for the player: the four aces (The computer could tie by choosing any of the three remaining Kings, but those Kings would be replaced in the deck and the computer would choose again, rendering the original choice of a King irrelevant). Thus, the probability of winning given that a player chose the King is given by the ratio of the number of computer choices that would lead to a player win – here, 42 – to the total number of available (in the deck and not presented on-screen) and relevant (not the cards that would lead to a tie) computer choices – here, 46 – and is equal to 91.3%.

The rules for each hand were chosen such that the optimal choices carried a chance of winning between 70 and 90%. Further, the percent likelihood of winning with the next best choice was approximately 20% less than that of the optimal choice. The suit of the card had no effect on the outcome of a trial. In order to limit biases toward any specific rule or group of rules, we balanced the number of hands used with each rule. This resulted in each rule being represented by seven hands, except for "Closest to 8," which had six representatives (with 90 total hands, one rule had to have one fewer, and we chose that rule because it was the mean of the values in a suit).

Hands were arranged in a script that allowed for repetitions at the retention intervals to be studied, in all, 45 of the hands were repeated. Those hands that were repeated appeared four times in total. We examined five different retention intervals: 0 lag, 1 lag, 4 lag, 15 lag, and 30 lag. In the 0 lag condition, the same hand was repeated four times consecutively. In the 1 lag, there was one intervening trial, in the 4 lag, four intervening trials, and so on. As a basis for comparison, 45 the script also included 45 "filler" hands that were not repeated. Overall, there were 225 trials in the experiment.

Figure 1 provides a screenshot from a single trial of the game. Participants were shown an array of cards – the “hand” - on screen. The five cards at the top of the screenshot represent the hand for this trial. At the beginning of the trial, the Hole Card was represented as face-down (as a blue field surrounded by a white border). Underneath each card was an integer corresponding to the position of the card. The game asked participants to pick a card, which they did by pressing the number corresponding to their choice and pressing the “enter” key. Their choice was presented on screen next to "Your Card:" (in this case, the participant chose the second card: the five of spades). The Hole Card was then turned over on-screen (here, the Hole Card turned out to be the Jack of Hearts). The rule for the hand was then presented, participants were informed if they won or lost, and the running score of the game was posted. The computerized opponent randomly chose a card from the rest of the deck. On a given trial, the player - participant or computer - who chose the card closest to the target won, and received one point. If the computer chose a card that resulted in a tie, the program automatically chose another card (participants were aware of this and that they would never see a tie).

At the end of the game, participants were thanked, shown the final score of the game, and asked to fill out a four-question survey about their experience playing the game. These responses were not coded: feedback was used to inform us about the experimental paradigm and subsequent studies. Participants competed for prizes: a $10 gift card to a local bookstore to be given to the individual with the highest game score in each experimental condition.

## *Results and Discussion*

### *Optimal choice as a function of repetition.*

We analyzed choice behavior in terms of four conditional choice outcomes: wins given that a participant made the optimal choice, wins given suboptimal choice, losses given optimal choice, and losses given suboptimal choice. Figures 2, 3, and 4 are plots of optimal choices as a function of repetition and retention interval for the three-card, four-card, and five-card conditions, respectively.

Optimal choices for first presentations was averaged across fillers and all retention intervals. In both fillers and first presentations of to-be-repeated hands, participants had been given no information about the rule for the hand they were seeing at the time they made their choice. In the three-card condition, participants chose the optimal card at first presentation at levels that could be predicted by chance. In the four-card and the five-card conditions, participants chose the optimal card at rates significantly greater than what would be expected by chance [Condition 2: $t(39) = 6.49, p < .001$; Condition 3: $t(39) = 4.97, p < .001$). Table 1 compares these averages with the expected values. This discrepancy indicates that participants seek and in some cases can gain choice advantages from the features of the game itself – a form of strategy. As an example, consider the first presentation of a four-card hand consisting of the Two of Clubs, the Five of Spades, the Nine of Spades, and the Queen of Diamonds. Given that the rule for the hand cannot indicate a target card that is presented on screen, there are nine possible rules for that hand. It is reasonable to assume that each of these nine candidate rules is equally likely to be the actual rule (over the ninety hands, each of the thirteen possible rules occurred 7 times, except for "Closest to 7," which occurred 6 times). Of these nine rules, only "Closest to 3" would make the Two of Clubs the optimal choice, meaning that it would be the right choice one out of nine times. The rational player, therefore, could eliminate the Two of Clubs and choose among the remaining three cards.

Table 1

*Mean Proportion of Optimal Choice at First Presentation Across Game Types.*

| | Condition 1: 3 Cards | Condition 2: 4 Cards | Condition 3: 5 Cards |
|---|---|---|---|
| Mean proportion | .338 | .320* | .248* |
| 95% Confidence interval | (.312, .365) | (.304, .337) | (.225, .271) |
| Expected Proportion | .333 | .250 | .200 |

*Indicates significant departure from chance, *a* = .05

The overall averages for first presentations and fillers are denoted by horizontal dashed lines in Figures 3-5. This value can be thought of as a baseline responding level.

*Learning curves.*

Results of each of the first three experiments show similar patterns in learning. For Experiments 1, 2, and 3, the data show a dramatic dropoff in learning rate from the 0 lag retention interval to the 1 lag retention interval, indicating that even the most minute interruption between learning and test had devastating effects on the association between hand and rule. Participants displayed a steep learning curve with a classic negatively-accelerating pattern in the 0 lag condition, wherein there were no intervening trials between initial learning and test. For retention intervals of 1, 4, and 15 lag, we see nonzero increases in learning performance with repetition, but with a more linear shape. Learning in these conditions appears to be much more gradual. In the greatest retention interval, 30 lag, the curve is flat, indicating no significant increases in performance. Optimal responding data as a function of repetition were treated as learning curves. Table 2 shows parameter value estimates for learning curves constructed using the Weibull distribution (Sloboda & Chechile, 2009). This approach is based on hazard probability, and measures the likelihood of state change (in this case, from unlearned to learned) conditioned on the fact that that state change had not previously occurred. The Weibull distribution is given by the equation , where, in this case, *Pn* represents the proportion of optimal choices, *a* indicates initial knowledge state, and *c* is a learning-rate parameter.

Table 2

*Parameters for the Model* $P_n = 1 - e^{-an^c}$

| | | Model Parameters | |
|---|---|---|---|
| Game | Retention Interval | *A* | *C* |
| 3-card | 0 lag | 0.397 | 1.98 |
| | 1 lag | 0.385 | 0.771 |
| | 4 lag | 0.466 | 0.432 |
| | 15 lag | 0.377 | 0.183 |
| | 30 lag | 0.492 | 0.051 |
| 4-card | 0 lag | 0.369 | 1.858 |
| | 1 lag | 0.405 | 0.834 |
| | 4 lag | 0.299 | 0.668 |
| | 15 lag | 0.397 | 0.276 |
| | 30 lag | 0.418 | -0.034 |
| 5-card | 0 lag | 0.251 | 2.343 |
| | 1 lag | 0.323 | 0.976 |
| | 4 lag | 0.244 | 0.774 |
| | 15 lag | 0.323 | 0.185 |
| | 30 lag | 0.283 | 0.137 |

Across conditions, the learning-rate parameter *c* is greater than one for the 0 lag retention interval, indicating that the probability of learning increases with repetition. The shape parameter *c* is less than 1 for all retention intervals greater than 0. This is an indication that the hazard probability for learning is monotonically decreasing, that is, if the memory association is not formed right away, it is decreasingly likely that it will be made as the learning process continues. At the longest lag, the retention interval of 30 intervening trials, the *c* parameter is close to 0, indicating that learning is essentially negligible. These parameter values are in line with those found in analyses of more than two dozen learning studies using a variety of stimuli (e.g., words and letter triads; Sloboda & Chechile, 2009). As hypothesized, the learning rate parameter decreases in an almost uniform manner as retention interval increases. This analysis supports the behavioral results found in the study.

Figures 5, 6, and 7 demonstrate the relationship of values predicted by Weibull learning models with observed levels of optimal responding. Note the dramatic flattening of the curves for retention intervals greater than 0, that is, the deceleration of learning with any intervening trials.

*Rule learning vs. reinforcement learning.*

The paradigm allowed us to directly track choice behavior across repetitions of the same hand. This ability gave us a window onto what learning strategies may have informed choices. For example, if a participant chose a suboptimal card and won that trial and then chose the same suboptimal card the next time that same hand was repeated, we took that as evidence that reinforcement learning took place. Similarly, if a participant chose an optimal card and lost but chose the optimal card again on the next repetition of that hand, we took that as evidence that the participant learned the rule and was choosing according to it. Figure 8 depicts a decision tree representing the possible combinations of outcomes in successive presentations, along with interpretations of the learning strategies likely to lead to each combination. Each number on the decision tree represents a frequency cell.

For example, cell 1 indicates that a participant chose the optimal card and won on the first iteration of a hand, and then chose the optimal card again on the second iteration and won again. Cell 2 indicates that a participant chose the optimal card and won on the first iteration of a hand, but chose a different card when the hand was repeated - and won anyway. Cell 3 indicates that a participant chose the optimal card on both the first and second iterations of a hand and won the first time but lost the second time. Other outcome combinations are represented on the tree diagram. Certain combinations cannot be explained by learning strategy, such as following a win with an optimal choice with a loss with a suboptimal choice. This outcome pair we attributed to choosing by chance: it is unlikely in these cases that either rule or previously outcome had been properly stored or was able to be retrieved. In our analysis, we used the frequency of this outcome pair as a correction for random choosing. We also monitored consistency, that is, the tendency of a participant to choose a particular card on repeated presentations regardless of rule or reinforcement. This was also factored into our corrections.

With this information, we constructed contrasts to compare rule-based learning to reinforcement learning. Contrasts were calculated by adding the frequencies of choices that represented either rule- or reinforcement- based behavior, and then subtracting the frequencies of choices attributable to other pure strategies. The resulting differences were divided by the total number of choices made. For the rule contrast, we summed the cells that indicated rule-learning (simply, those that showed optimal choice on the second iteration regardless of the outcome of the first) and subtracted cells that indicated pure reinforcement, pure consistency, or pure random chance. Thus, the frequencies in cells 1, 3, 5, 7, 11, 13, 15, and 17 were summed, and the total frequencies of the cells 2, 4, 6, 7, 9, 10, 17, and 20 were subtracted from this total. For the reinforcement contrast, we summed the cells that indicated reinforcement learning, that is, those that showed repetition of a choice that led to a wins regardless of the optimal choice. For this contrast, the frequencies in cells 1, 3, 7, and 10 were summed, and other frequencies in cells 11, 13, 17, and 20 were subtracted. Contrasts could have been constructed for strategies such as consistency - a tendency to pick the same card regardless or rule or outcome, or chance - choices that are explained by no other strategy - but these values were generally small and not germane to the current analysis.

Contrast values took on values between -1 and 1. Larger values indicate that a learning strategy dominates others, while negative values indicate that other learning strategies are dominant. Figures 9, 10, and 11 display the contrasts between rule-learning and reinforcement for the transition between the first and second presentation of each hand as a function of retention interval. We examined individual contrast scores to check for irregularities that might indicate that some participants might have been better rule learners, or perhaps that others might make more powerful associations following reinforcement, but we found individual contrast scores to be scattered around the overall mean contrast scores without any discernible pattern.

These contrasts indicate that rule-based learning was the primary motivation for choice in the 0 lag retention interval condition, in which rules were presented immediately before the relevant trial. Rule learning also appears to play a role in other short-retention intervals, for example, participants appear to be using rules after a lag of 1 in the 4-card game. With more than one intervening trial, that is, in conditions with retention intervals greater than 1, rule-based learning was dominated by other learning strategies. At longer lags, available evidence suggests that rule-learning does not occur for this task at long lags. Reinforcement learning, on the other hand, was robust to increases in retention interval for early iterations: when comparing the second iteration to the first, the proportion of reinforcement-based choices was significantly greater than 0 at the *a* = .05 level for each interval.

The relative contributions of rules and reinforcement to learning are more difficult to separate beyond the first pair of iterations of each hand. A choice on the fourth iteration of a hand, for example, could have been influenced by a number of combinations of reinforcing events on the three preceding iterations. To learn about the effect of repeated reinforcement on choices, we looked at instances where individuals won by making the same choice on each of the first three instantiations of a given hand. Just as we examined the proportion of optimal choices made on the fourth iteration after the same rule had been presented three times, we can examine the proportion of repeated choices after the same outcome had occurred three times.

Table 3 shows the overall number of instances in which participants chose the same card on each the first three iterations of a given hand and the frequency with which they chose the same card again on the fourth iteration for each of the three conditions. These frequencies are further divided into instances where the repeated choice was of an optimal card and where the repeated choice was of a suboptimal card.

*Table 3*. Proportions of repetitions of three-time reinforced choices

| | All Instances | | | Optimal Choices | | | Suboptimal Choices | | |
|---|---|---|---|---|---|---|---|---|---|
| **3-Card Condition** | | | | | | | | | |
| | *n* | Prop. | *p* | *n* | Prop. | *P* | *n* | Prop. | *p* |
| 0 Lag | 104 | 0.913 | < .001 | 90 | 0.956 | < .001 | 14 | 0.643 | < .001 |
| 1 Lag | 76 | 0.763 | < .001 | 60 | 0.85 | < .001 | 16 | 0.438 | .127 |
| 4 Lag | 75 | 0.587 | < .001 | 51 | 0.765 | < .001 | 24 | 0.375 | .254 |
| 15 Lag | 67 | 0.448 | .018 | 37 | 0.568 | < .001 | 30 | 0.567 | .002 |
| 30 Lag | 66 | 0.424 | .047 | 38 | 0.553 | .002 | 28 | 0.500 | .022 |
| **4-Card Condition** | | | | | | | | | |
| | All Instances | | | Optimal Choices | | | Suboptimal Choices | | |
| | *n* | Prop. | *p* | *n* | Prop. | *P* | *n* | Prop. | *p* |
| 0 Lag | 111 | 0.991 | < .001 | 108 | 1.0 | < .001 | 3 | 0.667 | .016 |
| 1 Lag | 75 | 0.840 | < .001 | 65 | 0.908 | < .001 | 10 | 0.5 | .020 |
| 4 Lag | 76 | 0.724 | < .001 | 60 | 0.767 | < .001 | 16 | 0.25 | .370 |
| 15 Lag | 54 | 0.593 | < .001 | 36 | 0.778 | < .001 | 18 | 0.389 | .057 |
| 30 Lag | 51 | 0.333 | .066 | 27 | 0.407 | .022 | 24 | 0.5 | .002 |
| **5-Card Condition** | | | | | | | | | |
| | All Instances | | | Optimal Choices | | | Suboptimal Choices | | |
| | *n* | Prop. | *p* | *n* | Prop. | *P* | *n* | Prop. | *p* |
| 0 Lag | 81 | 0.926 | < .001 | 69 | 0.986 | < .001 | 12 | 0.583 | < .001 |
| 1 Lag | 63 | 0.603 | < .001 | 41 | 0.780 | < .001 | 22 | 0.409 | .006 |
| 4 Lag | 37 | 0.459 | < .001 | 14 | 0.857 | < .001 | 23 | 0.478 | < .001 |
| 15 Lag | 47 | 0.468 | < .001 | 33 | 0.576 | < .001 | 14 | 0.500 | .002 |
| 30 Lag | 62 | 0.326 | .007 | 25 | 0.600 | < .001 | 37 | 0.486 | < .001 |

Participants who are reinforced three consecutive times are overwhelmingly likely to choose the same card a fourth time. In all possible scenarios but one (30

Lag in the 4-Card condition), participants chose the same card a fourth time at rates significantly greater than would be predicted by chance. Particularly powerful is the combined effect of rule-based learning and reinforcement learning: participants were significantly likely to make the same choice in every possible situation. Since wins are less likely following suboptimal choices, there were relatively few instances where individuals were reinforced three consecutive times for cards not preferred by rules. However, even with this relatively small sample there is evidence from conditions that reinforced suboptimal choices are repeated even at the longest lags.

We may conclude from these results that rule-learning by itself is fragile and is eroded significantly by intervening trials. There is little evidence of pure rule-learning at long lags. What does seem to be retained - and what may be explaining increases in optimal behavior across iterations with longer retention intervals - is a single, reinforced card choice in a given card context.

As noted earlier, in Experiments 2 and 3, the mean proportion of optimal choices on the first iterations of hands was significantly above chance (see Table 1). This, along with subjective reports from participants about their game-play experience via the post-experimental surveys, indicated that information about optimal choice could be gleaned from features of the options themselves. As noted earlier, the optimal choice was unlikely to come from a cluster of values (in Experiment 1, where the baseline optimal choice rate was not significantly different from chance, clusters were far less likely because of the added space afforded by presenting just three cards at a time), a pattern likely to be noticed. This phenomenon led us to question how much of performance was due to the development of general strategies versus episodic memory. Further, increases in retention interval had systematic and devastating effects on optimal performance. Taken together, these questions led us to explore component memory processes involved in game play. We considered the possibility that remembering hands and their outcomes may be extraordinarily different because of the demands of remembering and distinguishing combinations of cards. We addressed this question with three more experiments that comprised Experiment 2.

