There Goes the Neighborhood: Does Tipping Behavior exist among Income Groups?

A thesis

submitted by

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In partial fulfillment of the requirements

for the degree of

Master of Science

in

Economics

TUFTS UNIVERSITY

May 19, 2013

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Abstract: The segregation models of Schelling (1969) stand as a cornerstone for understanding how people arrange themselves in space and how such arrangements can be inherently unstable. Extensions of such 'tipping' models concerning race have been the subject of much empirical testing, most notably in Card, Mas and Rothstein (2008). However, little attention has been paid to whether or not similar unstable spatial arrangements occur between income groups when considered separately from race. I propose a model in which social interaction effects lead to low income groups causing tipping behavior amongst high income groups. Using US census data normalized to the tract level from 1970-2000, I uncover compelling evidence for tipping behavior amongst income groups. It is particularly strong for people below/above the 25th percentile of income where tipping points range between 10 and 35 percent. When the proportion of people below the 25th percentile in a neighborhood becomes larger than said tipping point I estimate that the expected decrease in those above the 25th percentile in the neighborhood will be between 10 and 25 percent. Robustness checks indicate that these are not racial tipping points misinterpreted as tipping points for income and that the speed of tipping is fast enough that a discontinuous model is appropriate.

Acknowledgements:

Jeff Zabel, Yannis Ioaniddes, Emilia Simeonova, RZA, GZA, ODB.

Table of Contents:

1. Introduction

Questions of neighbourhood effects and locational choice have long been a central component of urban economics. Models dating back as early as the inter-jurisdictional sorting of Tiebout (1956) and the models of racial segregation found in Schelling (1969, 1971) have provided attempts at formalizing answers to these questions. In these models and their successors the interactions of agents within the models have played a central role, as the utility from one locational choice is partially dependent on the attributes and/or preferences of others who choose the same location. It is the interplay of these endogenous spill overs and the exogenous natural and geographic attributes that can be used to explain stratification within and between locales on the basis of race, education and, in the case of this thesis, income.

Though both the Schelling model and the Tiebout model have had numerous follow ups few studies attempt to combine the ethos of both models. Schelling's tipping model had the original application of segregation by race. Despite Schelling's insistence that race was simply an example for his model (inspired by the policy and social issues at the time) future empirical research has not investigated the implications of the model for groups besides race. In fact, large scale empirical work on tipping points models is very rare. The only studies I am aware of are Card, Mas and Rothstein (2008) and Easterly (2009), both of which use the same dataset as this thesis. The Tiebout model carries with it an implication of sorting based on income, which has sometimes been referred to as Tiebout sorting in the literature. It has received many empirical follow ups that both test and theoretically refine the model. The most notable of such literature has been the works of Epple and Platt (1998) and Epple and Sieg (1999). However, it should be pointed out that no large, country-wide scale empirical investigations have been made into

Tiebout sorting. This analysis will offer such an investigation; looking at neighbourhood tipping by income.

Unlike race, income is an economic characteristic that has a natural order to it, one person's income is always greater than, less than or equal to the income of others. Thus, at its simplest level one would group their neighbourhoods are either in the same income group as themselves, a higher one, or a lower one. This leads to the central question of this thesis; "what do the current occupants of a neighbourhood do when people of an income group lower than their own move in?" Most importantly, I am concerned with the effect the new people moving into the neighbourhood has on the mobility of the current occupants. In the case of lower income groups being the new arrivals, we expect to see a tipping point at which the current occupants decide to leave the neighbourhood en masse. A theoretical model is presented to provide exposition for this point.

I use US Census data from 1970-2000 that has been normalized to the tract level to search for tipping points (that is, discontinuous jumps in the change of upper income groups between two consecutive censuses). Using a variety of techniques for detecting these jumps I find compelling evidence that tipping does exist and is particularly strong when the population is divided by the 25th percentile. In this case being above a particular tipping point in a base year as determined by the percent of residents in the lowest quartile is associated with a 10-25 percent decrease in residents in the top 75 percent of income for a tract. The tipping points found are robust to the possibilities of fuzzy tipping (non-linear relationships being interpreted as discontinuous relationships) and these tipping points could simply be tipping points for race rather than for income.

The rest of the paper proceeds as follows; section 2 gives a literature review, section 3 provides the theoretical model, section 4 describes the data, section 5 presents the strategy for estimating tipping points and their size, section 6 explains the different divisions for income that I use, section 7 presents the results and robustness checks. The paper is concluded and summarized in section 8.

2. Literature Review

Schelling's theoretical works regarding neighborhood effects were published in two articles in 1969 and 1971. Since there is considerable overlap between the two, I will focus on the more extensive 1971 article in this review. The article offers two models to describe the segregation of two different groups when they must arrange themselves within some limited space. The context of these model concerns black and white people forming neighborhoods. The models can, however, be abstracted to any situation where a group consisting of people who are different in only one binary aspect must arrange themselves somehow, so it is appropriate to extend the model to income groups. Both models are explained graphically and not using set theory and higher mathematics, even though they could be.

The first model is the Spatial Proximity Model. Here, the key fact is how one defines their neighborhood, for example a person could define their neighborhood as every space adjacent to where their own space is, or every space within a certain distance to their own space. People then have preferences within these neighborhoods for what mix they would like amongst the groups. For example, a person could have a preference for at least 30 percent of their neighborhood being their own race. Once the initial allocation has been set they rearrange themselves according to some algorithm until they find equilibrium. The model was expanded to become 2 dimensional in the 1971 article, but could also be expanded to 3 dimensions, if one chose to do so.

Schelling provides few examples of his model, but some general inferences about sorting can be drawn from them. For example, in extreme race ratios the proportion of satisfied people will be so small that nearly everyone in the minority will move. This principle means that newcomers will cluster together into areas. Also, the more demanding people are, the higher the density of the clusters of people will be, a relatively simple equilibrium to reach. Integration requires far more complex and difficult patters than segregation. The smaller the minority the more efficient in their patterning they are required to be.

The second model Schelling develops is presented through two dimensional graphs where one axis measures the number of the first group and the other axis the number of the second group. The curves show the maximum number of blacks/whites that that many whites/blacks can tolerate. People's preferences are not homogenous in this model; the most tolerant whites are nearest to the origin. By looking at the resulting parabolas one can find the equilibria and determine whether or not they are stable. This model can also introduce limits on the amount of a certain group allowed in the neighborhood, leading to what Schelling calls "Bounded Neighborhood Models". The main cost they have is that people now must have a common definition of a neighborhood. The model also makes the assumption that a person knows who everyone else in their neighborhood is. The 1971 article expanded the model to include the PDF and CDF for each group's preferences, as shown in the summary of Easterly (2009) later in this section.

Schelling notes two important inferences from the model. First, if there are two locations, the least desirable of them will be filled by almost entirely one group only. The second, and most important for this thesis, is that getting from one equilibrium to another can be achieved by a small perturbation if the initial equilibrium is unstable. This is the 'tipping point' point hypothesis. Congruent with the assumptions of the model, Schelling supposed that the level at which any tipping occurred would be dependent on what people consider being their own neighborhood being common amongst different individuals. Schelling provides some anecdotal evidence of tipping, noting that once neighborhoods have gone from white to black they have rarely gone back. The phenomenon goes beyond neighborhoods too; Schelling recalls the same thing occurring at an ice cream stand in Lexington, MA.

