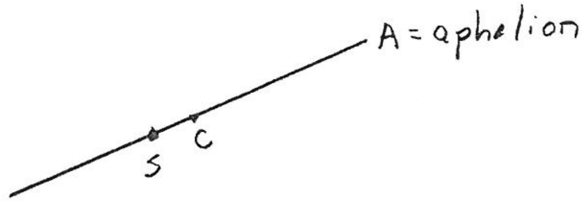
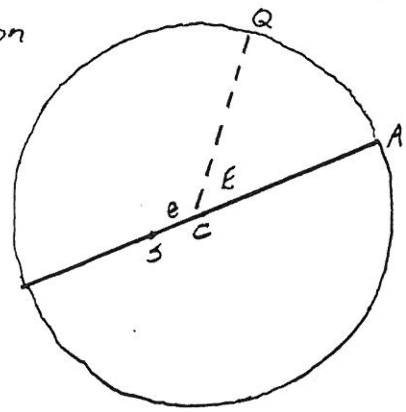
The Keplerian Ellipse



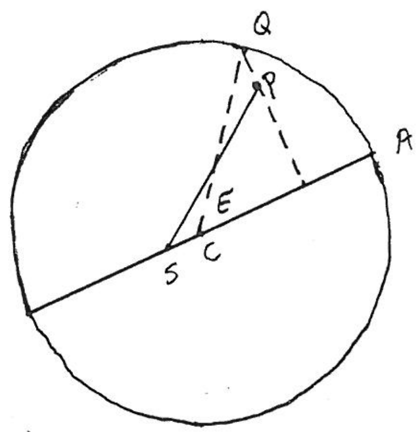
CA = mean distance $= a$

$\frac{CS}{CA}$ = eccentricity $= e$

Start with the eccentricity, the length of line of apsides (2CA), and its orientation, as given by the (heliocentric) longitude of its aphelion



Next draw a circle of radius $r = CA$, with the line of apsides as a diameter. Then, for any angle $ACQ = E$, the so-called "eccentric anomaly," the location of the planet on the elliptic trajectory is given as follows



$$SP = a\left(1 + e\cos E\right)$$

$$\angle ASP = \sin^{-1}\left(\frac{a\sqrt{1-e^2}\,\sin E}{SP}\right)$$

$$= \sin^{-1}\left(\frac{\sqrt{1-e^2}\,\sin E}{1 + e\cos E}\right)$$

The elliptic trajectory is then given by the locus of points P (as E goes from 0 to 360 deg)

Elements of Keplerian Orbits

Orientation of the line of apsides – i.e. longitude of aphelion: *ώ*

Length of the semi-major axis – i.e. mean distance of planet from Sun: *a*

Eccentricity: *e*

Period: *P* – or mean motion in longitude *n*

Orientation of the line of nodes – i.e. longitude of ascending node: *Ω*

Inclination: *i*

Mean longitude at epoch: *L*, or time last at aphelion, *T*

An Assessment of Kepler's Orbital Elements

Based on Those Inferred for 1600 from Newcomb

		Eccentricity	Apehelion	Mean Distance
Mercury	N	0.20555	251°14′09″	0.38710
	K	0.21005	252°49′58″	0.38808
Venus	N	0.00697	305°55′51″	0.72333
	K	0.00692	301°14′22″	0.72413
Earth	N	0.01688	276°04′02″	1.00000
	K	0.01800	275°44′08″	1.00000
Mars	N	0.09304	148°41′58″	1.52369
	K	0.09265	148°59′54″	1.5235
Jupiter	N	0.04784	187°53′27″	5.2027
	K	0.04822	186°52′01″	5.20
Saturn	N	0.05693	265°13′24″	9.546
	K	0.05693	264°57′36″	9.51

$\tilde{\omega}$ - longitude of perihelion

Ω - longitude of ascending node

Appendix C

Numerical Values

Numerical values continue to be refined, and further objects will be found that might be included in tables such as these. Don't be surprised if quite different values appear in other references (and even within the same reference). In some instances there are "official" values, so that different computations share a common set, making comparison possible. The following values are included partly for their interest, and partly so that you can include them, if you wish, in your computations.

C.1 Orbital Elements of Planets

The data for the planets (excluding Pluto) were kindly supplied by L. E. Doggett from data of P. Bretagnon (Reference 72). They are mean elements, referred to the equinox J2000.0. T is measured in Julian centuries from that epoch. The semimajor axes have been derived from the terms in L (the mean longitude at epoch) factored by T, with allowance first made for the precession in longitude. (See Appendix C7.)