## EXPERIMENT 2

The results from the first set of experiments showed monotonically decreasing learning rates with increases in retention interval. Predictably, learning was rapid when identical trials were presented in immediate succession. For shorter retention intervals, learning was shown to be slow, and more likely a function of reinforcement than the declarative rule-learning that would lead to more optimal choicemaking. For our longest lag, no learning was demonstrated. One possibility for this apparent lack of explicit learning regarded the stimuli themselves. It was hypothesized that the ability of participants to remember randomly generated combinations of cards presented on screen could have been a limiting factor in memory for the stochastic information associated with those combinations. In order to test whether participants were capable of accurately recognizing card combinations after the delays used in the game, we developed a direct memory task that mirrored the presentation and timing of the game task. In response to questions of difficulty arising from the continuous nature of the task and the number of items required to be held in memory, we sought to demonstrate that with other, previously examined stimuli, participants could perform well on even a seemingly difficult memory task. Thus, the fourth condition in Experiment 2 arranged pictures from the Snodgrass-Vanderwart (1980) set into subsets of 13 categories in an analogue to the 13 types of each card found in a deck of playing cards. We numbered the pictures so as to correspond to the numbers we assigned to each playing card for coding purposes, and then inserted the line-drawings into the continuous yes-no recognition paradigm used elsewhere in Study 2. Thus, participants in Study 2 saw a pattern of repeated trials that was identical to those seen in the 4-card game in Study 1 and in the 4-card recognition task in Study 2.

### *Participants*

In the Experiment 2, participants were recruited via the same means and from the same pool as in Experiment 1. Participants in Condition 1 (*n* = 26) saw hands of three cards, participants in Condition 2 (*n* = 26) saw four cards, and participants in Condition 3(*n* = 26) saw five cards.

### *Design and Methods*

Experiment 2 was written with the same software and presented with the same hardware as Experiment 1. Each condition in Experiment 2 corresponded to one of in Experiment 1. The scripts were altered slightly: in the second set of experiments, three non-coded hands were added to the beginning of the script. These were added because in the recognition paradigm, any participant should be able to correctly identify the first few hands as not previously presented. Otherwise, participants in the second set of experiments saw the same hands in the same order as in the first set. Therefore, participants in these three experiments saw 228 total trials, 225 of which were coded for analysis. As before, behavioral data was collected automatically by the experimental software.

In each trial, hands of cards were presented in exactly the same way they were in the first set of experiments. However, in place of the gameplay instructions, participants were given a yes/no recognition task. Figure 13 provides a screenshot. Underneath the hand, the question "Have you seen this hand before Y/N?" was presented. After entering a response, participants were asked to rate their confidence in their answer on a three-point scale. These data have not been analyzed, but help to keep the timing of the second set of experiments close to the timing of the first. In this case, the time it took participants to enter the confidence rating on each trial was designed to mirror the time it took participants to read the rule associated with each trial in the first set of experiments. Upon completion, participants were informed of their accuracy score (correct rejections plus hits, divided by total trials and multiplied by 100) and thanked for their participation (surveys were not given for this set of experiments). Participants competed for prizes of $10 gift cards to a local bookstore to be given to the individual with the highest accuracy score in each experimental group.

### *Results and Discussion*

The retention-interval effects for the association between hands and rules found in the game paradigm led us to further investigate memory for elements of the game. To this end, we measured memory for hands themselves in the memory task. Figures 14, 15, & 16 present hit rates as a function of retention interval for the three-card, four-card, and five-card yes-no recognition task, respectively. False alarm rates are included in the appendix.

The data show similar retention-interval effects across different stimulus-set sizes. In each stimulus-set group, participants recognized hands presented three times earlier at a rate significantly greater than chance after retention intervals of 0, 1, and 4 lag [$t(26) = 2.58, p < .05$]. In the 3-card and 4-card conditions, participants recognized hands following lags of 15 [$t(26) = 2.58, p < .05$]. Additionally, participants in the 3-card [$t(26) = 1.73, p < .05$] and in the 5-card [$t(26) = 2.58, p < .05$] memory task were able to recognize hands after a retention interval of 30 lag significantly more frequently than would be predicted by chance [$t(26 = 1.73, p < .01$].

As expected, randomized block ANOVA's show main effects for retention interval and presentation number for each of the three experiments [3-card: retention interval $F$ (4, 350) = 88.76, $p$ <.0001, presentation $F$(2, 350) = 37.03, $p$ <.0001, $p$ = .0334; 4-card: retention interval $F$ (4,350) = 142.55, $p$ <.0001, presentation $F$(2,350) = 33.58, $p$ <.0001; 5-card: retention interval $F$(4,350) = 95.98, $p$ <.0001, presentation $F$(2,350) = 28.34, $p$ <.0001]. For the 3-card and the 4-card conditions (but not the 5-card), there was also an interaction between retention interval and presentation that probably stem from ceiling effects in the 0 lag condition. (3-card: interaction $F$(8,350) = 2.12, $p$ = .0334; 4-card: interaction $F$(8,350) = 4.16, $p$ < .0001)

The results from Experiment 2 suggest that individuals can remember combinations of cards immediately, at short retention intervals, and often at intermediate and long retention intervals. Memory for card hands shows retention-interval effects similar but somewhat less pronounced than memory for optimal choices involving those card hands in the game paradigm. Participants demonstrated even greater recall in Condition 4 of Experiment 2. Figure 17 shows mean hit percentage across participants listed by lag condition and by repetition number.

In Condition 4, participants reached 100% accuracy on the fourth presentation in the 0 Lag condition, and responding was significantly above chance for every repeated item in every condition (for the lowest hit rate, the first presentation at 30 lag, $t(26) = 2.48, p < .01$). A randomized block ANOVA shows main effects for retention interval ($F(4,627) = 33.48, p < .0001$) and presentation number ($F(3,627) = 35.55, p < .001$), two unsurprising effects. It also reveals an interaction between retention interval and presentation ($F(12,627) = 5.27, p < .0001$), which, as before, likely arises from ceiling effects in the 0 lag condition.

Taken together, the results of the four conditions in Experiment 2 indicate that it is not inherently difficult to remember combinations of items, even when there are conceptual similarities between the items. In fact, memory for combinations of items appears to be remarkably good. Using playing cards, with their even higher similarity levels, makes the task more difficult, but still possible at retention intervals longer than one. In light of these results, it appears that poor memory for probability judgments in Experiment 1 accounted for much of the difficulty in game performance and optimal choice.

## EXPERIMENT 3

Experiments 1 and 2 provided indirect measures of memory for probability. To be successful, participants playing the card game needed to compute relative probabilities in order to compare their options and remember these probabilities after a delay and a varying number of intervening trials. This experiment directly measures memory for probability via a paradigm based on the Brown-Peterson paradigm (Brown, 1958; Peterson & Peterson, 1959).

In each trial, participants viewed a number of monochromatic circles – meant to represent poker chips – appearing to hover over a can. There were two different colors of chips in each trial. The chip remained above the can for a period of one second before appearing to fall into the can. In the each of the five retention interval conditions – 1, 4, 15, 30, and 60 seconds, respectively – participants performed a shadowing task as a distractor. In this shadowing task, participants will hear a series of letters read aloud (presented over headphones) and were asked to repeat them aloud as they were heard. Letters were presented at a rate of two letters per second.

Following the task, participants were asked to identify the probability of choosing a chip of a given color. Participants saw percentages from 5% to 95% displayed on the screen and were asked to click on the percentage that reflects their judgment. In all, there were 50 coded trials – 10 per retention interval– using a balanced number of candidate probabilities (we also included two "practice" trials that were not coded at the beginning of the experiment to ensure that participants were familiar with the task and had opportunities to ask questions before the first coded trial).

This study introduced a novel paradigm in order to examine two primary hypotheses. First, we expected subjective probability judgments to mirror the departures from objective probability measures that have been previously demonstrated and described by Prelec (1998). As previous research in probability judgments has shown that individuals overweight small probabilities and underweight large probabilities in uncertain conditions when making choices with real (e.g., Hertwig et al, 2004; Stott, 2006 ) or imagined (e.g., Tversky & Kahneman, 1992) consequences for utility, we predicted that individuals would follow the same pattern when estimating probabilities based on an easily viewed sample space. We further hypothesized that this pattern would be increasingly exaggerated with increased time between the presentation of the sample space and the elicitation of the judgment; that is, that individuals would show more pronounced misjudgment of probabilities the longer they had to wait to make their judgments.

### *Participants*

Participants in this study (*n* = 41) were recruited via the same means as in studies 1 and 2. All participants viewed the same experimental materials, and again participated in order to fulfill part of the course requirements for either Introductory Psychology or Statistics for the Behavioral Sciences.

### *Design and Methods*

This paradigm was designed using e-Prime software and was presented via a computer monitor interface and headphones which were worn by the participants throughout the experiment. After giving informed consent, participants were presented with the instructions on the monitor. Participants were told that they were to complete two tasks: to judge the probability of choosing a poker chip of a given color from a can and to repeat to the best of their ability the letters that they would hear over the headphones.

In each trial, participants would first see a representation of 40 poker chips hovering above a gray can. Two different colors of chips would be presented at a time (Figure 18 is a screencap from a trial in which 36 red chips and 4 blue chips were represented). After one second, the chips were animated to fall into the can.

Once all of the chips were removed from sight, an instruction was given on the screen to repeat aloud letters as they were spoken. The letters as presented over the headphones were spoken in a male voice at a rate of two letters a second. This distractor task lasted for one, four, 15, 30, or 60 seconds (corresponding to two, eight, 30, 60, or 120 letters, respectively) depending on the condition being assessed by a given trial. The scripts for the distractor task were generated with a random letter generating program written in QuickBasic. None of the scripts used featured spellings of recognizable English words. Each participant complied with the distractor task.

Following the distractor task, participants were asked to judge the probability of pulling a certain color of chip from the can based on the last event they had viewed. Participants were allowed to choose probability values between .05 and .95 in steps of .05, and were instructed to do so by clicking the appropriate box, as depicted in Figure 19. Participants were given a two-minute break after approximately half of the trials were completed. In all, there were 57 trials, with each objective probability represented three times.

*Results and Discussion*

Given the performance of participants in the first two studies, memory for probabilities with clearly presented sample spaces was surprisingly accurate. Figure 20 plots median probability estimates from all participants across retention-interval conditions. The median probability estimates show a remarkably high linear correlation with the presented probabilities ($R^2 = .9966$).

In fact, shifts in objective probability account for greater than 96% of the variance in all five temporal conditions when compared to mean probability estimates, as demonstrated in Figures 21-25. The smallest of these correlations occurs in the sixty-second condition ($R^2 = .9695$); the highest correlation occurs in the four-second condition ($R^2 = .993$). Subjective probability in this task correlates linearly with objective probability, and the regression lines based on median responses intercept the y-axis near or just below the origin. The intercepts for the five conditions are -2.40, -1.20, -1,05, -1.14, and 1.473, respectively. Thus, median responses do not indicate the hypothesized overweighting of small probabilities and underweighting of large probabilities. We did not see the anticipated curvilinear pattern in the data, and analyzing medians collapsed across conditions yielded both a strong linear correlation and a near-zero intercept (-0.483). Follow-up studies may further explore the relationship between objective and subjective probability to see if the flattening of the regression line is an artifact of fluctuations in estimation, or, if a consistent curvilinear pattern could be uncovered by a finer-grained task (i.e., asking for probability judgments in steps smaller than 0.05).

Generally, our participants, given a perception-based task, appeared to be accurate judges of probability. Future investigations of the effect of retention interval on memory for probability should examine longer retention intervals. As currently constituted, the nature of the task suggests that fatigue effects may arise with longer intervals, thus, subsequent experiments should consider different distractor tasks that would better lend themselves to greater duration. This study thus presents evidence that individuals are extraordinarily good at estimating probabilities, so long as the sample spaces associated with events are presented visually.

## GENERAL DISCUSSION

These studies suggest a complex relationship between memory and probability judgments. On the one hand, individuals show apparent difficulty encoding relative probability judgments and applying them to decisions. On the other, individuals do appear to be remarkably accurate in formulating probability judgments based on remembered (and presumably unrehearsed) perceptual information.

Participants in Experiment 1 consistently used features of the task that were present at time of test to assist in their decision-making. Optimal responding levels were significantly above chance in all forms of the game and for all retention intervals. Moreover, optimal responding was significantly above chance even before any kind of feedback or rule information was offered. This rate was still far less than could be achieved with perfect memory. In a task where this rate were higher, though, we may surmise that baseline responding might represent a "good-enough" strategy, and possibly interfere with memory for probabilistic information by offering a computationally easy and less cognitively demanding alternative.

Following a framework established by Sloboda and Chechile (2009), we analyzed learning curves for optimal choices using a two-parameter model based on hazard probability. When relevant feedback was given immediately before a trial – that is, in the 0 lag retention interval – learning was rapid and accelerated with repetition. The model showed that the probability of learning – in hazard terminology, a "state change" – increased as identical trials were presented in succession. In short and intermediate retention interval conditions, the shape parameter of the curve dipped below one, indicating a monotonically decreasing probability of state change. In other words, if learning did not occur on the first trial, it was decreasingly likely to occur on successive trials. Finally, in the longest retention interval condition, the probability of state change was nearly 0 on all repetitions, an indication of the difficulty of maintaining an association between a stochastic event and relevant rule instruction over a sustained delay.

Our paradigm allowed us to track choice behavior across trial repetition. In this way, we could see how outcomes from trials affected selections made on their respective repetitions. Taking this information, we constructed contrasts to reveal which learning strategy – rule-learning or reinforcement learning – was more valuable in predicting choice. From these contrasts, it appears that rules are applied in choice only when the interval between learning and application is short, implying that instruction about the features of a stochastic event may be useful as a predictor only when distraction is minimal. However, reinforcement learning was robust with regard to increasing retention interval for all versions of our task. Indeed, even at the longest retention interval, the reinforcement contrast was significantly positive, indicating that reinforcement had a significant effect on choice. Repeated reinforcement proved to be especially effective. Participants who were reinforced three times for a given choice were quite likely to make that choice a fourth time. This effect was particularly strong when these choices were optimal choices, indicating that rule learning, though fragile, is much stronger when bolstered by reinforcement.

Strength of stimulus recognition, as predicted, appears to be related to optimal choice behavior. In direct memory tests, participants generally showed recognition memory above chance for repeated hands at retention intervals of 0, 1, and 4, and in some cases 15, and 30. Thus, optimal responding tended to degrade at a rate faster than simple recognition. In order to make optimal choices at a rate that exceeds that which is possible simply by analyzing features of the task at time of test, it appears that more than the hands themselves must be remembered. This may appear an obvious conclusion, but recall that evidence from the rule/reinforcement contrasts indicated that, at all retention intervals and for all stimuli sizes, participants chose based on reinforcement, demonstrating memory for elements - but not the entireties - of stimulus sets. Because optimal choices bore a relatively high rate of reinforcement, many choices made by participants could be attributable to either strategy.

These results reveal interesting features of the role of memory in decision-making. Although remembering hands helped improve optimal choice, at no retention interval did participants choose as optimally as they did in the 0 lag condition, in which relevant information about optimal choices were presented immediately before the choice. That relatively long lags between encoding information about a decision trial and its repetition leads to consistent forgetting is unsurprising.  However, even the smallest amount of distraction we could create in this paradigm - a single intervening trial - led to significant dropoffs in the use of rules regarding stochastic events. Mnemonic associations made between features of a choice and the rule related to that choice appear to be fragile. Whether this fragility is a product of insufficient encoding or retrieval remains a question to be investigated in future experiments.

More direct tests of the relative probability judgments implicit in the task may be necessary in order to assess how participants reformulate their choices when given relevant information about optimal decision-making. It was assumed that participants, when given rules, would review the presented options and calculate which choice would yield the highest probability of reinforcement over time. Evidence from the study, particularly in the 0 lag retention interval, suggests that this process may not always take place. Participants may instead pay greater attention to the outcome of the trial, or employ a strategy they find sufficient based on the features of the task. Future experiments will likely explicitly investigate those judgments. It is hypothesized that the act of asking about those judgments will increase attention to the reformulation of relative probabilities and likely strengthen the associations between options and rules and increase proportions of optimal choosing. Further, it may be informative to alter the paradigm in such a way as to remove those features of the task that allowed a baseline level of optimal choosing that was significantly above chance. As the game was played against a randomly-selecting opponent, to choose a card near the center of the distribution of cards between the 2 and the Ace, that is, close to 8, was to make a safer choice than to choose a more extreme card. Modifications of the paradigm to remove such features, for example, choosing from proportions obscured as marbles in a series of opaque urns, may provide a clearer picture of the association between stochastic events and the probability judgments made about them.

Experiments 1 and 2 were designed in order to explore memory for probability and the effects of this memory on decision-making. Previous studies of how memory influences probability judgments have indicated that win-loss contingencies can, over time, create at least an implicit understanding of relative probabilities (e.g., Bechara, et al., 1997; Knowlton, Mangels, & Squire, 1996). The results of the first study in this package indicate that, given all the information necessary to identify an optimal option in an array during a stochastic event, individuals can remember and apply rules instead of reinforced choices. However, this ability is short-lived, and the habit-learning processes explored in earlier studies take over after relatively brief intervals.