Schelling’s tipping models are entirely theoretical. The most relevant empirical point of reference for this paper is Card, Mas and Rothstein (2008; henceforth CMR). It investigated tipping behavior for race in MSAs and PMSAs in the United States using census data found in the Neighborhood Change Database (NCDB) covering 1970 to 2000. The tipping point is identified within an MSA over a ten year window, that is, tipping points are found for 1970-1980, 1980-1990 and 1990-2000. With proportion of non-whites as the independent variable and change in whites as the dependent variable, CMR uncover tipping points in the range of 5% to 20%. When checked against data from the National Opinion Research Center’s ‘General Social Survey’, it is found that neighborhoods with higher tipping points on average have more tolerant racial attitudes. Tipping points are also found to increase in each successive 10 year window, indicating an increased tolerance in racial attitudes over time. To uncover tipping points two methods are used, the first is a dubbed a ‘search method’ the second a ‘fixed point method’. I use both these methods to identify tipping points in this thesis, the details of which can be found in the estimation strategy section.

The tipping points rise by about a percentage point or two each window, which CMR attribute to increasing racial tolerance. There are, however, large standard deviations to the estimates so the increases could be considered incidental. The search method revealed a lower estimate of the tipping point than the fixed point method in each time period and will, by construction, always recover a tipping point. The fixed point method failed to find a tipping point in 4 of 104 MSAs in the 1970-1980 window and 3 of 113 MSAs in the 1980-1990 window. The fixed point method recovered a tipping point in all 114 MSAs used in the 1990-2000 window.

The tipping points found in CMR may be not be robust to different methods, however, Lee, Seo and Shin (2012) developed a very sophisticated threshold estimation technique that utilizes sup-likelihood-ratio (LR)-type statistics to find the tipping points for Boston, Chicago, New York and Philadelphia in the 1980-1990 window also using the NCDB dataset. The tipping points found are 51.75%, 48.45%, 23.70% and 39.65% respectively. These estimates are all much higher than those presented in CMR. Likewise Wang (2009) found that CMR underestimated tipping points in Los Angeles drastically, attributing this to CMR's methods only being able to find 'sharp' tipping points and not 'fuzzy' tipping points. That is, CMR mistake a non-linearity for a discontinuity in their methods used. In my estimation I address such issues by adding two other methods, the 'segment method' and the 'jump method', both of which are detailed in the estimation strategy section.

Although they do use a vector of control variables in their estimation, CMR note that this may not be sufficient to absorb the effects of covariates related to neighborhood demographic (including income) and housing characteristics. To address this, they present a series of extended quadratic polynomials to extend the specifications of the vector of controls. The estimators are robust to this inclusion, thus they conclude omitted variables are unlikely to be the source of their results.

CMR also suggest that the results could be a result of suburbanization and white exodus from the central city reflecting a preference for lower-density neighborhoods. They do not find that there are any systematic differences in the magnitude of the tipping discontinuity in suburban and urban areas, meaning that a preference for suburbanization is unlikely to be the cause of tipping behavior, as opposed to white preferences for the race of their neighbors.

Further investigations are run later in the paper, finding that housing prices experience only a modest change around the tipping point discontinuity when non-linearity is accounted for. Tipping in the racial compositions of schools is also found using the National Center for Education Statistics' 'Common Core Data' (CCD) from 1990-2000. The presence of dynamic tipping in both housing and school racial compositions indicates that racial preferences are likely to be the cause of the process.

In a companion paper to their 2008 publication CMR (2008b) consider whether or not tipping was 'two-sided'. That is, can tipping occur in the opposite direction from their initial paper? Will an influx of whites cause minorities to leave a neighborhood? They find that tipping is in fact only one-sided, using the same methods and data that they used in their original paper.

The only other thorough investigation of the tipping point model of neighborhood sorting is by Easterly (2009), he uses the same NCDB data as CMR. The two papers are closely related; CMR cites a working version of Easterly's paper and Easterly cites the final published version of

CMR several times and is in some parts an attempt at direct refutation of CMR findings. Easterly notes that the parametric assumptions in CMR come at the cost of being able to find global versus local instability at the tipping points. Local instability is a necessary but not sufficient condition in Schelling (1969, 1971). Global stability, on the other hand, is both a necessary and a sufficient condition for tipping. Easterly's intention is to test the global dynamics necessary to confirm the tipping story.

Easterly first gives an explanation of Schelling's model. In this case w_j denotes the minimum proportion of whites in a neighborhood that each white individual j will be willing to tolerate. That is, as long as the percent of whites in the neighborhood is above w_j then the individual will stay in the neighborhood, if not then they will move out. In this model, whites are heterogeneous in w_j and their preferences can be summarized in the cumulative density function for w. The point where the CDF crosses the 45 degree line is where the fraction of whites is represented as:

$$w = F(w) \tag{2.1}$$

This is the point where the proportion of whites willing to live a neighborhood that is w percent white is equal to w. This is the tipping point, without an equilibrium point such as the one above and two more such that $F(1) = 1$ and $F(0) = 0$.

The dynamics of the white share are given by the distance between $F(w)$ and the 45 degree line:

$$\Delta w = F(w) - w \tag{2.2}$$

This is the equation Easterly actually estimates using a fourth order polynomial.

For a tipping point, one further assumption is needed:

$$F'(w) > 1 \qquad (2.3)$$

Evaluated at a point strictly less than 1 and strictly greater than 0 where (2.1) holds. If (2.3) holds this implies that one of the points where (2.1) holds is an unstable equilibrium. Thus, (2.1) and (2.3) together define a tipping point. A graphical description of this model is provided in Figure 1, which models the 45 degree line and a CDF showing two stable equilibria at F(w)=0 and F(w)=1 and an unstable equilibrium (tipping point) at the fixed point F(w)=w.

Easterly restricts his sample to census tracts that had a population of over 100 in both 1970 and 2000. Easterly also analyzes the data in one 30 year window of 1970-2000, unlike CMR who looked at 3 different ten year windows. In his estimation Easterly uses a fourth order polynomial to estimate equation (2.2) and (2.3), that is, he regresses the change in white share in a census tract from 1970 to 2000 against the initial white share in that census tract in 1970. He uses two specifications, one allowing a different constant for different MSAs and another where the constant is the same in all MSAs. There is no meaningful difference between the two specifications. All the polynomial terms are significant in both regressions. Specifying the regressions in this manner makes the questionable assumption that there is only one nationwide tipping point and not different tipping points in different areas. Despite the significance of the terms there is only one point on the estimated equations where the conditions of equation (2.2) are met. That is, Easterly uncovers just one tipping point at a level of very low white share, unlike the tipping points at a high level of white share predicted by CMR. One interesting result from Easterly's estimates is that condition (2.1) only comes close to being met at points close to zero initial white share and 100% initial white share, indicating very polarized preferences amongst whites, unlike the bell-shaped preferences that Schelling assumed.