For detailed information on osculating elements, refer to the *Astronomical Almanac*. For low-precision formulas, see Reference 38.

Mercury ☿

$$\tilde{\omega} = 77.4561\ 19 + 1.5564\ 77 \times T + 0.0002\ 96 \times T^2$$
$$\Omega = 48.3308\ 93 + 1.1861\ 88 \times T + 0.0001\ 76 \times T^2$$
$$i = 7.0049\ 86 + 0.0018\ 21 \times T - 0.0000\ 18 \times T^2$$
$$e = 0.2056\ 3175 + 0.0000\ 2041 \times T - 0.0000\ 0003 \times T^2$$
$$L = 252.2509\ 06 + 149474.0722\ 49 \times T + 0.0003\ 04 \times T^2$$
$$a = 0.3871\ 0353$$

P= 87.969 days

Venus ♀

$$\tilde{\omega} = 131.5637\ 07 + 1.4022\ 21 \times T - 0.0010\ 73 \times T^2 - 0.0000\ 05 \times T^3$$
$$\Omega = 76.6799\ 20 + 0.9011\ 20 \times T + 0.0004\ 07 \times T^2$$
$$i = 3.3946\ 62 + 0.0010\ 04 \times T - 0.0000\ 01 \times T^2$$
$$e = 0.0067\ 7188 - 0.0000\ 4777 \times T$$
$$L = 181.9798\ 01 + 58519.2130\ 30 \times T + 0.0003\ 11 \times T^2$$
$$a = 0.7233\ 074$$

P= 224.701 days

Earth ♁

$$\tilde{\omega} = 102.9373\ 48 + 1.7195\ 39 \times T + 0.0004\ 60 \times T^2$$
$$e = 0.0167\ 0862 - 0.0000\ 4204 \times T$$
$$L = 1000.4664\ 49 + 36000.7698\ 23 \times T + 0.0003\ 04 \times T^2$$
$$a = 1.0000\ 1161$$

P= 365.256 days

Mars ♂

$$\tilde{\omega} = 336.0602\ 34 + 1.8410\ 44 \times T + 0.0001\ 35 \times T^2$$
$$\Omega = 49.5580\ 93 + 0.7720\ 96 \times T + 0.0000\ 16 \times T^2$$
$$i = 1.8497\ 26 - 0.0006\ 01 \times T + 0.0000\ 13 \times T^2$$
$$e = 0.0934\ 0062 + 0.0000\ 9048 \times T - 0.0000\ 0008 \times T^2$$
$$L = 355.4332\ 75 + 19141.6964\ 75 \times T + 0.0003\ 11 \times T^2$$
$$a = 1.5237\ 1069$$

P= 686.980 days

Jupiter ♃

$$\tilde{\omega} = 374.3313\ 09 + 1.6126\ 38 \times T + 0.0010\ 31 \times T^2 - 0.0000\ 04 \times T^3$$
$$\Omega = 100.4644\ 41 + 1.0209\ 54 \times T + 0.0004\ 01 \times T^2$$
$$i = 1.3032\ 70 - 0.0054\ 97 \times T + 0.0000\ 05 \times T^2$$
$$e = 0.0484\ 9485 + 0.0001\ 6324 \times T - 0.0000\ 0047 \times T^2$$
$$L = 34.3514\ 84 + 3036.3027\ 89 \times T + 0.0002\ 24 \times T^2$$
$$a = 5.2102\ 1558$$

P= 4332.589 days

Saturn ♄

$$\tilde{\omega} = 93.0567\ 87 + 1.9637\ 61 \times T + 0.0008\ 38 \times T^2 + 0.0000\ 05 \times T^3$$
$$\Omega = 113.6655\ 24 + 0.8770\ 94 \times T - 0.0001\ 21 \times T^2 - 0.0000\ 02 \times T^3$$
$$i = 2.4888\ 78 - 0.0037\ 36 \times T - 0.0000\ 15 \times T^2$$
$$e = 0.0555\ 0862 - 0.0003\ 4682 \times T - 0.0000\ 01 \times T^2$$
$$L = 50.0774\ 71 + 1223.5110\ 14 \times T + 0.0005\ 20 \times T^2$$
$$a = 9.5380\ 7012$$

P= 10759.22 days