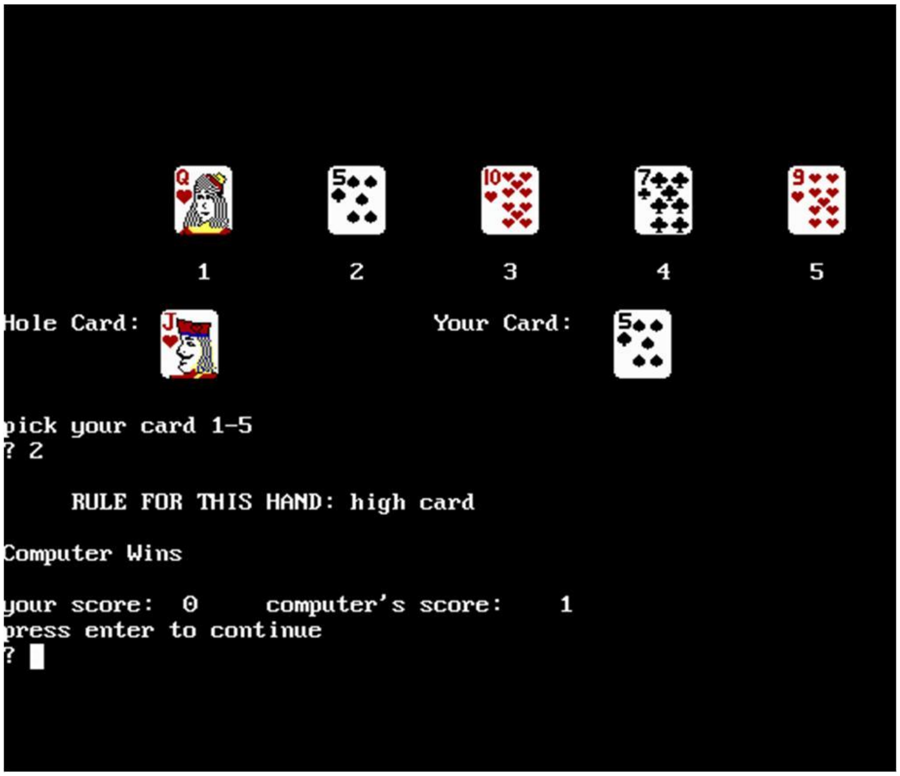
In Experiment 1, we also observed the spontaneous development of basic strategy. Participants consistently used features of the task that were present at time of test to assist in their decision-making. Optimal responding levels were significantly above chance in all forms of the game and for all retention intervals. Moreover, optimal responding was significantly above chance even before any kind of feedback or rule information was offered. This rate was still far less than could be achieved with perfect memory. In a task where this rate were higher, though, we may surmise that baseline responding might represent a "good-enough" strategy, and possibly interfere with memory for probabilistic information by offering a computationally easy and less cognitively demanding alternative.

Finally, Experiment 3 demonstrated conditions under which individuals are particularly adept with probabilities. When the full sample spaces of stochastic events were presented visually, individuals demonstrated remarkable accuracy in probability estimation, even as they had to rely on memory without opportunity for rehearsal. The working hypothesis for this study – that probability judgments reliant on memory would demonstrate a curvilinear relationship with objective probability – was inspired by previous research that showed similar patterns for decisions made under uncertainty (e.g., Hertwig et al, 2004; Stott, 2006; Tversky & Kahneman, 1992). Presumably, the difficulty in making risky decisions and the difficulty in relying on memory would result in analogous judgments. Based on the results of Experiment 3, however, it appears that the processes involved in the two tasks are qualitatively distinct. It may be informative to deploy the paradigm in a risky context in order to allow a direct comparison of probability judgments and, further, to investigate a possible interaction when risky judgments rely on memory for probability.

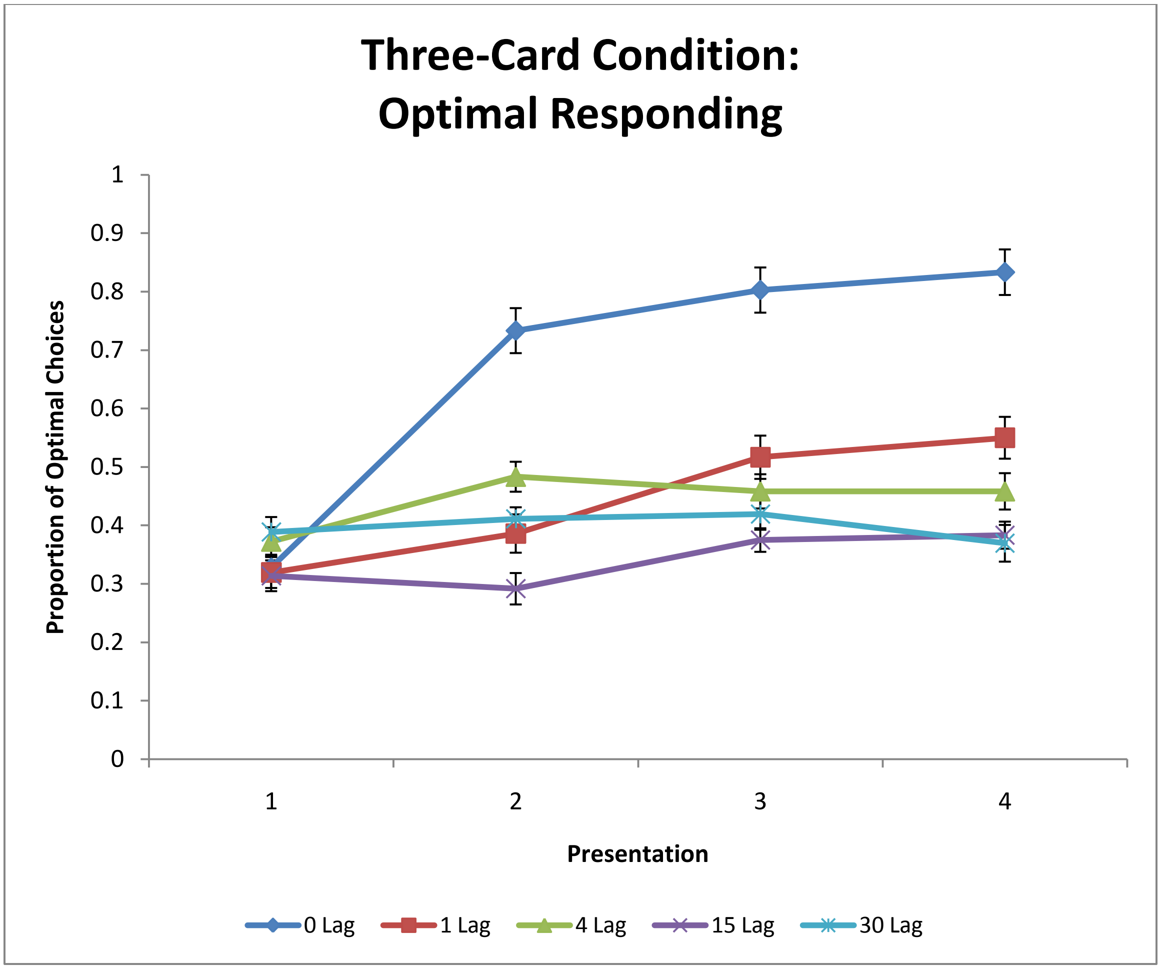
Experiment 3 used a Brown-Peterson-type paradigm (Brown, 1958; Peterson & Peterson, 1959). The distractor task required participants to repeat letters as they heard them. Our immediate follow-up study will use spoken-aloud numbers instead to maximize interference with the memory task and minimize the possibility for participant rehearsal. A second planned follow-up study will replace the Brown-Peterson-type paradigm with a continuous recall paradigm that is similar in some respects to that used in Experiment 1 and 2. In this planned experiment, chips will be animated to fall into marked cans, and the relative proportions of chips in each can will remain the same across trials. For example, a participant will be asked about the probability of retrieving a blue chip from can 1, will see chips fall into cans 2 and 3, and then be asked again the probability of retrieving a blue chip from can 1 (the answer, normatively speaking, should be the same).

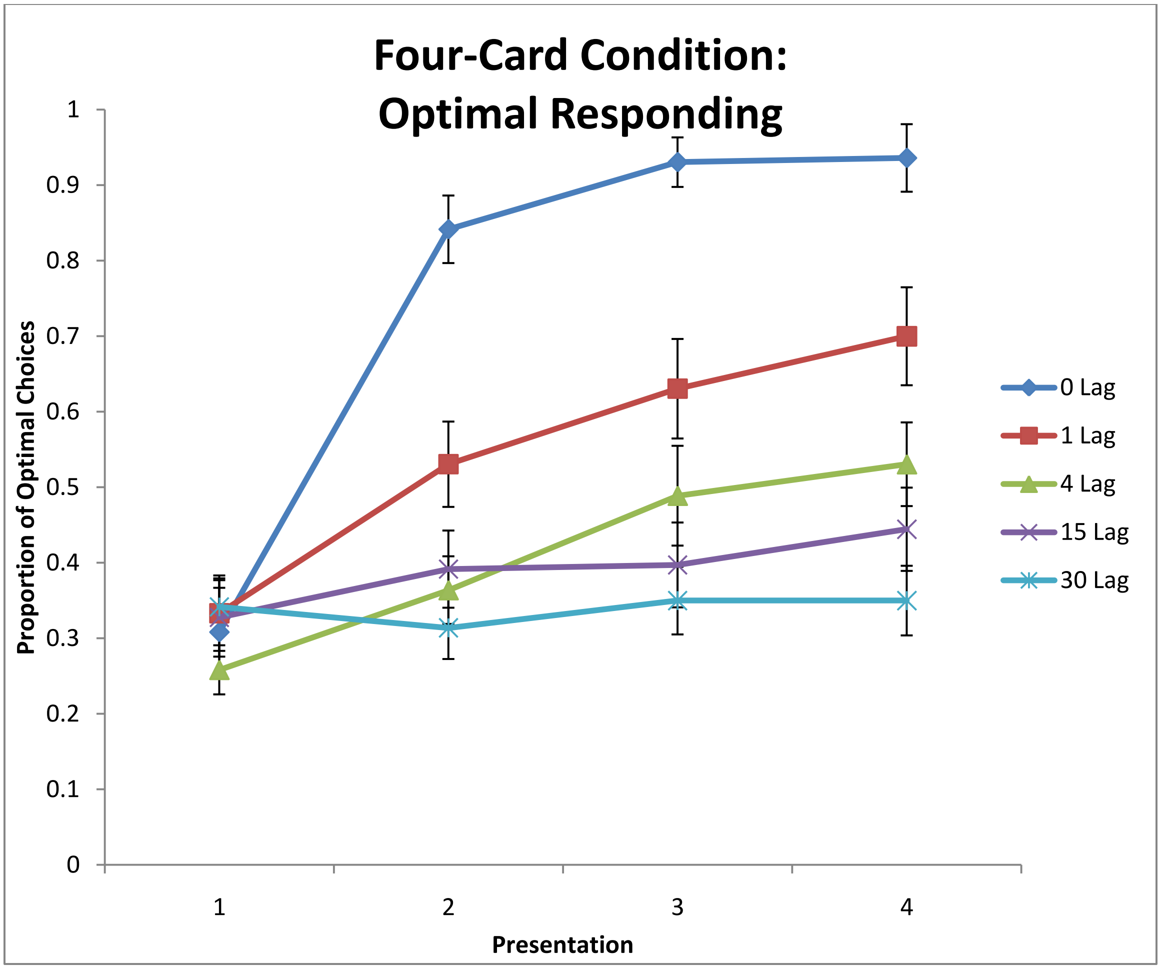
Taken together, the results of these three experiments present a complex picture of the ability to process, remember, and apply probabilities. Given a complex task that involves processing of relative probabilities and reinforcement of possibly suboptimal selections, optimal performance tends to drop sharply with increased retention interval, an effect thrown into stark relief by the ability of individuals to remember combinations of items at the same intervals in a non-competitive setting. On the other hand, when stochastic events are presented in such a way that the full sample space is presented visually, individuals are remarkably accurate in their estimates of relative probability, even after delays during which they are unable to rehearse the information. Further investigation of the disparity between and the interaction between these two abilities is warranted: such work may contribute to a better understanding of the relative strengths and weaknesses of individuals with regard to probabilistic reasoning and thus how to improve decision-making.