Easterly then runs the model with fixed effects for an MSA, allowing tipping points to vary across location and also introduces regressions with controls for income and clustering of standard errors on different levels to check if the results are biased due to spatial autocorrelation. His final sets of regressions analyze the data in ten year windows like CMR. In all additional regressions, the results do not appear to have any meaningful differences; the dynamic behavior predicted by Schelling's tipping model is not present. Easterly concludes that while Schelling's model remains a classic theoretical milestone, it is a slippery model to estimate. By the simplest tests of the model using the most obvious control variables the model cannot be confirmed. While the paper finds some interesting results, CMR seems to be the superior article as its results economize on any assumptions about functional forms, unlike Easterly whose assumptions appear to be prohibitive to his results. In particular he assumes that global stability requires the data to be analyzed on a nationwide level, not adequately acknowledging that MSAs could be self-contained housing markets.

The most notable theoretical expansions of the Schelling Models in the literature are the works of Zhang (2004, 2011). Zhang (2004) is a mathematically sophisticated take on Schelling’s checkerboard model. By employing a theory of stochastic stability and an equilibrium concept from evolutionary game theory Zhang shows that in the long-run residential segregation will prevail most of the time (speaking in terms of probability), that vacancies are more common in black neighborhoods than in white neighborhoods and that whites will pay more for the same housing than blacks. Zhang (2011) expands upon this by working in the tipping model. Zhang finds an equilibrium to be resistant to tipping if it takes fewer than expected periods to return to the stochastic equilibrium it started at. The main contribution to tipping theory here is its definition in terms of stochastic stability.

Sethi and Somanathan (2004) decompose the effect of racial disparity and income inequality in residential segregation and in Schelling's tipping phenomenon in particular. They develop a model that considers a city with a continuum of houses and two disjoint neighborhoods that are the same size. Taking into account income, race and rent, the model predicts that income disparity between ethnicities partially causes tipping, but tipping will still exist when income between the different groups converges. However, the result is purely theoretical and I do not know of any empirical works that have tested this theory.

Wang (2009) also investigated tipping amongst race; however, his analysis is restricted to the Los Angeles MSA. Using data from the Los Angeles County Union Tract Data Series, the paper estimates tipping points and the speed at which tipping occurs in the same 3 ten-year windows as CMR. Wang does this using a “fuzzy set function of right shoulder sigmoidal form” to compensate for the absence of covariates in his estimation and to allow for adequate flexibility so that heterogeneity of the functional form does not ruin his estimates. Wang also estimates spatial autocorrelations, not finding any evidence that this could be a source of bias in the estimates. As noted before, by allowing for the speed of tipping in the model or ‘fuzzy’ tipping, Wang estimates much higher tipping points in Los Angeles than CMR did.

Interestingly Wang states that the CMR has underestimated the tipping points for Los Angeles for each of the 3 ten year windows that he analyzes. If tipping occurs, change in the upper group as a function of proportion of the lower income group in a base should be a cubic with a sharp range of transition in between two flat portions. Wang’s choice of a sigmoidal function to model designates the middle of the range of transition as the tipping point, rather than the start of it. Thus, when the speed of tipping is slow (that is, the cubic has a wide range of transition) the tipping point will be much higher than those in CMR, which due to its sharp tipping specification will designate the start of the range of transition as the tipping point. Conceptually, CMR is correct and Wang is not, a tipping point should be the start of the range of transition, not the finish. However, it is still a well put together paper and its awareness of the speed of tipping makes does raise an important issue of whether or not a discontinuous model is even appropriate. If the range of transition is wide and relatively flat then to specify the model as discontinuous could still be a mistake. This is acknowledged using a robustness check for 'fuzzy' tipping specified later.

Bruch and Mare (2006) conducted empirical investigations into Schelling's theories using data on Detroit from 1976 -1992 to show a large increase in the racial tolerance of whites and small decrease in racial tolerance of blacks. Then in an addition to the Detroit data, they then use data from Los Angeles and Boston from 1992-1994 to show that neighborhoods do not react to racial makeup in terms of a threshold function, but instead in terms of a non-linear continuous function. This, once again, raises the concern that CMR (and, by the common use of techniques, this thesis) mistake non-linear relationships ('fuzzy tipping') for discontinuous jumps ('sharp tipping').

Tiebout (1956) is the most notable theoretical model of sorting based on income in the literature. This is a reply to Musgrave (1939) and Samuelson (1954) that both denied "market type" solutions exist to the problem of determining the optimal level of expenditure of public goods. Tiebout shows that this is only true on a federal level but not a local level.

Tiebout reckons that if, at the local level, revenue and expenditure patterns are fixed then consumers will move to the community where the local government will best satisfy their preferences. As such, the greater the number of communities and the greater the variance between them the closer the consumer will come to recognizing their preferences fully.

The model Tiebout uses has 7 assumptions; full mobility, full information about government revenues and expenditures, a large variety of communities, employment is the same everywhere, there are no externalities between communities for public services, there is a fixed factor that restricts a community from reaching optimal size and communities will attempt to reach this optimum size. The model is this form is acknowledged as unrealistic and extreme, but with these in place the market for public goods will be the same as if they were private. Relaxation of some of these assumptions can find a more realistic solution, but within this model we find sorting is based completely on one's preferences for publicly provided goods and how much they can afford to contribute (that is, income). If agents are assumed to have the same preferences, sorting is based totally on income.

The most significant expansion to Tiebout (1956) has been the works of Epple and Platt (1998), Epple and Sieg (1999) and Epple, Romer and Sieg (2001). These papers provide a framework for testing models characterizing equilibrium in a system of local jurisdictions. The framework involves communities with fixed jurisdictional boundaries that constitute a MSA. Each one of these communities provides a public good, the cost of which is covered by a local property tax. What exactly the public good is will be determined by the collective choice of the neighborhood. If there are assumed to be no migration costs, the equilibrium conditions are that budgets are balanced, markets clear and no household wishes to change its residence.

The goal of the second step in the process is to identify factors that influence locational choice and quantify their importance since the Tiebout hypothesis implies at least some sorting based on local public goods. The approach of this paper is to provide a model of community choice and public choice that would suggest the identifying restrictions. Utility is a function of taste, public goods, local housing goods and private goods, this function is subject to income being equal to the sum of property tax, the amount spent on private goods and the amount spent on housing. In equilibrium, the following conditions will hold; boundary indifference, stratification and increasing bundles. If tastes are homogeneous then each community will have only individuals whose income lies along a certain interval.

Epple and Sieg use 1980 census data from the Boston MSA to carry out their empirical investigation. This is because since there are no overlapping jurisdictions in MA. It models the quantiles of income distribution very well. The tendency of prices to ascend with community income provides positive evidence for the general validity of the equilibrium model. The empirical findings reveal that within community income variance is greater than between community income variance. However, such analysis is not carried out on the neighborhood level. There are also large differences in public good provision along communities. The size of jurisdiction may mean that there is no Tiebout effect on the neighborhood level.

Creative expansions to the Epple-Sieg approach can be found in Banzhaf and Walsh (2008, 2013). Their 2008 paper tests Tiebout's sorting mechanism using a locational equilibrium model, similar to those of Epple-Sieg. The model predicts that as population density increases, so will public goods provision. If the provision increase is large enough, average income will also increase. They test this in the context of environmental quality using census data on California from 1990-2000 and define neighborhoods as half-mile diameter circles. They find strong evidence that people "vote with their feet" in response to environmental quality, supporting Tiebout's initial hypothesis. The paper raises a concern for this thesis; a census tract may not correctly approximate a neighborhood.

Banzhaf and Walsh (2013) is the only empirical work I know of that combines both Tiebout's and Schelling's theories on sorting, though the paper is still mostly theoretical. Once again using an Epple-Sieg style model of locational equilibrium, this time customized so as to account for group segregation. They characterize the equilibria of the model and then derive the comparative statics of improvements in public goods. They find that the dynamics of tipping are dependent on the level of public good provision. Most interestingly public good provision to low-income neighborhoods is shown to actually increase segregation. An empirical test using the same data as in their 2008 paper and the 'fixed point' method from CMR finds that a great deal of the noise in tipping point estimation comes from not accounting for public good provision. They conclude that more precise estimates of tipping points can be found if one includes measures of this. This is not included in this thesis, but could be a useful expansion in future work.

3. Theoretical Framework

I use a simple model of the housing market adapted from Becker (1991) where the buyers are individual households and do not consider suppliers. Consider a neighborhood where housing is homogenous and there are two groups of buyers; upper income households (U) and lower income households (L). I denote the inverse demand function for homes as:

$$P_g(n_g, L) \tag{3.1}$$

Where $g \in \{U, L\}$ is the households' grouping as either upper income or lower income. So there will be n_g households who are willing to pay at least P_g to live in a neighborhood where the share of low income households is L. By construction, we have the following derivatives:

$$\partial P_U / \partial n_U < 0 \quad (3.2)$$

$$\partial P_L / \partial n_L < 0 \quad (3.3)$$

$$\partial P_L / \partial L = 0 \quad (3.4)$$

$$\partial P_U / \partial L < 0 \text{ if L exceeds some threshold T} \quad (3.5)$$

(3.5) represents the social interaction effect that leads to tipping. Note that the inverse demand function does not include U, thus we are assuming that while the upper income group has a social interaction effect the lower income group does not and thus will not experience tipping behavior in reaction to either income group.

For a mixed equilibrium there must exist some lower income group share L such that the L^{th} highest lower income household has the same willingness to pay as the $(1\text{-}L)^{th}$ highest upper income household. Thus, the equilibrium condition is:

$$P_U(1-L, L) = P_L(L, L) \quad (3.6)$$

The derivative of the left hand function in the above equality with respect to the lower income group share is:

$$\partial P_U(1-L, L) / \partial L = \partial P_U / \partial L - \partial P_U / \partial n_U \quad (3.7)$$

The second component on the right hand side of (3.7) is positive, I assume the social interaction effect is small and positive at L=0 and after the threshold T becomes more and more negative. Thus, the left hand side of equation (3.6) has an inverted parabola shaped graph while the right hand function of the equality is assumed to be downward sloping and linear.

Now we consider various positive shocks to L, if the current equilibrium is on the left hand side of the parabola the shock will have no long run effect, since the marginal willingness to pay of the upper income group here exceeds that of the lower income group and transactions will occur to revert the allocation back to the original point. Thus, this is a stable equilibrium. If a shock occurs to any potential equilibrium on the downward sloping side of the parabola, however, there would be a greater reduction in upper income groups' willingness to pay by more than the lower income groups so the neighborhood will instead reallocate itself at all lower income groups' equilibrium. From this we can see that any equilibrium on this side of the curve is unstable and we can then call the lower income share at which the point of tangency lies a tipping point. Figure 2 provides a graphical description of this, with equilibria to the left of the point of tangency shown as stable and equiliria at and to the right of it being shown as unstable. Thus, the point of tangency is the tipping point.

The framework is extreme and makes several big assumptions, homogenous housing, no transaction costs to moving, there is no accounting for housing supply either and although it states that people differ in income but does not integrate this into their willingness to pay. Because of such assumptions, the theory exists simply for exposition of the concept of tipping. In reality we should only expect to find rapid declines in upper income groups at tipping points but not the completely divided equilibrium.

4. Data Description

The data used in the analysis is collected from census data and covers the 1970, 1980, 1990 and 2000 censuses. Up until the 2000 census, the database was called the Underclass database and was collected by the Urban Institute, a non-partisan think tank based in Washington, D.C. Since the 2000 census it has been updated by the commercial firm, Geolytics Inc. and has been renamed the Neighborhood Change Database (NCDB) and made available on CD-ROM. The lowest unit of observation in the dataset is the census tract, a division that is supposed to approximate a neighborhood that will usually contain between 2,500 and 8,000 people. Tract boundaries are chosen with the intention of capturing people with similar socioeconomic characteristics. This is a fact to be aware of in the analysis, as it could exaggerate segregation. Tracts never cross state, MSA or county lines. Thus, since I carry out my analysis at the MSA level, the NCDB can in this case be thought of as 362 different datasets; one for each MSA. Note that due to restrictions placed on the dataset the actual amount of MSA's used will be much less.

When analyzing data over multiple censuses the problem of changing boundaries must be addressed. Geolytics done this in several ways, depending on what resources they had for each census. They used geographic information software (GIS) to overlay 2000 tract boundaries on earlier boundaries. They then used 1990 census block data to estimate the proportion of old tracts in various demographic measures that went into the new tract and then recalculated the 1990 tract data using the 2000 tract boundaries. Block data was not available for 1970 and 1980, so the tracts were matched using only spatial changes in tract boundaries. Thus, the 1970 and 1980 data are less accurate.

Census data brings with it the possibility of undercounting the population. This is of core concern to this analysis as high income people should be more likely to be enumerated than low income people. I cannot offer a solution to this problem and can instead only state that results are contingent on this 'undercounting' not being large enough to have any significant impact on my results. At worst, my results will have a downward bias on the estimation of tipping points and my estimates can simply be viewed as a lower bound for the tipping point.

A further problem with these data is the unit of observation being the census tract itself. It would be preferable to have individual level data, given that the theory of tipping would be expected to happen on the household level since these are the units that move between areas. Such data are available. However, it is not useful for my purposes because, unlike the NCDB, there is not any such dataset where individuals are normalized across multiple censuses. Thus, if such data were to be used, the results would be nonsense.

The next problem with the census tract as a unit of observation is the accuracy of its approximation to a neighborhood. As stated before, census tracts are products designed to collect a similar group of individuals. Many researchers consider this to be a virtue, arguing that by doing this tracts provide a good approximation to one's neighborhood or community. Such a view is, I believe perfectly reasonably, based more on data availability than any empirical justification. There are several reasons that make the census tract a questionable unit of observation. First, if it is does not align with what the people in a tract consider to be their own neighborhood, my results will be compromised. Likewise, the census tract could be too aggregate of a measure and thus mitigate the effects found in the analysis. Anderson et al (1994) and Hersh (1995) show that the correlation between demographics and exogenous factors can be very sensitive to community definitions, meaning that if a census tract is not a correct approximation of a neighborhood, results could be highly misleading.