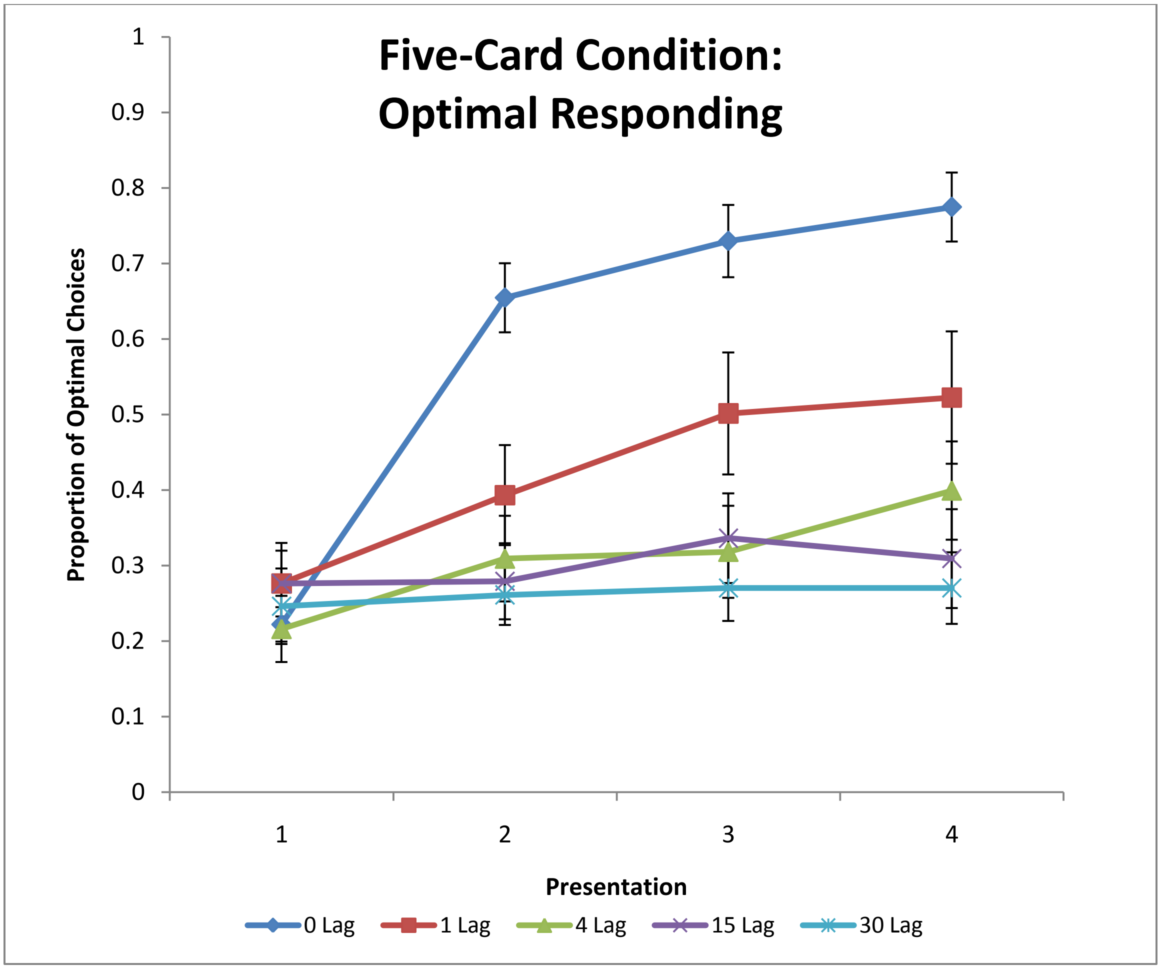
# FIGURES

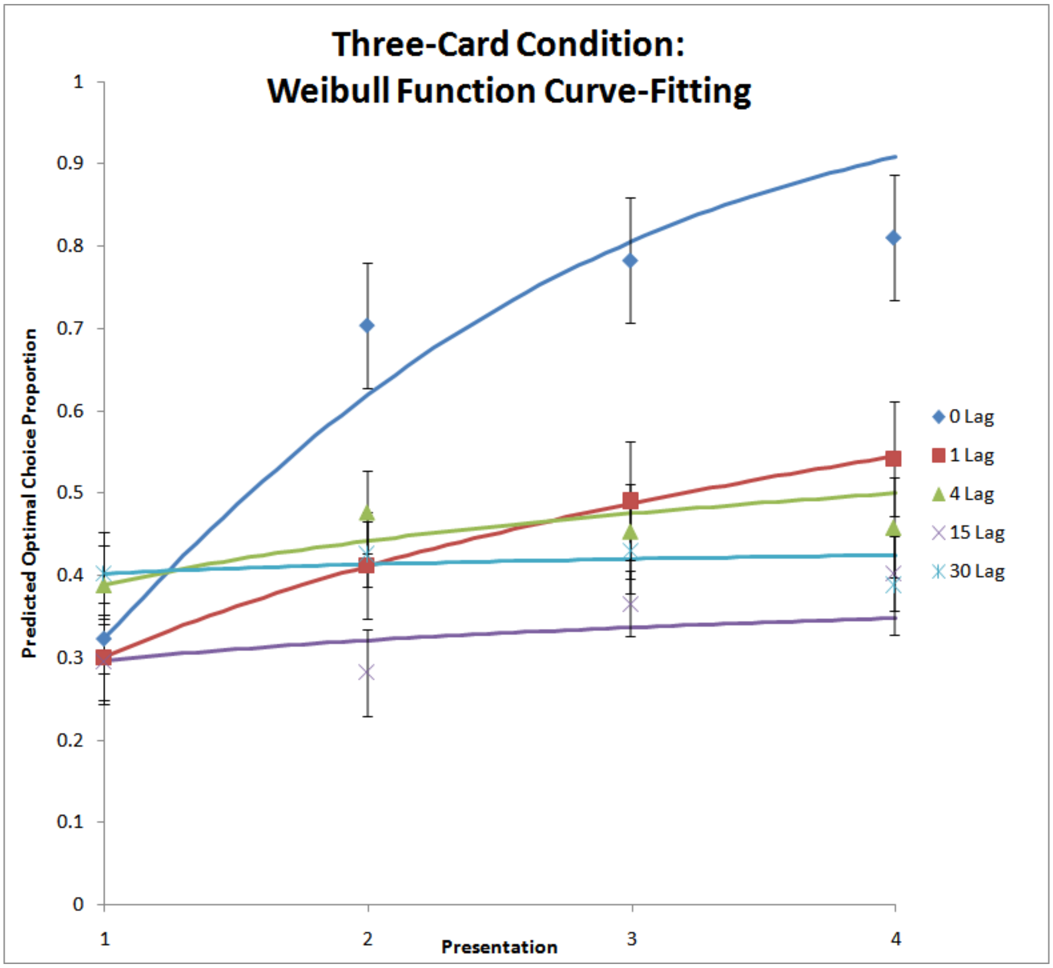


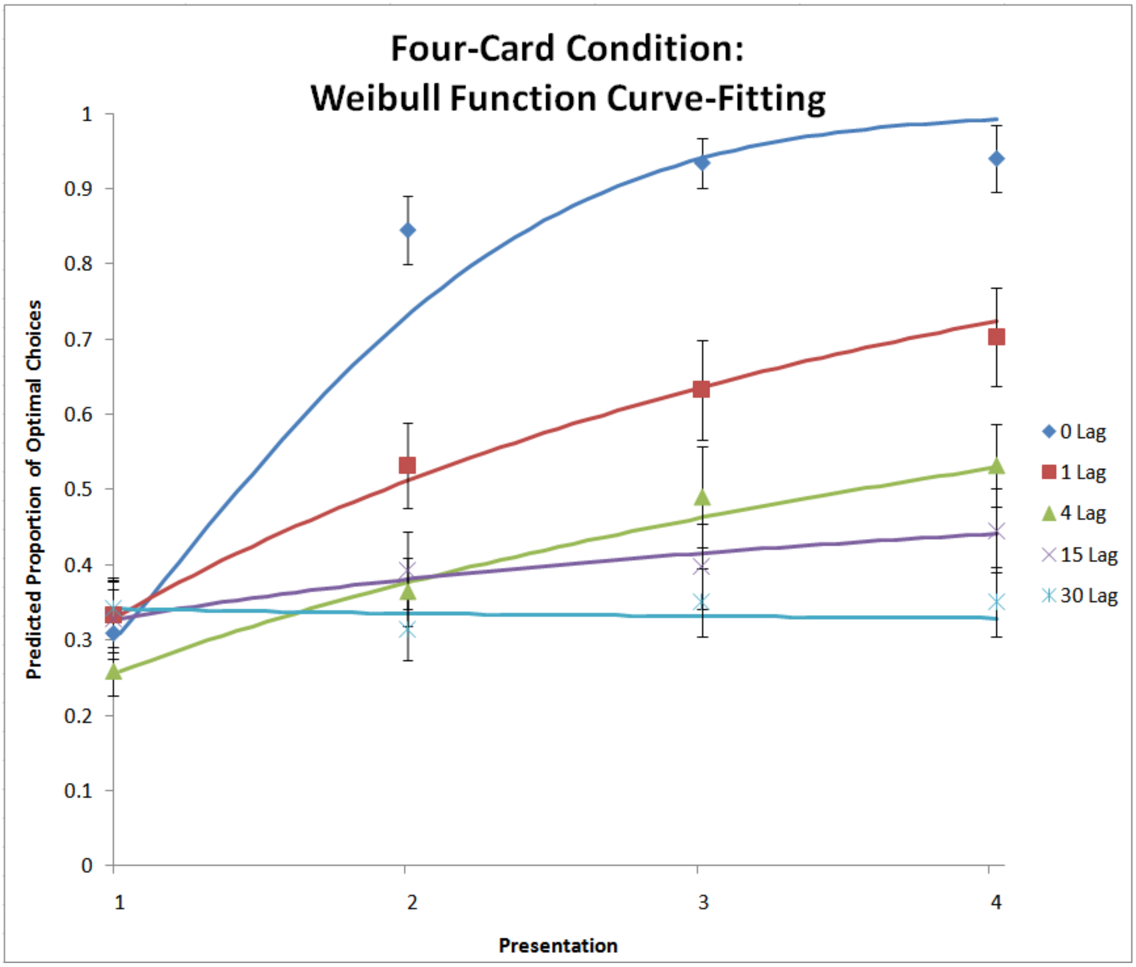
```
Hole Card:                 Your Card:

pick your card 1-5
? 2

     RULE FOR THIS HAND: high card

Computer Wins

your score:  0     computer's score:    1
press enter to continue
? 
```

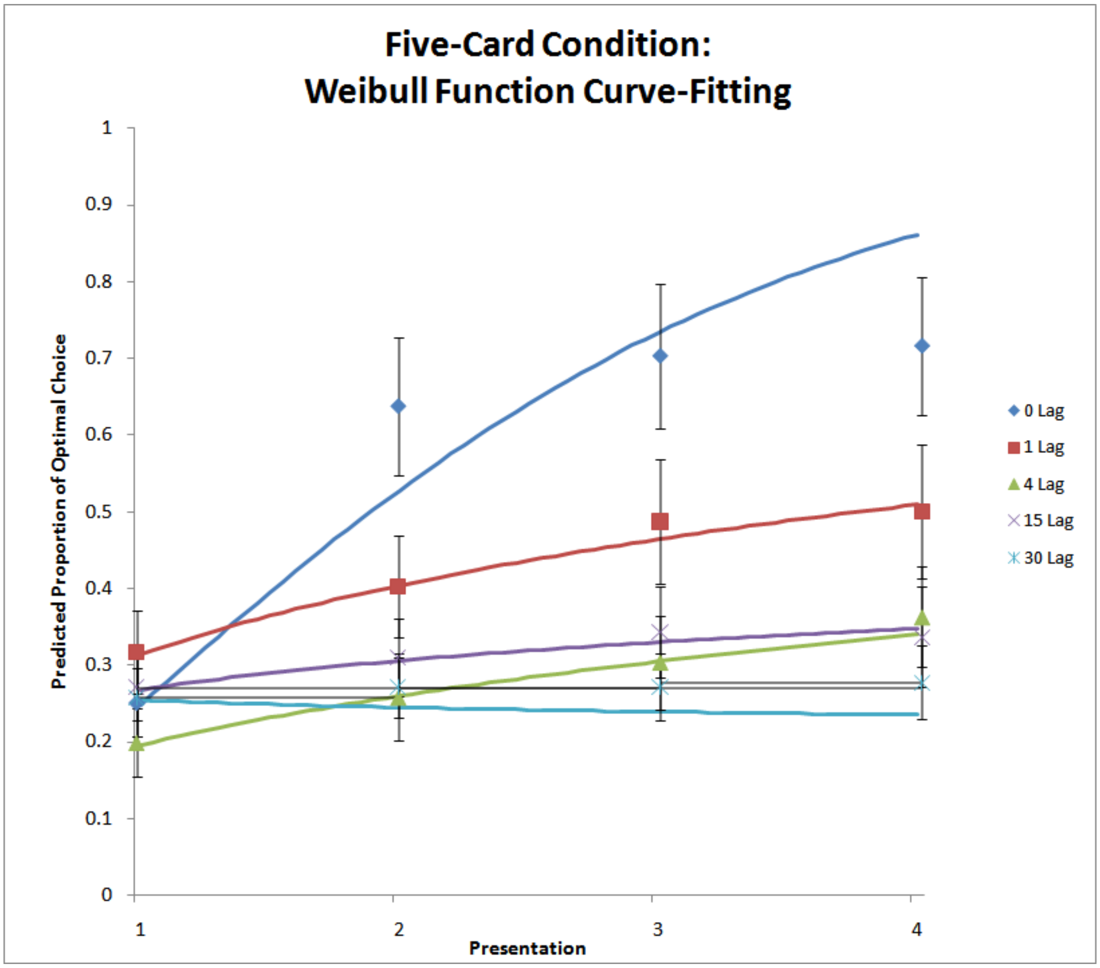

Three-Card Condition:
Optimal Responding
Proportion of Optimal Choices
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
Presentation
0 Lag
1 Lag
4 Lag
15 Lag
30 Lag

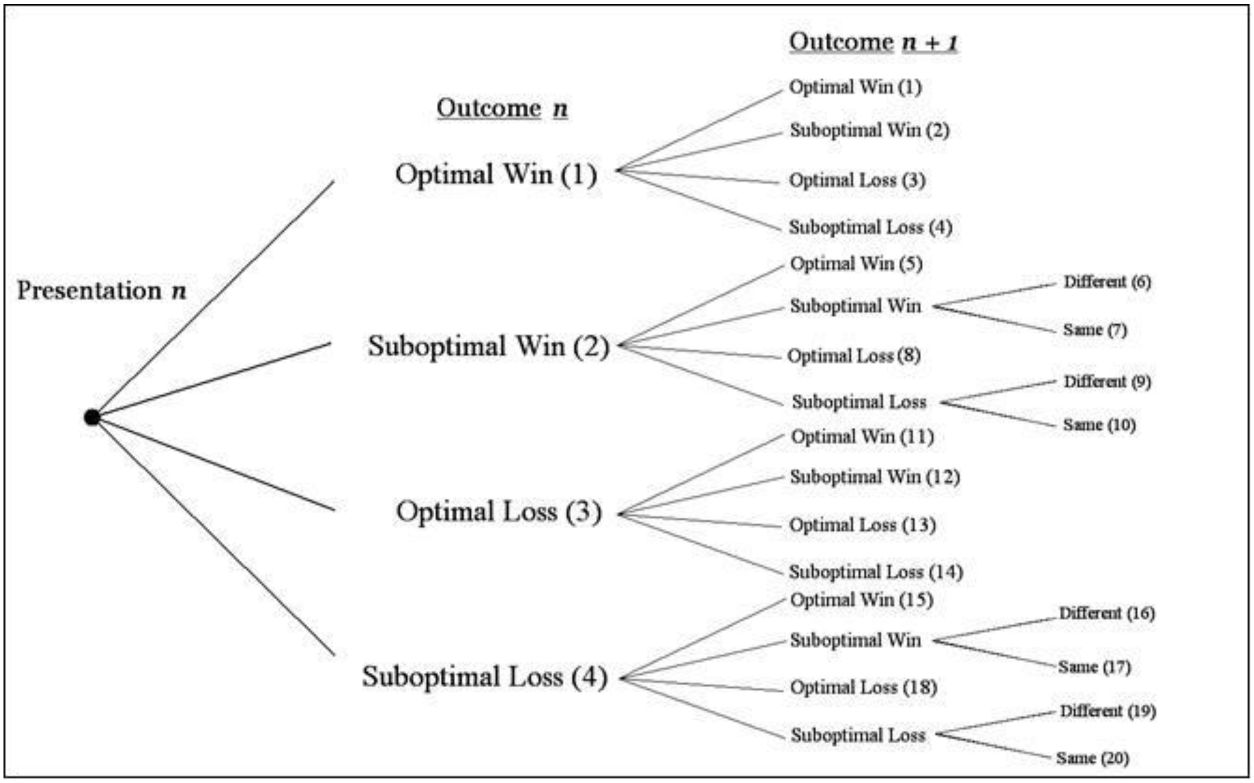

Four-Card Condition:
Optimal Responding
Proportion of Optimal Choices
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
Presentation
0 Lag
1 Lag
4 Lag
15 Lag
30 Lag

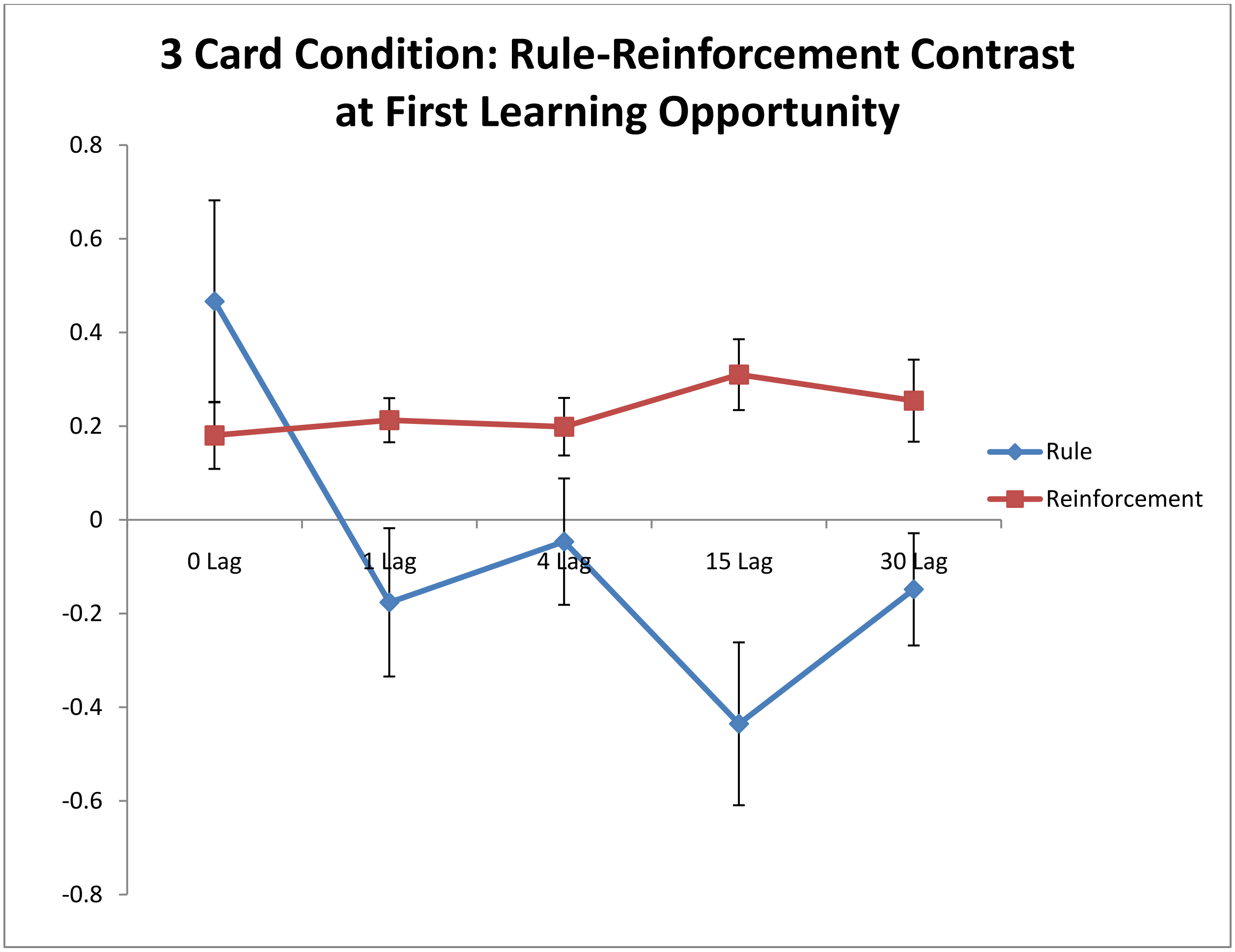

Five-Card Condition:
Optimal Responding
Proportion of Optimal Choices
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
Presentation
0 Lag
1 Lag
4 Lag
15 Lag
30 Lag