Several different studies have given reason for us to question whether a census tract is actually an accurate approximation of a neighborhood. The majority of this work comes from the sociology literature. Sampson (1988) suggests that Electoral Wards (the English and Welsh equivalent of a census tract) where inferior approximations to community compared to neighborhoods created using surveys to find out about peoples' usual behavior within a certain geographic area (that is, where they go to visit friends and relatives, for leisure entertainment, participation in sporting events, participation in organizations and clubs etc.). Matthews et al (2006) found that the best approximation to geographic areas such as neighborhoods used to study problems such as the income issues that this thesis is concerned with can be found using combinations of demographic data, surveys on behavior and GIS techniques. Banzhaf and Walsh (2008) used a less elaborate but similar approach of combining ancillary data with GIS techniques to define a neighborhood when studying public goods provision and tipping.

It would be optimal to combine the census data I have with other data sources to reach an accurate definition of a neighborhood in the context of this study. Such a task is only permissible when the analysis is restricted to a small area though. Thus, I chose to sacrifice some of the validity of my results for breath when I use census tracts as my unit of observation, allowing me to make my analysis nationwide, rather than only a very small area. Such methods for identifying neighborhoods could be useful in future elaborations on this study as robustness checks.

Table 1 shows the summary statistics for the data. I present data on tract population, education, age, average family income (not inflation adjusted), gender ratios and race. We see expected increases in tract population and average family income within the tract both also carry large standard deviations, but this is also to be expected. Looking at the proportion of people in different age brackets we can see that the distribution of age has shifted to older groups, also what we'd expect given the baby boomer generation. Male to female ratio in a tract is tantamount to constant across all censuses in terms of both mean and variances. The result revealed in this table that is not commonly known is that the proportion of people below the poverty line has been increasing over time.

5. Empirical Strategy

The original Schelling and Tiebout models are static, assuming both a fixed supply of housing in a neighborhood and a fixed total population. To allow for the increasing population and increases in the supply of housing in census tracts, I use changes in the population of income groups within a tract, each expressed as a fraction of the base period population. Similarly, the full panel capabilities of the dataset will not be exploited; instead the data will be evaluated in the 3 ten year windows available (1970-1980, 1980-1990, 1990-2000). I only look for tipping points in MSAs and not on a state or national level. Thus, I am assuming that any tipping points found will only be for a single city and that the interactions between different MSA's housing markets are not large enough to influence results. The main dependent variable is the ten year change in the high income group within the tract, relative to the initial population, while the main dependent variable is the proportion of the population in the tract who were in a low income group in the base year. The regression that I wish to estimate is:

$$Y_{ic,t} = \sigma(m_{ic,t-10}) + d1[m_{ic,t-10} > 0] + f_c + X_{ic,t-10}B + \varepsilon_{ic,t} \quad (5.1)$$

where $Y_{ic,t}$ is the ten year change in the income group of interest in tract i, in city c and in year t. $\sigma(m_{ic,t-10})$ represents a spline fitted with knots at each quartile of $m_{ic,t-10}$. $m_{ic,t-10}$ is defined as the difference between the proportion of people in the lower income group and the tipping point, that is $m_{ic,t-10} = p_{ic,t-10} - T_{c,t}$. d is a coefficient on a dummy variable that takes on the value 1 if $m_{ic,t-10}$ is positive. $X_{ic,t-10}$ represents a vector of covariate values and $\varepsilon_{ic,t}$ is a standard error term.

The coefficient of interest in equation (5.1) is d which tells us the size of the tipping points for each MSA in the sample. The null hypothesis is that d=0, in which case there is no detectable difference in the change of the high income group between census tracts that are above the tipping point for their respective MSA compared with those that are below the tipping point.

In order to estimate equation (5.1), I need to know the tipping point. Such information is not known a priori so the point must instead be estimated from the data. For this, I employ four different methods. Two are taken from CMR, while the other two are adapted from the statistics literature. In every case, to ensure independence, a two-thirds random subsample of the data is used to estimate the tipping point and the remaining one-third subsample is used to estimate equation (5.1). This is because of the result found in Leamer (1978) that when the same data are used to identify the location of structural break and to estimate its magnitude, conventional test statistics will reject the null hypothesis d=0 too often.

The first method used is taken from CMR; the authors call it the 'search' method. Similar to the tests used to identify structural breaks in time series data such as Bai (1997), this technique looks for the biggest t-statistic at the 'jump'. The technique ignores covariates and approximates the parameters with a constant function in the range *[0,N]* so that equation (5.1) becomes:

$$Y_{ijk} = c_{jk} + T_{jk} 1[n_p > n_p^*] + \varepsilon_{ijk} \qquad (5.2)$$

where N is set equal to the highest proportion of people in the second income group found in all of the tracts in the city. The tipping point is then found by simply selecting the value of n_p^* that maximizes the R-squared of equation (5.2). In practice, this involves iteratively running regressions at different values of n_p^* with and seeing which one creates the best fit for the model. This value of n_p^* is the tipping point that is used in equation (5.1) with the remaining third of the dataset.

The first of the two non-linear methods used to detect tipping points is the CMR 'fixed point' method. The fixed point technique works on smoothed approximations to

$$E\left[Y_{ic,t} \mid a, m_{ic,t-10}\right] \tag{5.3}$$

for different cities. Typically this function resembles a cubic with a sharp range of transition, but otherwise being relatively flat over the range of values for $m_{ic,t-10}$. Using the fixed point method, if there is a tipping point at n_p^* then the following condition will be met:

$$E\left[Y_{ij,t} \mid c, x_{ij,t-1} = m^* - \varepsilon\right] > E\left[Y_{ic,t|a}\right] > E\left[Y_{ic,t} \mid a, x_{ic,t-10} = m^* + \varepsilon\right] \tag{5.4}$$

Under this condition, the tipping point will be the proportion of high income people at which the neighborhood high income population grows at the average rate for the city. To identify this point, a fourth order polynomial is fitted with the outcome and the dependent variable. The root of this function is selected and another fourth order polynomial is fitted to all the points within ten percentage points of this root, then the root of this function is selected as the tipping point. In the case that there is more than one real root, I select the root at which the slope of the function is the most negative. If there is no real root, a tipping point is not identified.

The fixed point method is preferred over the search point method because it can fail to return a tipping point and behaves well in smaller sample sizes, not reflecting obvious outliers (a prevalent problem in the case of the poverty estimator). It still does not condition on covariates, however, and has no real awareness of the speed of tipping.

The second non-linear method I use is extremely similar to that presented in Bowman et al (2006) and follows on research from Hall and Titterington (1992) and Muller (1992). I will refer to it as the jump method. Made available in an R package in 2012, this works by comparing left and right handed nonparametric regression curves and uses a hypothesis test to indicate how appropriate the point is for a discontinuity. This is done iteratively over the data by fitting the following two regressions:

$$Y_{ij,t} = \hat{m}_L(n_p^-) + \varepsilon_{ij,t} \tag{5.5}$$

$$Y_{ic,t} = \hat{m}_R(n_p^+) + \varepsilon_{ic,t} \tag{5.6}$$

where $\hat{m}_L(n_p^-)$ and $\hat{m}_R(n_p^+)$ are smooth loess regression functions fitted to either side of the point n_p. Thus, the rudimentary information on the presence of any discontinuity at this point is contained in:

$$\{\hat{m}_L(n_i) - \hat{m}_R(n_i)\}^2 \tag{5.7}$$

A tipping point is selected at the point n_i where equation (5.7) is maximized. Hypothesis tests would usually be run to assess the significance of the discontinuity; however, I ignore these instead assessing the discontinuities significance using the coefficient d in equation (5.1).

The jump method follows a similar theme to the search method. However, I believe it has the advantage of flexibility. The search method runs the risk of misinterpreted fuzzy tipping as sharp tipping, given an adequate smoother this will not occur in a loess regression, which will model a fuzzy tipping point continuously and only detect sharp tipping as discontinuities. The method does not allow for covariates, a definite drawback when one considers the aforementioned concerns of Banzhaff and Walsh (2010). However, according to the theoretical properties of smoothing laid out in Loader (1996), if the smoothing parameter is small the effects of bias should be negligible.

The final technique I use has the intention of being a robustness check, rather than a primary result. It follows in the tradition of broken line relationships generally applied to the biostatistics literature. Having its origins in Ulm (1991) and Betts et al (2007) it was finalized and made available as a package for the statistical software R in Muggeo (2008). The technique is analogous to the CMR search technique in its linearity and goal of maximizing fit. However, it offers several advantages over the search technique to be detailed after the exposition of the technique. In line with the name of the R package it comes with, I refer to this as the 'segment' method.

This technique assumes that there are 3 different segments in the relationship between the change in an upper income group and the base year population of a lower income group. They consist of a relatively flat section where the independent variable has little effect on the dependent variable, followed by a relatively large effect after the tipping point, which is soon followed by another relatively flat section. Thus, the regression the technique estimates is:

$$Y_{ijk} = a + \alpha_1 n_p + \alpha_2 (n_p - n_p^*)_+ + \alpha_3 (n_p - n_p^{**})_+ + X_{ij,t-10} B + \varepsilon_{ij,t} \quad (5.8)$$

where n_p^* is the tipping point and n_p^{**} represents the proportion of income group 2 at which the tipping ends. In practice the method works by iteratively fitting the piecewise linear model with the linear predictors:

$$\alpha_1 n_p + \alpha_2 (n_p - n_p^*)_+ + \alpha_3 (n_p - n_p^{**})_+ + \gamma_1 I(n_p > n_p^*)_- + \gamma_2 I(n_p > n_p^{**})_- + B_{ij,t-10} X + \varepsilon_{ij,t} \quad (5.9)$$

Where $I(\cdot)_{-} = -I(\cdot)$ and γ is the parameter that can be interpreted as the re-parameterization of the tipping point and the point at which the tipping process ends. For each iteration a linear model is fitted and the points of interest are updated via $\hat{n}_p^{*} = \hat{n}_p^{*} + \hat{\gamma}_1 / \hat{\alpha}_2$ and $\hat{n}_p^{**} = \hat{n}_p^{**} + \hat{\gamma}_2 / \hat{\alpha}_3$. Note that in each case, $\hat{\gamma}$ measures the gap, at the current estimate of n_p^{*}, between the two straight lines fitted in equation (5.8). When the algorithm converges the gap is 'small', that is, $\hat{\gamma} \approx 0$. The standard error is obtained via the delta method.

The 'segment' approach offers two advantages over the search method. First, it can recognize that there is not a tipping point, (the search method will always obtain a tipping point whether there is one or not). This occurs when the algorithm does not converge. The technique also conditions on covariates, although according to Loader (1996) and Hansen (2000) this is not necessary to correctly find a tipping point if the functional form has been guessed correctly, so it not the most pertinent issue in terms of estimation. However, Banzhaff and Walsh (2013) suggests that CMR's tipping points could be inaccurate because of a failure to adjust for differences in public goods and it would also seem to be important to control for race in the estimation so that I do not simply detect the same effect as CMR.

Second, the segment method also offers more information about the nature of tipping. The search method only tells us about 'sharp' tipping and not 'fuzzy' tipping. That is, it does not tell us about the speed at which tipping occurs. Wang (2011) was aware of this issue, his solution was to use a sigmoidal fuzzy set function to find both the tipping point and the speed of tipping. Using a piecewise continuous function offers, I believe, adequate flexibility to achieve that same goal and has a more easily interpretable measure of the speed of tipping. One can simply look at the sum of the coefficients α_1 and α_2 (in equation (5.4)) to measure the slope over the range of tipping to find the speed of tipping.

There are disadvantages to the segment method too. The method requires one to select starting points for estimation and thus runs the risk of convergence at a local maximum different from the global maximum where the fit is the best. If this is the case the tipping point estimate could be incorrect.

6. Income Divisions

The investigation of tipping based on income involves the step of dividing people into two different income groups in order to check for tipping behavior. This is more nuanced than previous studies of tipping among different racial groups, where obvious dividing criteria exist. In light of the broad range possibilities for division among income groups; I choose to test for tipping among several divisions of income groups.

Several divisions into income groups are available in the data, with minimal manipulation required. These are the so-called 'vanilla' divisions and are described in Table 2, where we can see the income brackets found in the data for each census. Each bracket in the data gives the amount of households in that census tract who are in that income bracket. There are notably more brackets as time increases, creating finer divisions. Thus, my estimates are more accurate the further into the future they are. I also use poverty, which is a simple binary division that requires no manipulation to apply the estimation strategy to.

The 'vanilla' divisions come with several problems making them unusable in the form shown in Table 2. First, the brackets for comparison are changed for each census. For example, one can see from table 2 that the only common segments between the 1970 and 1980 are $50,000. 1980 and 1990 have 6 common segments between them and 1990 and 2000 have 12. Despite the increased number of common segments in later censuses, a second problem is that they are not in real terms. Also, using such a division does not account for the increases in real income that have occurred in each ten year window between censuses. Given these problems, the divisions as presented in table 1 are not suitable for estimating tipping as they are, a modification is needed. To overcome these problems, I choose to divide people by their income percentile, using the 10^{th}, 25^{th} and 50^{th} percentile as my divisions. However, any such percentiles can only be drawn from the information available in Table 2. The data only give us the amount of people in a tract within each of those divisions presented and, obviously, these divisions are not going to provide me with the percentiles I need. Thus, percentiles must instead be estimated. To do this, I first sum the total amount of people in each division for each MSA in the sample, then using linear interpolation; I find the percentiles of income for that particular MSA. Once the income percentiles for an MSA are determined, I then find the amount of people in each census tract who are within the percentile of interest. Then one can solve for the amount of people in that tract below whatever level of income is appropriate for the percentile of interest.

Using percentiles does not solve all these problems, however. The divisions described in Tables 2 use income as reported in a decennial census. This presents several problems. Theoretical models of tipping carry with them an assumption that an agent can observe what group everyone else is in as well as what group they themselves are in. In reality, it is rare that one know the income of neighbors precisely. However, despite not having access to other peoples pay cheques, it is possible to guess the income of one's neighbor based on several variables that are more easily observable to neighbors than a person's annual income.