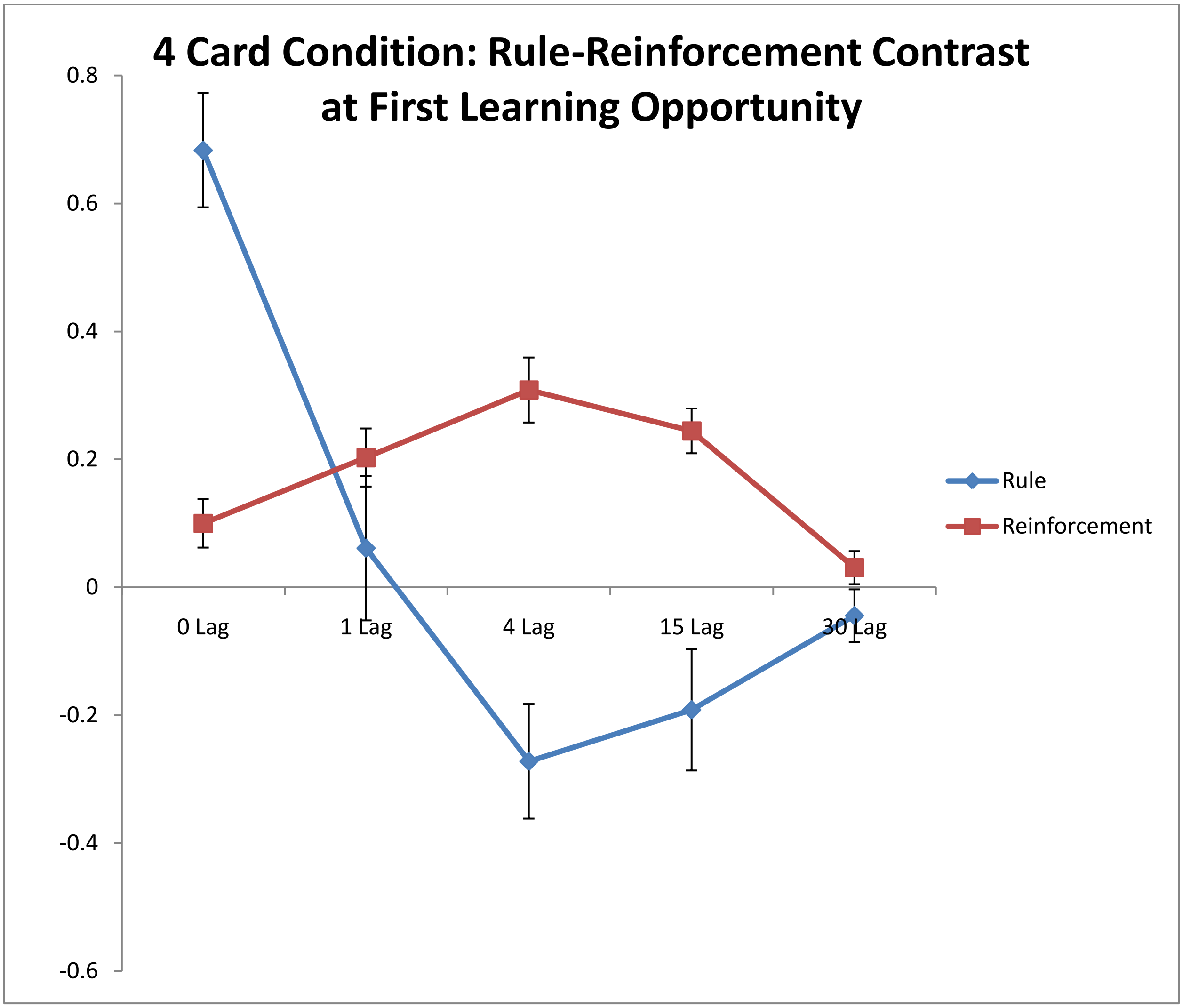

Three-Card Condition:
Weibull Function Curve-Fitting
Predicted Optimal Choice Proportion
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1
2
3
4
Presentation
0 Lag
1 Lag
4 Lag
15 Lag
30 Lag

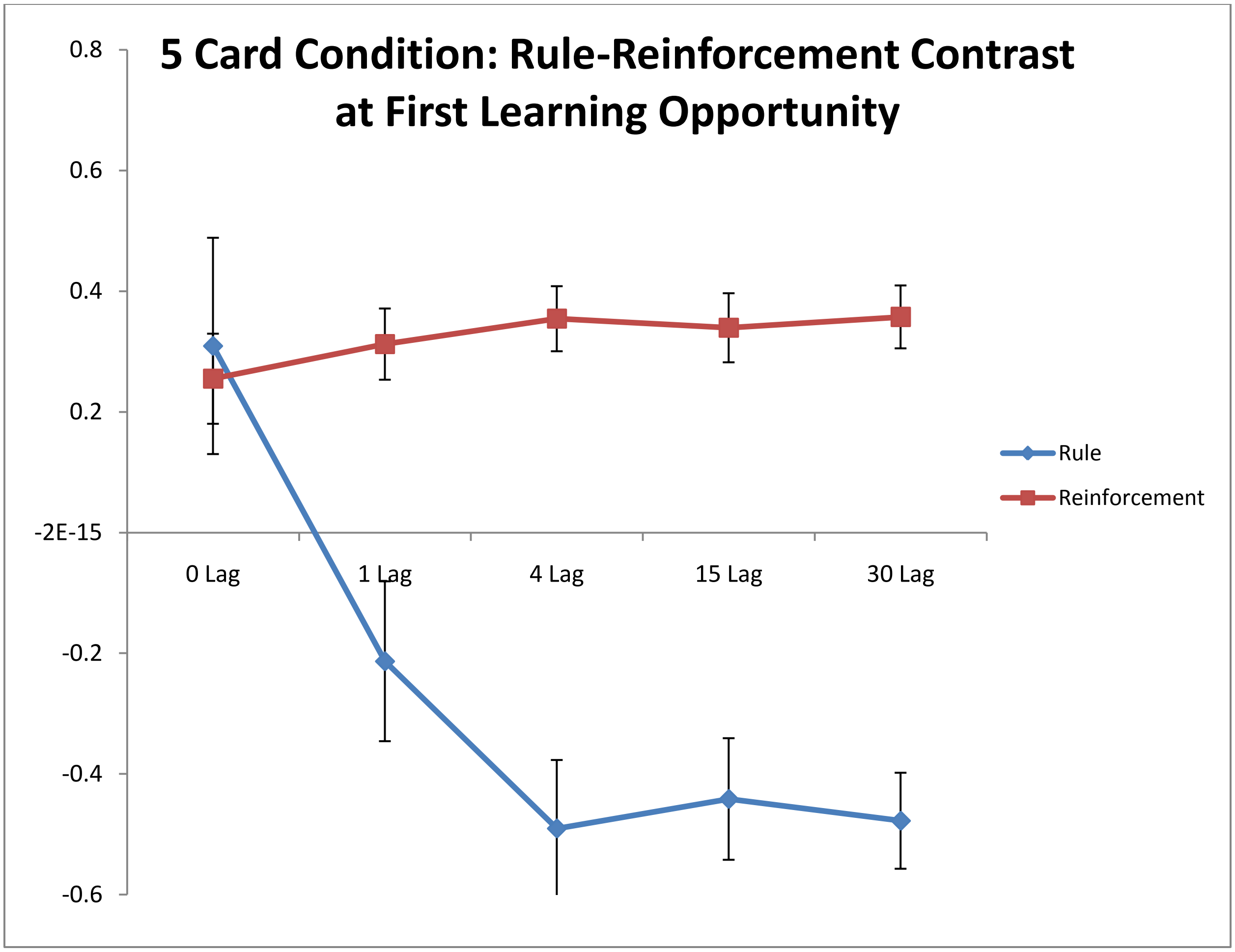

Four-Card Condition:
Weibull Function Curve-Fitting
Predicted Proportion of Optimal Choices
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
Presentation
0 Lag
1 Lag
4 Lag
15 Lag
30 Lag

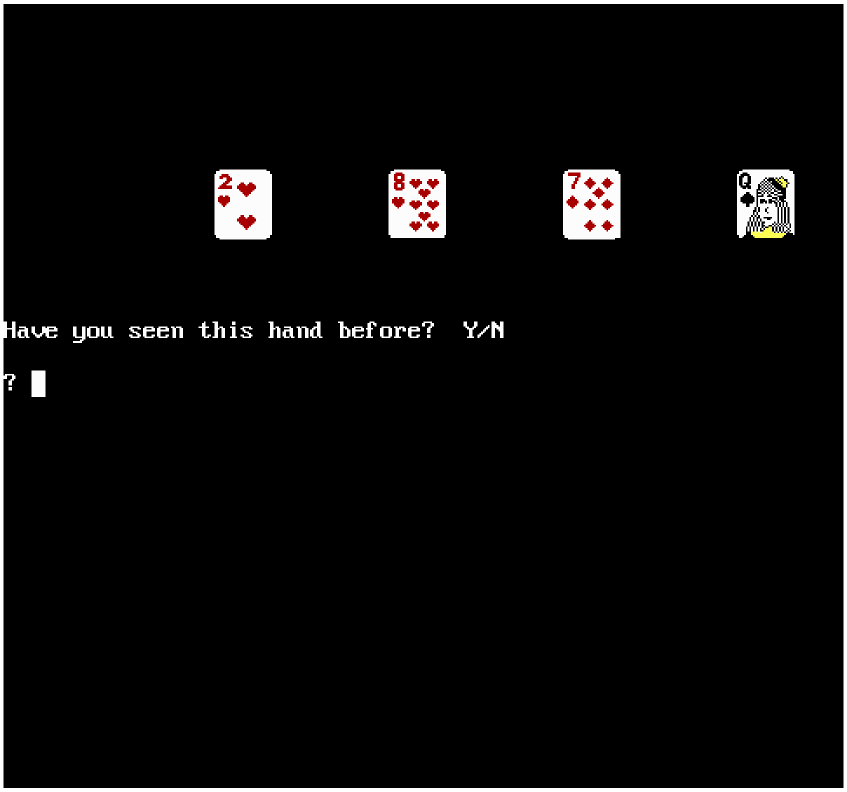

Five-Card Condition:
Weibull Function Curve-Fitting
Predicted Proportion of Optimal Choice
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
Presentation
0 Lag
1 Lag
4 Lag
15 Lag
30 Lag


Outcome n + 1
Outcome n
Presentation n
Optimal Win (1)
Suboptimal Win (2)
Optimal Loss (3)
Suboptimal Loss (4)
Optimal Win (1)
Suboptimal Win (2)
Optimal Loss (3)
Suboptimal Loss (4)
Optimal Win (5)
Suboptimal Win
Different (6)
Same (7)
Optimal Loss (8)
Suboptimal Loss
Different (9)
Same (10)
Optimal Win (11)
Suboptimal Win (12)
Optimal Loss (13)
Suboptimal Loss (14)
Optimal Win (15)
Suboptimal Win
Different (16)
Same (17)
Optimal Loss (18)
Suboptimal Loss
Different (19)
Same (20)

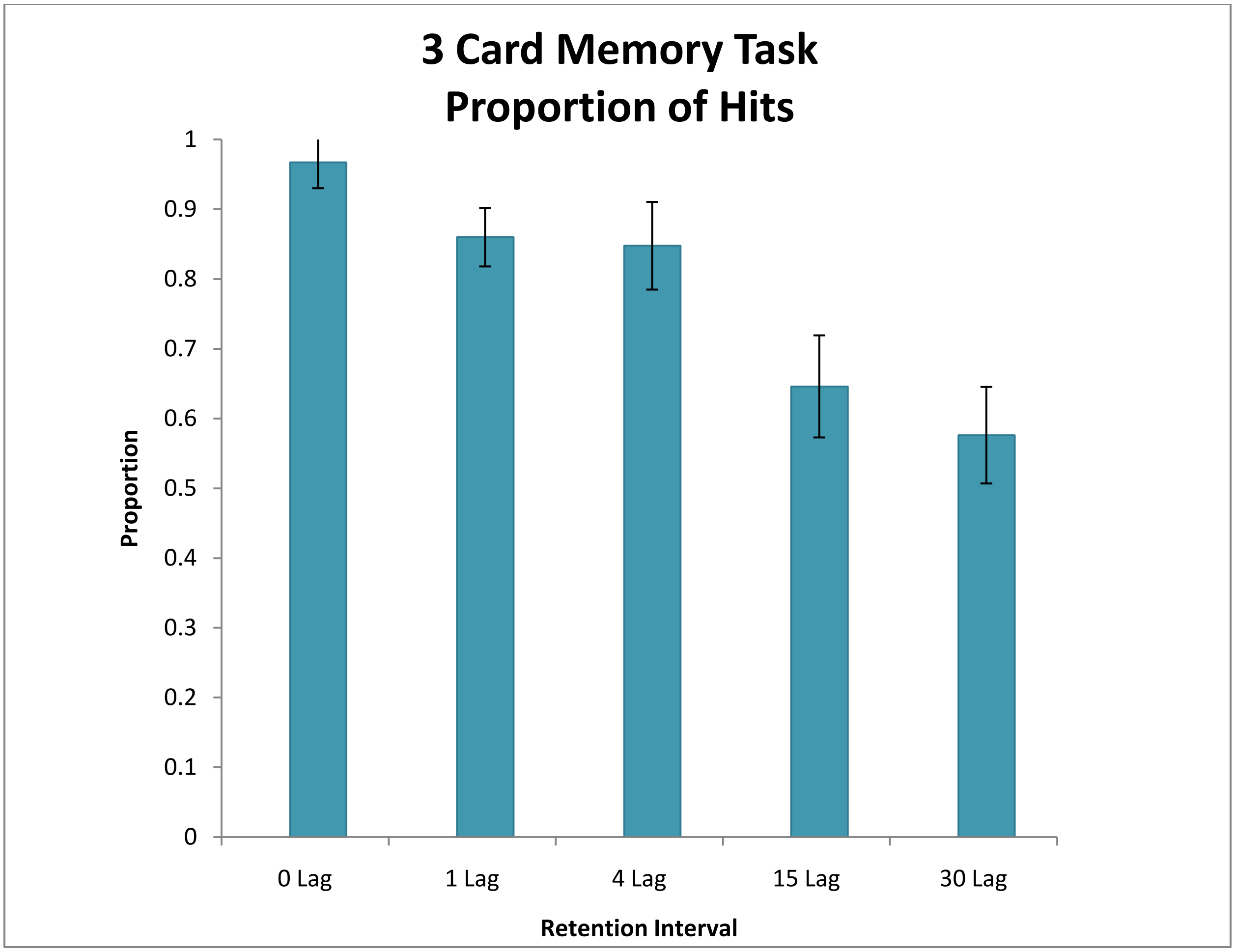

3 Card Condition: Rule-Reinforcement Contrast at First Learning Opportunity
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0 Lag
1 Lag
4 Lag
15 Lag
30 Lag
Rule
Reinforcement

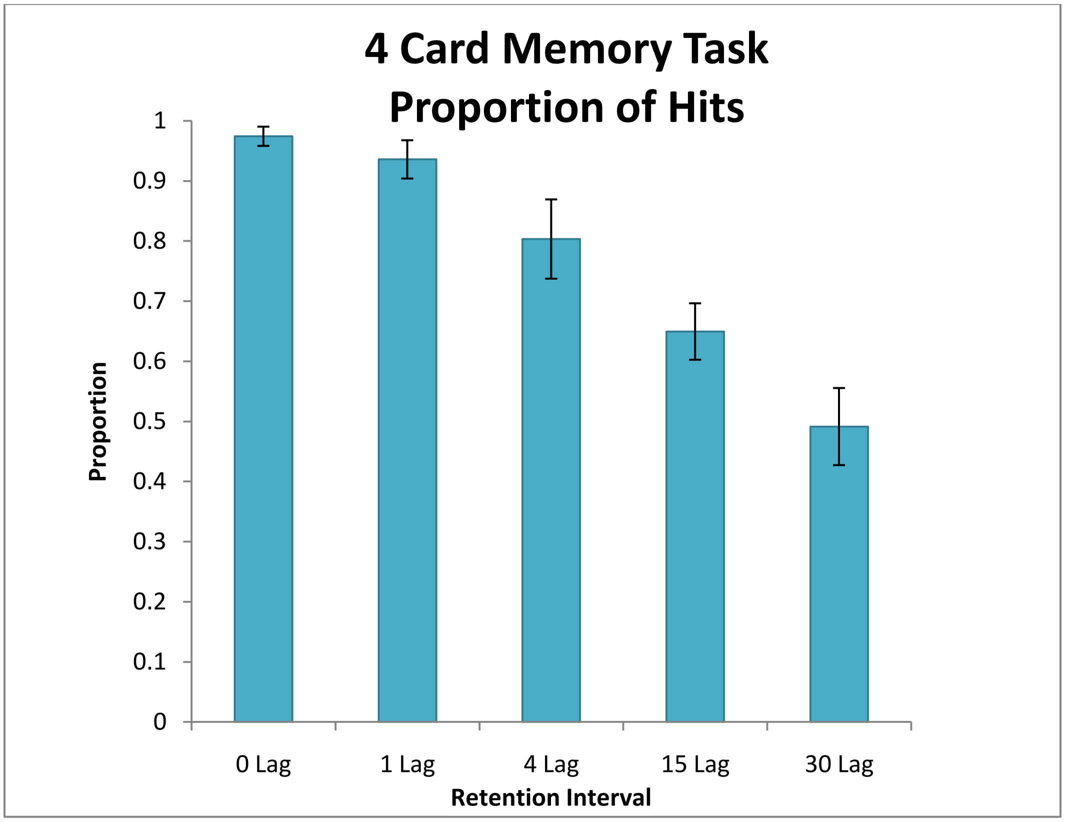

4 Card Condition: Rule-Reinforcement Contrast at First Learning Opportunity
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0 Lag
1 Lag
4 Lag
15 Lag
30 Lag
Rule
Reinforcement

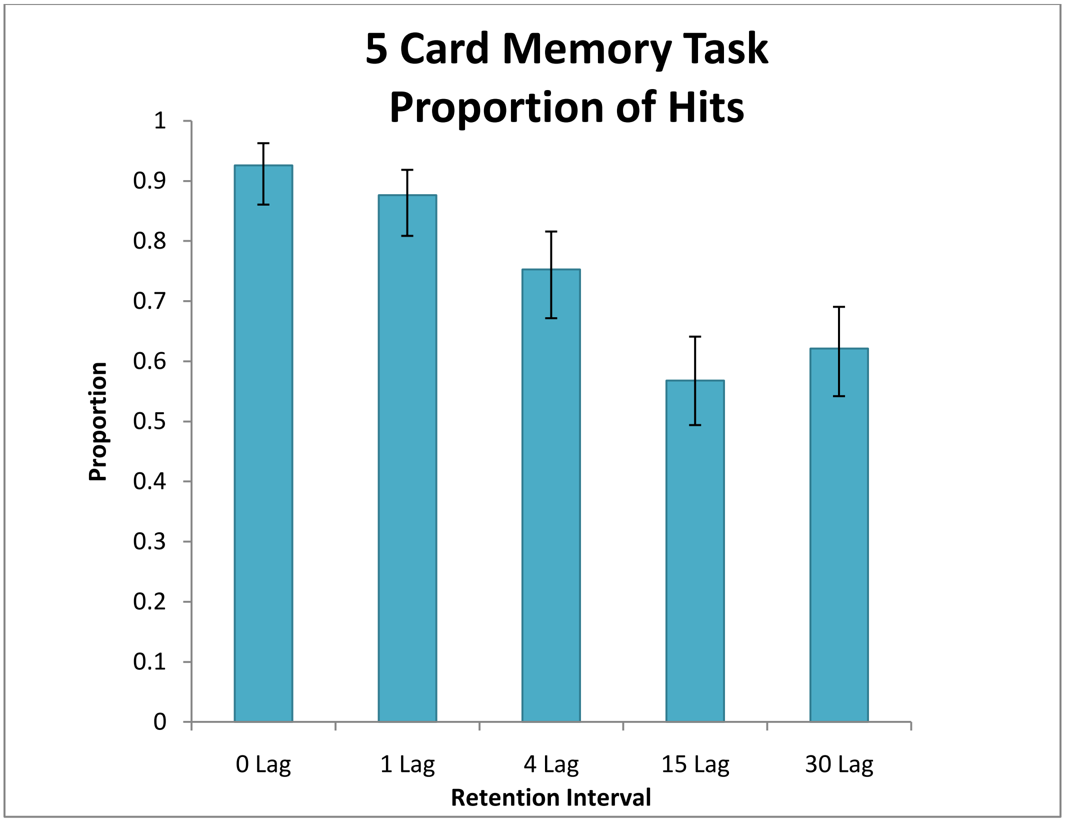

5 Card Condition: Rule-Reinforcement Contrast at First Learning Opportunity
0.8
0.6
0.4
0.2
-2E-15
-0.2
-0.4
-0.6
0 Lag
1 Lag
4 Lag
15 Lag
30 Lag
Rule
Reinforcement

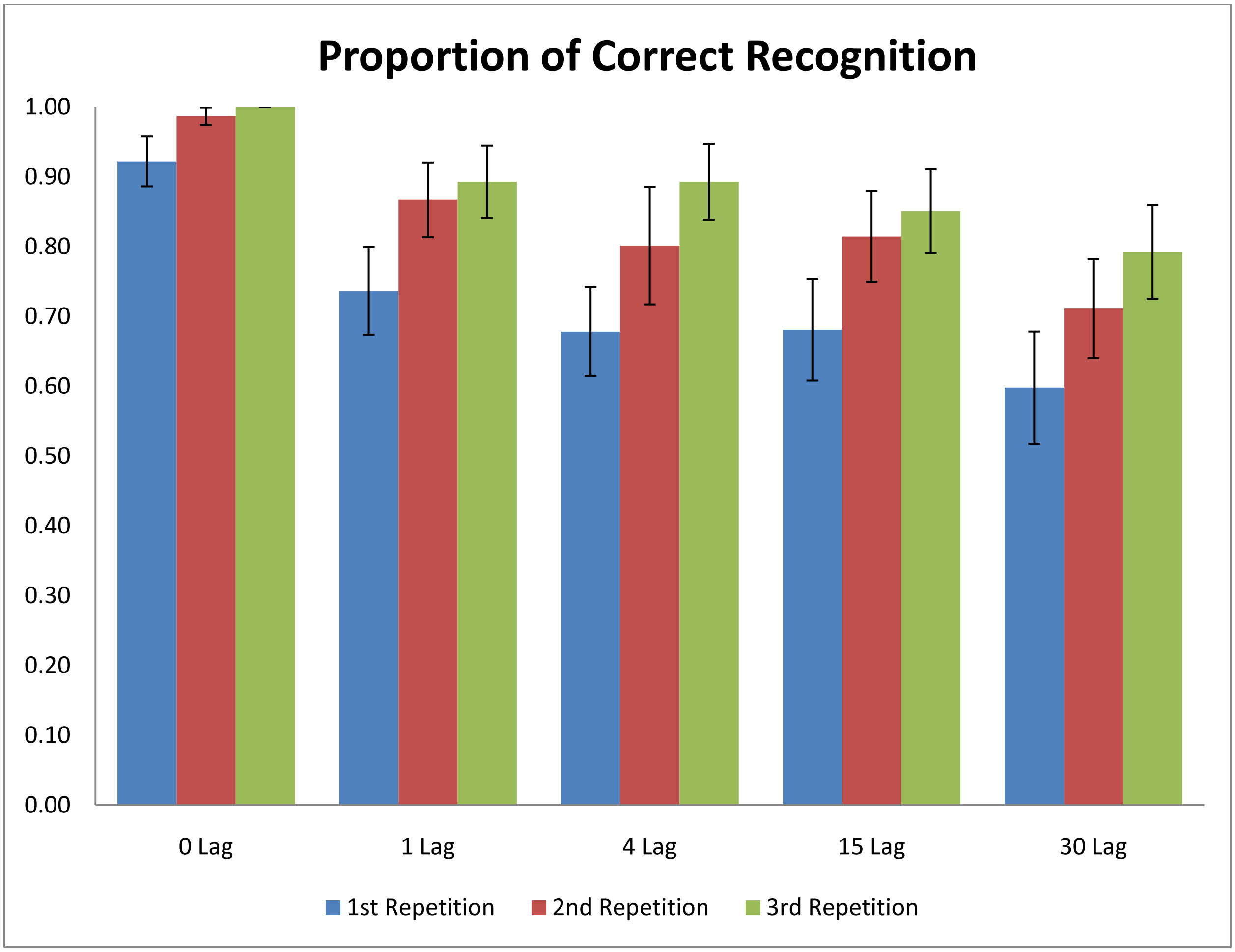


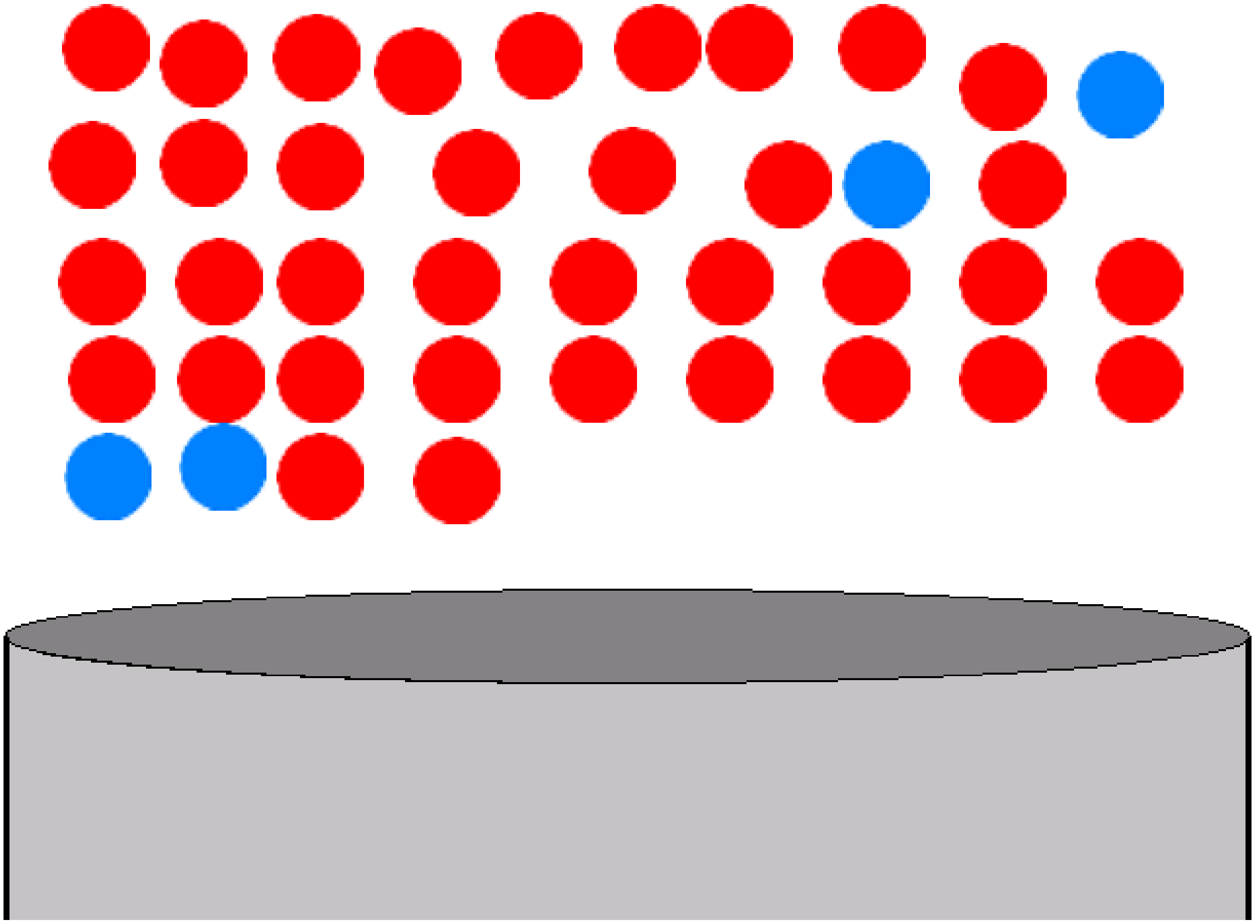
```
Have you seen this hand before?  Y/N

? 
```

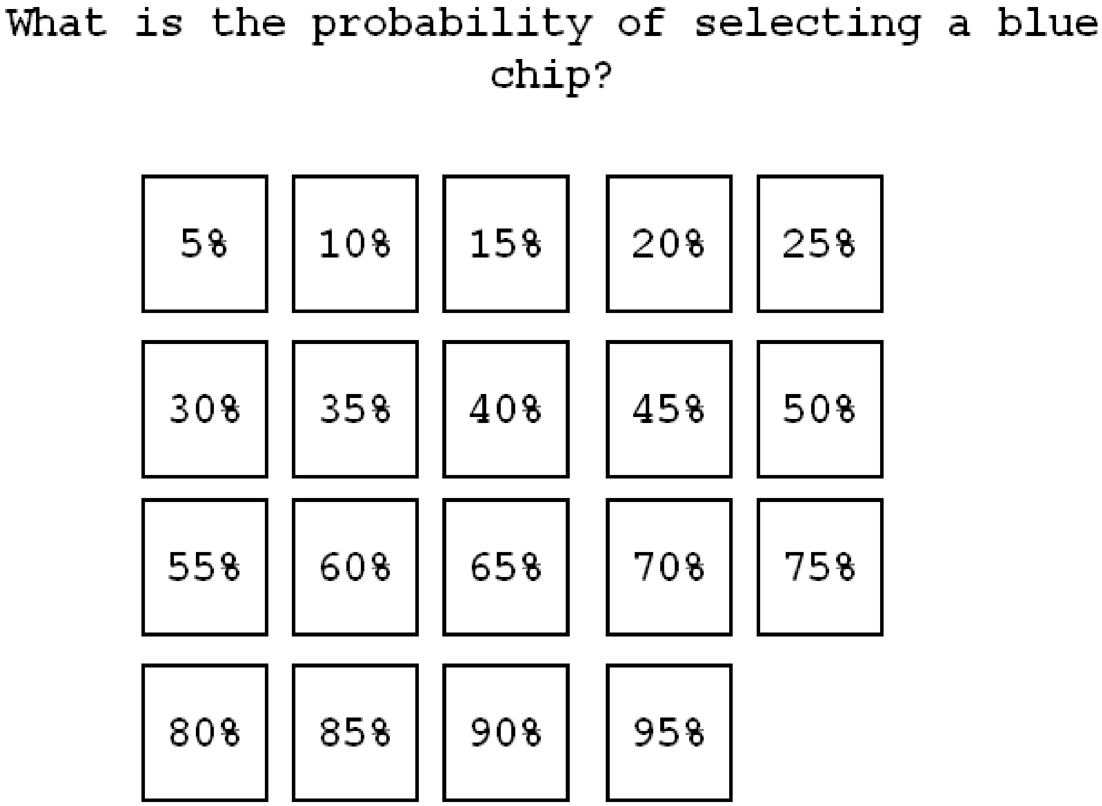

3 Card Memory Task
Proportion of Hits
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Proportion
0 Lag
1 Lag
4 Lag
15 Lag
30 Lag
Retention Interval


4 Card Memory Task
Proportion of Hits
Proportion
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 Lag
1 Lag
4 Lag
15 Lag
30 Lag
Retention Interval


5 Card Memory Task
Proportion of Hits
Proportion
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 Lag
1 Lag
4 Lag
15 Lag
30 Lag
Retention Interval

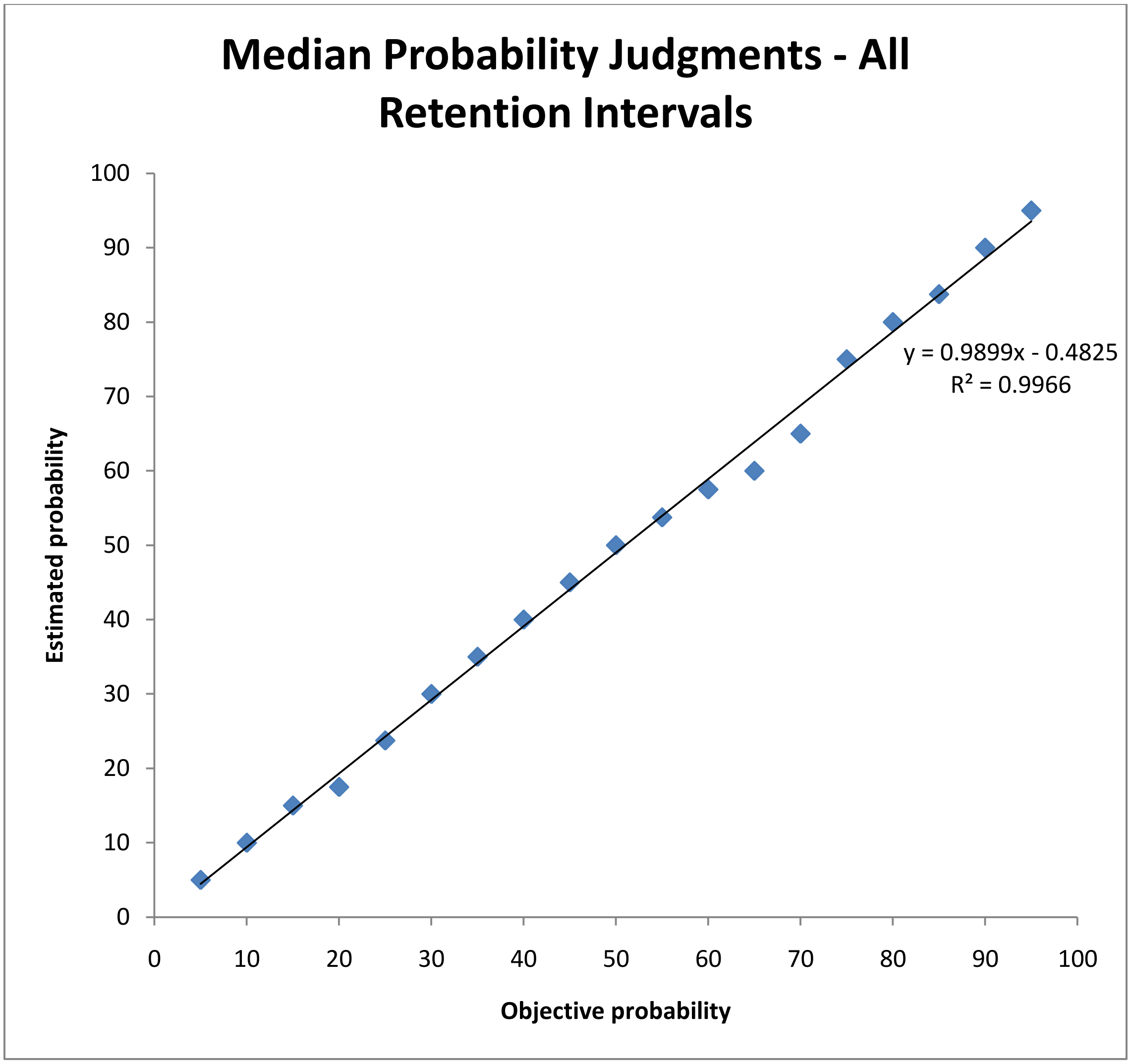

Proportion of Correct Recognition
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
0 Lag
1 Lag
4 Lag
15 Lag
30 Lag
1st Repetition
2nd Repetition
3rd Repetition