Usually, such observables can be regressed against a person's reported income and then the predicted values from such a regression can be used as an estimate of permanent income in the analysis. Not only does such an approach avoid the problem of unobservability, but it also avoids problems that could arise through the volatility of income. The census only captures a person's income at one point during a ten year span; this is not necessarily an accurate description of that person's income over the entire ten years. Permanent income, however, is a measure that represents an estimate of a person's income over a much greater time period. This stems from the components used to estimate permanent income having far lower volatility across time than income itself.

Unfortunately, the lowest unit of observation available to me is the census tract. Thus, a much more nuanced approach will have to be used to get to permanent income. In the same vein as the vanilla income divisions, I use percentiles. But instead of estimating percentiles of income, I estimate the percentiles for permanent income in an MSA. In order to estimate these I need to construct a permanent income distribution for each census tract, analogous to discrete distributions that can be constructed using the divisions in Table 2. This can be done using a series of logistic regressions. For each MSA I run a logistic regression where the dependent variable is the proportion of people in that tract in that division and the independent variables are the measures education, age and race for that tract then obtain the predicted values from the logit. It is imperative that logistic regression is used, since many of the proportions of people in a certain income brackets are extremely low (less than 2%) and using OLS can produce nonsensical negative values. From here the process is the same as that detailed for the vanilla percentiles.

7. Results and Robustness Checks

7.1 Ordinary Income Groups

Table 3 shows the means and standard deviations for tipping points found by method and income group division. Table 4 shows the correlations between tipping points found under the different methods. Tipping in this case concerns the change in the tract population of those not in the listed income group (for example, how the top 90% react to the bottom 10%). Under the methods used there appears to be no notable trend amongst any of the methods to reveal higher or lower tipping points than any other. Encouragingly, the tipping points appear to be fairly consistent across the 3 methods displayed, despite the lack of any real correlation between points found in the correlation tables in table 4. Thus, despite the different methods being unlikely to reveal the same tipping point (although the individual tipping points are not shown the difference in the tipping point found can vary by numbers as large as 30 points depending on the method used) it appears as the sample increases the average tipping point will converge to a similar result. This result combined with the large variances notable in table 4 show that, even more so than for race, a precise tipping point is extremely difficult to identify in a particular city, but with a large enough sample of cities it can be established that tipping does exist. Given the similar results across the different methods, this can be regarded as strong evidence that tipping does exist among income groups. There is little evidence of a time trend when looking horizontally across the tipping points. Looking vertically, however, there is an obvious trend at the higher the percentile being considered for tipping, the higher the tipping point will be until there is simply no tipping.

The results in tables 3 and 4 do not go higher than the 50th percentile. I experimented with the 75th and 90th percentile, however, there appears to be no evidence that tipping exists among these income groups. In the case of both percentiles the fixed point method failed to find tipping points in about half the cases. The search and jump method, which must find a tipping point by construction, returned nonsensical results finding an average tipping point of over 99%. Those cities that did return tipping points for the fixed point method had similarly high tipping points. Furthermore, in the case of several cities it was not possible to calculate a tipping point for these percentiles, as many tracts do not contain anyone within that percentile limiting sample sizes in procedures that are very data intensive.

In addition to finding tipping points, I also assess the size of tipping at the points summarized in table 3. A tipping point is not meaningful if the population of the neighborhood only tips by a minute amount. Table 5 provides information on the size of tipping points by reporting the coefficient on the dummy variable found in equation (5.1). This can be interpreted as the expected change in neighborhood population of those not in the income group listed if the neighborhood is above the tipping point found for that city. From the table we are not able to definitely say whether or not tipping is important in many cases. The jump method tends to identity points where the discontinuity is smaller than in the search and fixed point methods, in roughly half the cases the discontinuity found is not significantly different from 0. There are no apparent trends either horizontally across time or vertically across income groups. Ultimately, from table 5 we can infer that although tipping points can be identified at levels of poverty, 10th percentile, 25th percentile and the 50th percentile the size the discontinuities at the tipping points are only consistently non-trivial at the divisions of poverty and the 25th percentile.

7.2 Permanent Income Groups

As noted in section 5, ordinary income may not give us the full picture. Thus, table 6 shows the results for tipping points based on the permanent income regression also detailed in section 6. The tipping points for the 50th percentile show similar results in terms of the mean and the variance. However, in the 10th and 25th percentile we find tipping points that are generally lower than those found using the normal income group percentiles and exhibit much lower variance. Also, the horizontal and vertical trends are clear in this table. Tipping points increase as we raise the income percentile and have also been rising as time goes by. While the vertical trend has a clear a priori interpretation, the horizontal trend does not. Further investigation of this trend is outside the scope of this paper. It could, however, be an interesting avenue for further research, particularly when 2010 data is made available. Table 7 shows the correlations between the tipping points for a certain percentile, the tipping points appear much more consistent than for the normal income group percentiles typically displaying correlations of about 0.25. Because of the consistency and lower variance in the permanent income tipping points, I use these for further robustness checks, rather than the tipping points shown in table 3.

Table 8 assesses the size of the permanent income tipping points using the same regression as in table 5 simply with the new tipping points. As with the tipping points themselves, there is far more consistency in the permanent income estimates of the size of tipping as there was with the ordinary income estimates for the size of tipping. Unlike for ordinary income, in any particular percentile bracket within a decade no estimate is significantly different from any other at the 5% level. The 3 methods appear to agree about the significance of the size of the tipping point except for the 1980-1990 brackets in the 50th percentile.

Once again, the 25th percentile is where tipping is the largest. The coefficients on the discontinuous jump in table 8 can be seen to be notably larger for the 25th percentile than the 10th or 50th percentiles (or the 75th and 90th percentiles where once again tipping was not found). This agrees with the results found for ordinary percentiles where similar vertical patterns in the size of tipping were found. Thus, I say that the 25th percentile is the best binary division to make for calculating tipping points. A more sophisticated theory that incorporates stochastic stability as in Zhang (2011) is required to interpret and although that is beyond the scope of this thesis could be used in future expansions on this work.

The permanent income results add an interesting layer to the ideas of income mixing in neighborhoods; we can infer that it is not income itself that people are reacting to when they consider their neighbors, but the perception of income. It also provides evidence that this tipping is the result of social interactions rather than other things related to income, such as housing prices forcing people to move out in tipping like behavior as found in Dorn (2010).

7.3 Robustness Checks

There is a possibility that these are simply the racial tipping points that were found in CMR that appear to be income tipping points since the two are so highly correlated. To test for this possibility I run the same regression as in equation (5.1) but instead of using change in an income groups as the dependent variable, I instead use the change in the amount of whites in the neighborhood and add the change in the relevant income (formerly the dependent variable) to the vector of controls. The change in income group is fitted with splines to allow adequate flexibility. The results of these regressions are shown in the top row of each income group bracket in table (9). The coefficients are generally small compared with the ones found in table (7) and are, except for a couple of cases, not statistically significant. In some cases the coefficient is even positive, the opposite of what we would except if these had been racial tipping points mistakenly interpreted as income tipping points. Thus, neighborhoods beyond the tipping point do not experience any detectable change in the amount of white people in the neighborhood, indicating that these are not racial tipping points that have been detected in table (7).