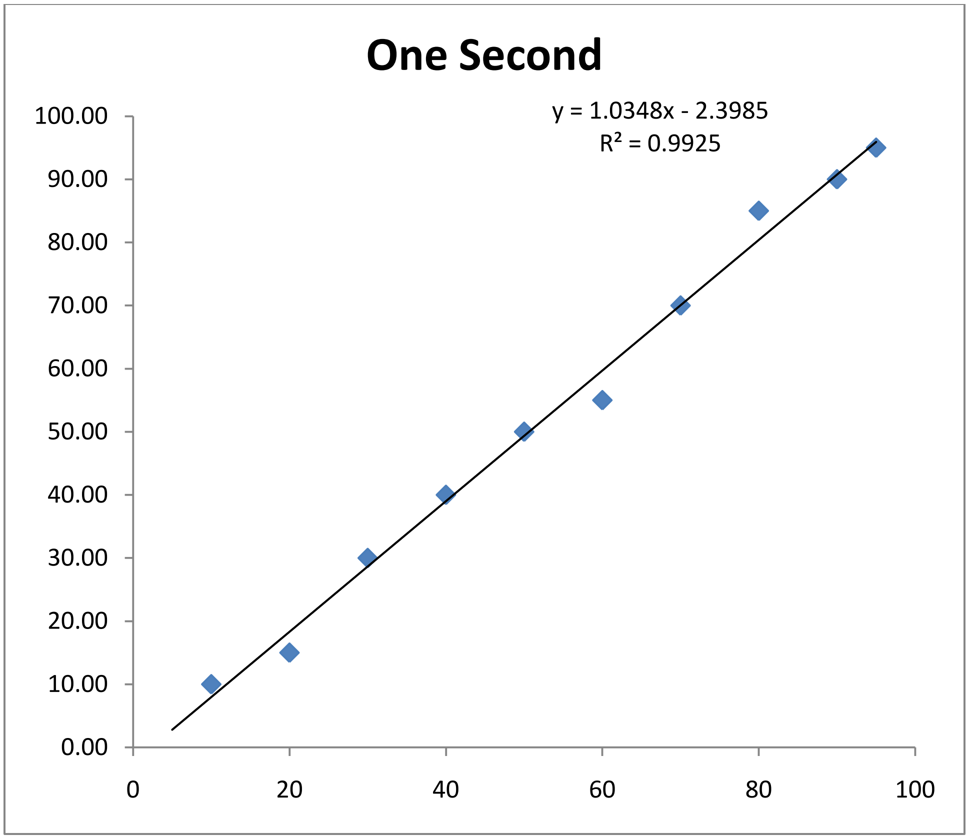


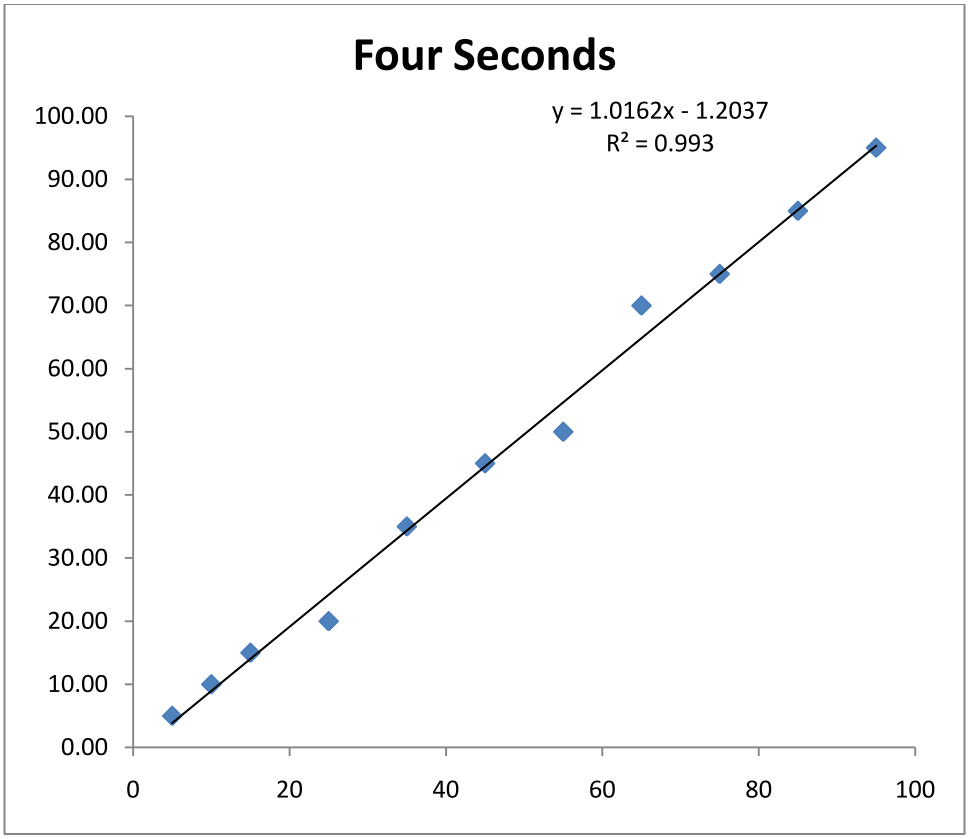


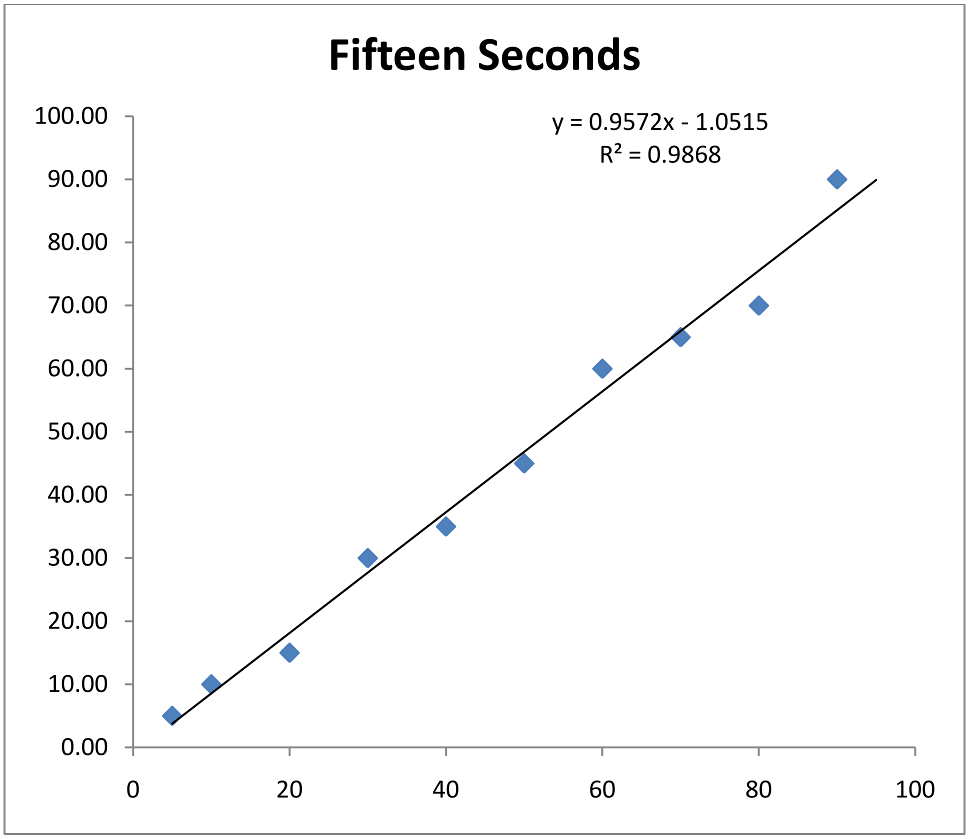
What is the probability of selecting a blue chip?

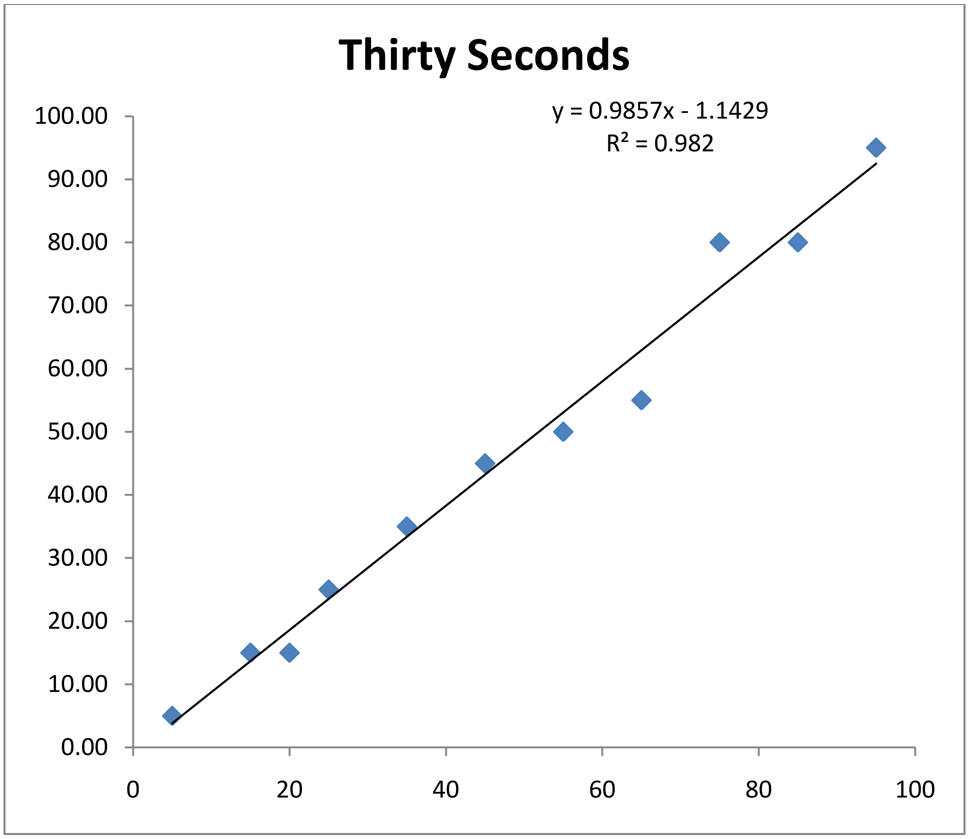
| | | | | |
|---|---|---|---|---|
| 5% | 10% | 15% | 20% | 25% |
| 30% | 35% | 40% | 45% | 50% |
| 55% | 60% | 65% | 70% | 75% |
| 80% | 85% | 90% | 95% | |

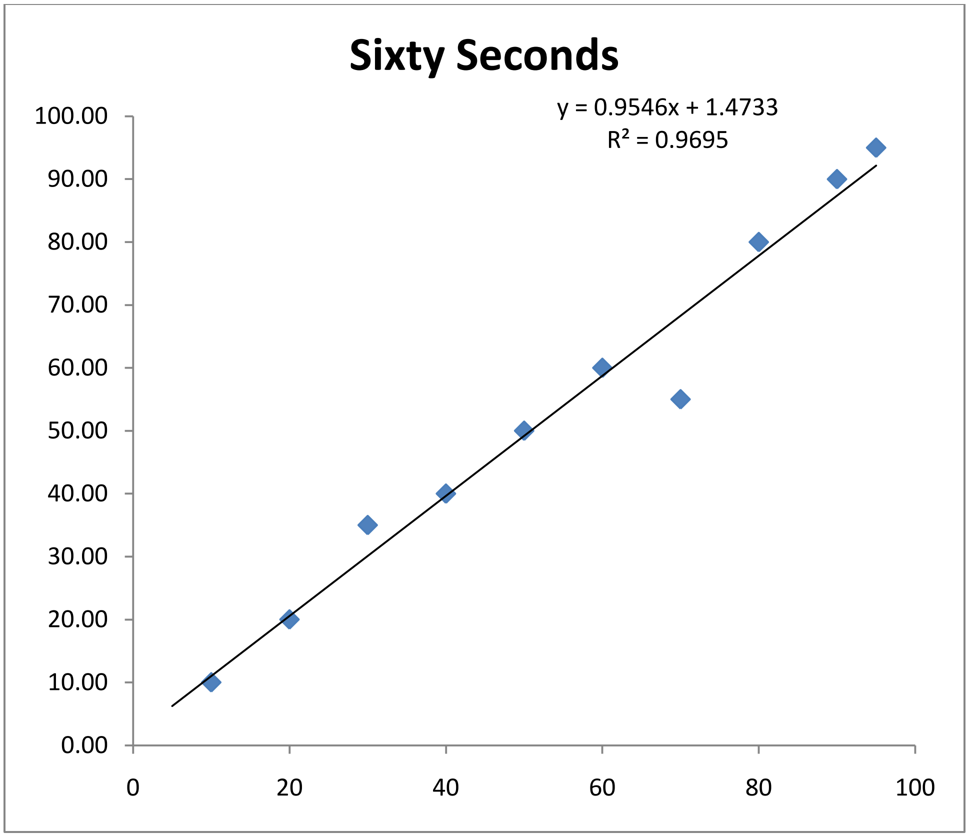

Median Probability Judgments - All Retention Intervals
Estimated probability
Objective probability
0
10
20
30
40
50
60
70
80
90
100
y = 0.9899x - 0.4825
R² = 0.9966


One Second
y = 1.0348x - 2.3985
R² = 0.9925
100.00
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
0
20
40
60
80
100


Four Seconds
y = 1.0162x - 1.2037
R² = 0.993
100.00
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
0
20
40
60
80
100


Fifteen Seconds
y = 0.9572x - 1.0515
R² = 0.9868
100.00
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
0
20
40
60
80
100


Thirty Seconds
y = 0.9857x - 1.1429
R² = 0.982
100.00
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
0
20
40
60
80
100


Sixty Seconds
y = 0.9546x + 1.4733
R² = 0.9695
100.00
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
0
20
40
60
80
100

# APPENDIX

False Alarm Rates for Experiment 2

| Condition | Mean False Alarm Rate | 95% Confidence Interval |
|---|---|---|
| 3 Cards | 0.306 | (0.259, 0.353) |
| 4 Cards | 0.262 | (0.224, 0.299) |
| 5 Cards | 0.314 | (0.271, 0.357) |

Sample Script for Trials: Four Card Memory Paradigm

| Trial # * | Card 1 | Card 2 | Card 3 | Card 4 | Retention Interval† | Presentation # | Hand ID # |
|---|---|---|---|---|---|---|---|
| 4 | Jh | 9d | 5s | 2s | 5 | 1 | 37 |
| 5 | Qh | 2d | Ks | 6s | 1 | 1 | 1 |
| 6 | Qh | 2d | Ks | 6s | 1 | 2 | 1 |
| 7 | Qh | 2d | Ks | 6s | 1 | 3 | 1 |
| 8 | Qh | 2d | Ks | 6s | 1 | 4 | 1 |
| 9 | Kc | Ad | 8s | 2h | 2 | 1 | 10 |
| 10 | 8d | 10h | 5d | Qs | 4 | 1 | 28 |
| 11 | Kc | Ad | 8s | 2h | 2 | 2 | 10 |
| 12 | 9s | Kc | Js | 5h | 3 | 1 | 19 |
| 13 | Kc | Ad | 8s | 2h | 2 | 3 | 10 |
| 14 | 3s | Jc | 9c | 5c | 5 | 1 | 38 |
| 15 | Kc | Ad | 8s | 2h | 2 | 4 | 10 |
| 16 | 4h | 7d | Jd | 9c | 4 | 1 | 29 |
| 17 | 9s | Kc | Js | 5h | 3 | 2 | 19 |
| 18 | 5s | 10h | 7c | 9h | 1 | 1 | 2 |
| 19 | 5s | 10h | 7c | 9h | 1 | 2 | 2 |
| 20 | 5s | 10h | 7c | 9h | 1 | 3 | 2 |
| 21 | 5s | 10h | 7c | 9h | 1 | 4 | 2 |
| 22 | 9s | Kc | Js | 5h | 3 | 3 | 19 |
| 23 | Ks | 2h | Kd | 10h | 4 | 1 | 30 |
| 24 | 9c | As | 7d | 3s | 0 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 25 | 2h | 10h | Qc | 4h | 0 | 1 | 0 |
| 26 | 8d | 10h | 5d | Qs | 4 | 2 | 28 |
| 27 | 9s | Kc | Js | 5h | 3 | 4 | 19 |
| 28 | 7d | 3h | As | 3s | 1 | 1 | 3 |
| 29 | 7d | 3h | As | 3s | 1 | 2 | 3 |
| 30 | 7d | 3h | As | 3s | 1 | 3 | 3 |
| 31 | 7d | 3h | As | 3s | 1 | 4 | 3 |
| 32 | 4h | 7d | Jd | 9c | 4 | 2 | 29 |
| 33 | Ac | 7d | Ks | 5h | 0 | 1 | 0 |
| 34 | As | 6c | 8c | Ac | 2 | 1 | 11 |
| 35 | Jh | 9d | 5s | 2s | 5 | 2 | 37 |
| 36 | As | 6c | 8c | Ac | 2 | 2 | 11 |
| 37 | 10h | Ad | 8d | Jc | 5 | 1 | 39 |
| 38 | As | 6c | 8c | Ac | 2 | 3 | 11 |
| 39 | Ks | 2h | Kd | 10h | 4 | 2 | 30 |
| 40 | As | 6c | 8c | Ac | 2 | 4 | 11 |
| 41 | 7h | Qh | 4h | 8h | 3 | 1 | 20 |
| 42 | 8d | 10h | 5d | Qs | 4 | 3 | 28 |
| 43 | 4s | 2c | Qd | 8c | 0 | 1 | 0 |
| 44 | 2d | 7d | As | Jd | 3 | 1 | 21 |
| 45 | 3s | Jc | 9c | 5c | 5 | 2 | 38 |
| 46 | 7h | Qh | 4h | 8h | 3 | 2 | 20 |
| 47 | 3c | 3d | As | 8s | 5 | 1 | 40 |
| 48 | 4h | 7d | Jd | 9c | 4 | 3 | 29 |
| 49 | 2d | 7d | As | Jd | 3 | 2 | 21 |
| 50 | 8c | 4d | Qc | 8s | 0 | 1 | 0 |
| 51 | 7h | Qh | 4h | 8h | 3 | 3 | 20 |
| 52 | 3s | Qh | 7c | 4c | 0 | 1 | 0 |
| 53 | Kd | 5c | 6c | Js | 0 | 1 | 0 |
| 54 | 2d | 7d | As | Jd | 3 | 3 | 21 |
| 55 | Ks | 2h | Kd | 10h | 4 | 3 | 30 |
| 56 | 7h | Qh | 4h | 8h | 3 | 4 | 20 |
| 57 | 5s | Kd | 8s | Qd | 0 | 1 | 0 |
| 58 | 8d | 10h | 5d | Qs | 4 | 4 | 28 |
| 59 | 2d | 7d | As | Jd | 3 | 4 | 21 |
| 60 | 4d | 7c | 10c | 4h | 5 | 1 | 41 |
| 61 | 8c | Ad | 2d | 5s | 0 | 1 | 0 |
| 62 | Ks | 4c | 2c | 6c | 0 | 1 | 0 |
| 63 | As | 9h | 3d | 2s | 2 | 1 | 12 |
| 64 | 4h | 7d | Jd | 9c | 4 | 4 | 29 |
| 65 | As | 9h | 3d | 2s | 2 | 2 | 12 |
| 66 | Jh | 9d | 5s | 2s | 5 | 3 | 37 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 67 | As | 9h | 3d | 2s | 2 | 3 | 12 |
| 68 | 10h | Ad | 8d | Jc | 5 | 2 | 39 |
| 69 | As | 9h | 3d | 2s | 2 | 4 | 12 |
| 70 | 4d | 9d | 8d | 9c | 0 | 1 | 0 |
| 71 | Ks | 2h | Kd | 10h | 4 | 4 | 30 |
| 72 | 9c | As | 7s | Ad | 1 | 1 | 4 |
| 73 | 9c | As | 7s | Ad | 1 | 2 | 4 |
| 74 | 9c | As | 7s | Ad | 1 | 3 | 4 |
| 75 | 9c | As | 7s | Ad | 1 | 4 | 4 |
| 76 | 3s | Jc | 9c | 5c | 5 | 3 | 38 |
| 77 | 7s | 10h | 4h | Jd | 2 | 1 | 13 |
| 78 | 3c | 3d | As | 8s | 5 | 2 | 40 |
| 79 | 7s | 10h | 4h | Jd | 2 | 2 | 13 |
| 80 | Kd | 10s | 5d | 2d | 3 | 1 | 22 |
| 81 | 7s | 10h | 4h | Jd | 2 | 3 | 13 |
| 82 | As | 9h | 5h | Qs | 0 | 1 | 0 |
| 83 | 7s | 10h | 4h | Jd | 2 | 4 | 13 |
| 84 | 2d | 9h | 9d | Ks | 4 | 1 | 31 |
| 85 | Kd | 10s | 5d | 2d | 3 | 2 | 22 |
| 86 | 6h | Qc | Qd | 4h | 1 | 1 | 5 |
| 87 | 6h | Qc | Qd | 4h | 1 | 2 | 5 |
| 88 | 6h | Qc | Qd | 4h | 1 | 3 | 5 |
| 89 | 6h | Qc | Qd | 4h | 1 | 4 | 5 |
| 90 | Kd | 10s | 5d | 2d | 3 | 3 | 22 |
| 91 | 4d | 7c | 10c | 4h | 5 | 2 | 41 |
| 92 | 10s | 7s | Jh | Jc | 2 | 1 | 14 |
| 93 | Qc | 5s | 4c | Ad | 5 | 1 | 42 |
| 94 | 10s | 7s | Jh | Jc | 2 | 2 | 14 |
| 95 | Kd | 10s | 5d | 2d | 3 | 4 | 22 |
| 96 | 10s | 7s | Jh | Jc | 2 | 3 | 14 |
| 97 | Jh | 9d | 5s | 2s | 5 | 4 | 37 |
| 98 | 10s | 7s | Jh | Jc | 2 | 4 | 14 |
| 99 | 10h | Ad | 8d | Jc | 5 | 3 | 39 |
| 100 | 2d | 9h | 9d | Ks | 4 | 2 | 31 |
| 101 | Kc | 6h | 6c | 9h | 0 | 1 | 0 |
| 102 | As | 6d | 9h | 2h | 5 | 1 | 43 |
| 103 | Kd | 3d | Qd | 7h | 3 | 1 | 23 |
| 104 | 6c | 10c | 4d | 3h | 4 | 1 | 32 |
| 105 | Jd | 7d | Jc | 2s | 0 | 1 | 0 |
| 106 | 2d | Js | Qd | 8s | 0 | 1 | 0 |
| 107 | 3s | Jc | 9c | 5c | 5 | 4 | 38 |
| 108 | Kd | 3d | Qd | 7h | 3 | 2 | 23 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 109 | 3c | 3d | As | 8s | 5 | 3 | 40 |
| 110 | Qd | 8d | 5s | 9h | 0 | 1 | 0 |
| 111 | 10d | 6d | 4c | 7h | 0 | 1 | 0 |
| 112 | 5h | 9h | 3c | 7h | 5 | 1 | 44 |
| 113 | Kd | 3d | Qd | 7h | 3 | 3 | 23 |
| 114 | 5s | Kh | 7c | 5c | 0 | 1 | 0 |
| 115 | Kh | 10d | 6h | 9s | 2 | 1 | 15 |
| 116 | 2d | 9h | 9d | Ks | 4 | 3 | 31 |
| 117 | Kh | 10d | 6h | 9s | 2 | 2 | 15 |
| 118 | Kd | 3d | Qd | 7h | 3 | 4 | 23 |
| 119 | Kh | 10d | 6h | 9s | 2 | 3 | 15 |
| 120 | 6c | 10c | 4d | 3h | 4 | 2 | 32 |
| 121 | Kh | 10d | 6h | 9s | 2 | 4 | 15 |
| 122 | 4d | 7c | 10c | 4h | 5 | 3 | 41 |
| 123 | 2h | 9h | 6s | 5d | 0 | 1 | 0 |
| 124 | Qc | 5s | 4c | Ad | 5 | 2 | 42 |
| 125 | 6d | 5h | Jd | 9h | 1 | 1 | 6 |
| 126 | 6d | 5h | Jd | 9h | 1 | 2 | 6 |
| 127 | 6d | 5h | Jd | 9h | 1 | 3 | 6 |
| 128 | 6d | 5h | Jd | 9h | 1 | 4 | 6 |
| 129 | 8c | Kc | 9h | 3h | 3 | 1 | 24 |
| 130 | 10h | Ad | 8d | Jc | 5 | 4 | 39 |
| 131 | 3c | 6s | 9d | 7d | 4 | 1 | 33 |
| 132 | 2d | 9h | 9d | Ks | 4 | 4 | 31 |
| 133 | As | 6d | 9h | 2h | 5 | 2 | 43 |
| 134 | 8c | Kc | 9h | 3h | 3 | 2 | 24 |
| 135 | 5c | 2h | Qh | Qs | 5 | 1 | 45 |
| 136 | 6c | 10c | 4d | 3h | 4 | 3 | 32 |
| 137 | 8c | 10d | 5h | 7c | 0 | 1 | 0 |
| 138 | 6c | Qs | 5c | Ac | 0 | 1 | 0 |
| 139 | 8c | Kc | 9h | 3h | 3 | 3 | 24 |
| 140 | 3c | 3d | As | 8s | 5 | 4 | 40 |
| 141 | 7d | 6s | 9h | 3s | 4 | 1 | 34 |
| 142 | 5h | 9d | 10s | 2c | 0 | 1 | 0 |
| 143 | 5h | 9h | 3c | 7h | 5 | 2 | 44 |
| 144 | 8c | Kc | 9h | 3h | 3 | 4 | 24 |
| 145 | 3c | 9s | 4d | Kd | 4 | 1 | 35 |
| 146 | 5h | 10s | 7c | Kc | 0 | 1 | 0 |
| 147 | 3c | 6s | 9d | 7d | 4 | 2 | 33 |
| 148 | 7s | 9d | 2c | 6d | 1 | 1 | 7 |
| 149 | 7s | 9d | 2c | 6d | 1 | 2 | 7 |
| 150 | 7s | 9d | 2c | 6d | 1 | 3 | 7 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 151 | 7s | 9d | 2c | 6d | 1 | 4 | 7 |
| 152 | 6c | 10c | 4d | 3h | 4 | 4 | 32 |
| 153 | 4d | 7c | 10c | 4h | 5 | 4 | 41 |
| 154 | 4d | 2s | Ad | 9d | 2 | 1 | 16 |
| 155 | Qc | 5s | 4c | Ad | 5 | 3 | 42 |
| 156 | 4d | 2s | Ad | 9d | 2 | 2 | 16 |
| 157 | 7d | 6s | 9h | 3s | 4 | 2 | 34 |
| 158 | 4d | 2s | Ad | 9d | 2 | 3 | 16 |
| 159 | 10d | 5s | 7s | Qh | 0 | 1 | 0 |
| 160 | 4d | 2s | Ad | 9d | 2 | 4 | 16 |
| 161 | 3c | 9s | 4d | Kd | 4 | 2 | 35 |
| 162 | 6h | 10h | 2s | 2c | 0 | 1 | 0 |
| 163 | 3c | 6s | 9d | 7d | 4 | 3 | 33 |
| 164 | As | 6d | 9h | 2h | 5 | 3 | 43 |
| 165 | 8h | 9c | Ad | 6c | 0 | 1 | 0 |
| 166 | 5c | 2h | Qh | Qs | 5 | 2 | 45 |
| 167 | 7h | 3d | Qh | Jd | 0 | 1 | 0 |
| 168 | As | 10h | 4c | Ah | 1 | 1 | 8 |
| 169 | As | 10h | 4c | Ah | 1 | 2 | 8 |
| 170 | As | 10h | 4c | Ah | 1 | 3 | 8 |
| 171 | As | 10h | 4c | Ah | 1 | 4 | 8 |
| 172 | Kc | 3d | 2d | Js | 0 | 1 | 0 |
| 173 | 7d | 6s | 9h | 3s | 4 | 3 | 34 |
| 174 | 5h | 9h | 3c | 7h | 5 | 3 | 44 |
| 175 | 2s | Jd | 7d | Kd | 0 | 1 | 0 |
| 176 | Jd | 7c | 4c | Qs | 0 | 1 | 0 |
| 177 | 3c | 9s | 4d | Kd | 4 | 3 | 35 |
| 178 | 6d | As | 8s | 6c | 4 | 1 | 36 |
| 179 | 3c | 6s | 9d | 7d | 4 | 4 | 33 |
| 180 | 6h | Qc | Qd | 9c | 1 | 1 | 9 |
| 181 | 6h | Qc | Qd | 9c | 1 | 2 | 9 |
| 182 | 6h | Qc | Qd | 9c | 1 | 3 | 9 |
| 183 | 6h | Qc | Qd | 9c | 1 | 4 | 9 |
| 184 | 8s | 4s | 10c | 2s | 0 | 1 | 0 |
| 185 | 4d | Js | 5d | Ks | 0 | 1 | 0 |
| 186 | Qc | 5s | 4c | Ad | 5 | 4 | 42 |
| 187 | Ac | 10s | 5d | Qs | 0 | 1 | 0 |
| 188 | Jd | Kc | 9d | Kd | 0 | 1 | 0 |
| 189 | 7d | 6s | 9h | 3s | 4 | 4 | 34 |
| 190 | 10d | Kh | 7c | 10c | 0 | 1 | 0 |
| 191 | 10d | 3h | 9h | Qs | 3 | 1 | 25 |
| 192 | 4c | Js | 6s | Qd | 0 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 193 | 3c | 9s | 4d | Kd | 4 | 4 | 35 |
| 194 | 6d | As | 8s | 6c | 4 | 2 | 36 |
| 195 | As | 6d | 9h | 2h | 5 | 4 | 43 |
| 196 | 10d | 3h | 9h | Qs | 3 | 2 | 25 |
| 197 | 5c | 2h | Qh | Qs | 5 | 3 | 45 |
| 198 | Ks | 8c | 4c | Ac | 2 | 1 | 17 |
| 199 | Qd | 2h | 9s | 6d | 0 | 1 | 0 |
| 200 | Ks | 8c | 4c | Ac | 2 | 2 | 17 |
| 201 | 10d | 3h | 9h | Qs | 3 | 3 | 25 |
| 202 | Ks | 8c | 4c | Ac | 2 | 3 | 17 |
| 203 | Ks | 4s | 3s | Js | 3 | 1 | 26 |
| 204 | Ks | 8c | 4c | Ac | 2 | 4 | 17 |
| 205 | 5h | 9h | 3c | 7h | 5 | 4 | 44 |
| 206 | 10d | 3h | 9h | Qs | 3 | 4 | 25 |
| 207 | Qd | 10d | 3d | 5d | 0 | 1 | 0 |
| 208 | Ks | 4s | 3s | Js | 3 | 2 | 26 |
| 209 | 6h | 7s | 8d | 10h | 0 | 1 | 0 |
| 210 | 6d | As | 8s | 6c | 4 | 3 | 36 |
| 211 | Jd | Ks | 6s | 8c | 0 | 1 | 0 |
| 212 | 3s | 5s | 9c | Kc | 3 | 1 | 27 |
| 213 | Ks | 4s | 3s | Js | 3 | 3 | 26 |
| 214 | 9c | Kd | 4h | 2s | 0 | 1 | 0 |
| 215 | 7d | Qs | 10s | 3s | 0 | 1 | 0 |
| 216 | Ad | 10h | 6d | Jh | 0 | 1 | 0 |
| 217 | 3s | 5s | 9c | Kc | 3 | 2 | 27 |
| 218 | Ks | 4s | 3s | Js | 3 | 4 | 26 |
| 219 | 6c | 3d | Ad | 4h | 2 | 1 | 18 |
| 220 | 2c | 4d | 10h | 8d | 0 | 1 | 0 |
| 221 | 6c | 3d | Ad | 4h | 2 | 2 | 18 |
| 222 | 3s | 5s | 9c | Kc | 3 | 3 | 27 |
| 223 | 6c | 3d | Ad | 4h | 2 | 3 | 18 |
| 224 | 8d | 10c | 4s | 2d | 0 | 1 | 0 |
| 225 | 6c | 3d | Ad | 4h | 2 | 4 | 18 |
| 226 | 6d | As | 8s | 6c | 4 | 4 | 36 |
| 227 | 3s | 5s | 9c | Kc | 3 | 4 | 27 |
| 228 | 5c | 2h | Qh | Qs | 5 | 4 | 45 |

*(first three trials were practice trials and thus not coded)

†Conditions are:  1 = 0 lag, 2 = 1 lag, 3 = 4 lag, 4 = 15 lag, 5 = 30 lag

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