In light of the criticisms brought up in Wang (2010), one should account for the possibility that the model detailed in equation (5.1) is not appropriate and that a fuzzy tipping model might be more appropriate than a sharp one (that is, it could be that non-linearity mistaken has been discontinuous jump). Although, qualitatively speaking, modeling a function as smooth or as discontinuous there is no real conceptual difference between the two types of functions as far as the tipping story is concerned, it is prudent to check that my model choice is appropriate.

In order to assess whether or not my tipping points have been misestimated because of this I fit a linear piecewise regression as detailed in equation (5.8). Table (10) details the results for this 'segment' method that was detailed in the estimation strategy. The first point to note is that the tipping points found are remarkable similar to those in for the permanent income percentiles, the only case in which there is any real difference is the (rather bizarre) tipping point of 85% found for the 1990-2000 period in the 50th percentile. The tipping points are statistically significant in every case, exhibiting very low standard errors in their estimation. More importantly the coefficients are taking on the signs and magnitudes we would expect if the tipping hypothesis was correct. The coefficient on B1 is either positive or negative and usually very small, indicating that prior to the tipping point the relationship between the change people above the percentile is largely unaffected by the proportion of people of people below the division in the tract. The range of tipping is usually fairly small too indicating that the discontinuous jump model that was shown in equation (5.1) was appropriate for modeling the tipping relationship. The coefficient on B2 is always negative and usually large compared with the other coefficients as well as been much more statistically significant. We can also get an idea of the speed of tipping through B2, looking we can see that in the range of tipping for every increase in the lower income groups tract share in the base year by 1% there is, on average, a decrease in the change in the upper income of between 1% to 3%. This fast speed of tipping is further evidence that the models used based on equation 1 were appropriate to use. Thus, as one might think, it makes little

8. Concluding Remarks

The tipping point hypothesis expressed by Schelling (1969) remains one of the most influential expressions of spatial arrangement in the urban economics literature. Empirical works preceding this one have chosen not to take the model outside of its original example of race; in the spirit of Tiebout (1956) I have chosen to test the tipping hypothesis with income groups instead.

The theoretical model used postulates that an social interaction effect exists between different income groups, by construction the derivative of an individual's utility function with respect to this is zero if they are in a low income group and can be positive, negative or zero if they are in the higher income group. This leads to there being 3 different equilibrium arrangements one where there the neighborhood is all a high income group, one (unstable) equilibrium where there is a mix of the high and low income groups and one where there is only the low income group. The instability of the mixed equilibrium leads to tipping behavior, where a slight perturbation about the equilibrium leads to the neighborhood reallocating itself at the equilibrium with no people in the upper income group. Put simply, the theory says that inverse-gentrification is a rapid, non-linear process. The model exists for exposition purposes and makes several extreme and unrealistic assumptions including not moving costs, that there is always somewhere for the person to move to and that income and can only be divided into groups. Thus, when testing for the tipping effect I do not expect to see complete flight of the upper income group, but still a large departure in that population group.

Using US census data from 1970-2000 that has been normalized to the level of census tract from the year 2000, I estimate tipping points for each MSA in the US with over 100 tracts in them using 3 different methods in each of the 3 ten year windows available in the data, then test the significance of these tipping points using a discontinuous regression where whether the proportion of people in the lower income group in a census tract in the base year of the window is above the tipping point or not as a dummy variable. The dependent variable in these regressions is the 10 year change in the upper income group. Income divisions used for the analysis are poverty status, 10^{th} percentile, 25^{th} percentile and the 50^{th} percentile. Unreported tests were also carried out using the 75^{th} and 90^{th} percentile divisions, but no real evidence for tipping was found. There is compelling evidence for tipping amongst the other groups, however, the tipping points appear to be particularly large for the poverty and 25^{th} percentile divisions and do not appear to have any real trends in the tipping point or the size of tipping across time. The tipping points also only exhibit small correlations with one another between methods and over time periods.

When permanent income estimates are used instead of the ordinary income variables offered in the data to calculate income percentiles more consistent tipping points are found. There is a much higher correlation of the tipping points across the 3 different methods, but not in cities over time. I also find the size of tipping to be more consistent across the 3 methods and it is generally found to be more statistically significant. The estimates remain the same when splines are used on the covariates, indicating the result is unlikely to be due to omitted variables. The size of tipping exhibits a pattern where it is largest at the 25^{th} percentile. The significance of the results provides compelling evidence that tipping behavior does exist amongst income groups.

There are alternative explanations that the results are robust to. The first is the possibility that the tipping points uncovered are simply racial tipping points since there is broad overlap between racial divisions and income divisions. I find that the share of whites in the census tract does not tip at the tipping points discovered when the change in the income group is controlled for. Results are rarely significant and are even show positive tipping in some cases. The second possibility that I check for is that the neighborhoods I found are simply neighborhoods in which a large group of people moved out from and thus high mobility neighborhoods have a bias to those with high shares of the lower income group. We see that the results are robust to this too, when I use change in the lower income group as the dependent variable it there is no evidence found that the lower income group also leaves the neighborhood en masse if they are beyond the tipping point. These results provide even further evidence that the tipping point is due to the hypothesized social interaction effect and not simply a tipping point for something else mistaken as a tipping point for income.

The results are also robust to the possibility of fuzzy tipping that was brought up in Wang. Tipping points found using the segment method are similar to those found using the fixed point, search and jump methods. Furthermore the range of tipping is very short and the speed of tipping is very fast, indicating that my specification of the tipping point as a discontinuous jump is an appropriate characterization of the tipping point.

These results are compelling, but there is fruitful ground for future work in the same area. US census data from 1970-2010 is expected in the near future and will provide further opportunity to test the tipping hypothesis. Also only one side of the theoretical framework has been tested. It postulates not only that inverse-gentrification is a rapid non-linear process but that regular gentrification is a linear process. Testing for tipping in the opposite direction would serve as a useful companion piece to this work.

The theory is currently very simple, expansions can be made to it to better reflect the continuous nature of income using the stochastically stable equilibrium theories of Zhang (2011), were one can view the probability of moving as dependent on the difference between your income and your neighbors income. Such a theory would be useful for providing predictions in how tipping behavior would effected by changes in the income distribution and income inequality.

Further work can also be done with more sharply defined percentiles, since the size of the tipping points rise before the 25th percentile and then begin to fall afterwards, it may be more accurate to model tipping through a probability distribution if clear shape is given to the size of tipping. Furthermore, we do not have to include the entire population when estimating tipping points, it could be informative to see how the tipping points changes as we take different subsets of the population (for example how does the top 25% react to an influx of the bottom 25%).

Methods similar to the search that can be adapted to estimate tipping points using instrumental variables have also been presented in Caner and Hansen (2003). If an appropriate instrument were discovered, the adaptation and application of such methods would be another useful companion piece to this work.

Appendix A: Figures

Figure 1: Visual Description of the Intuitive Tipping Model

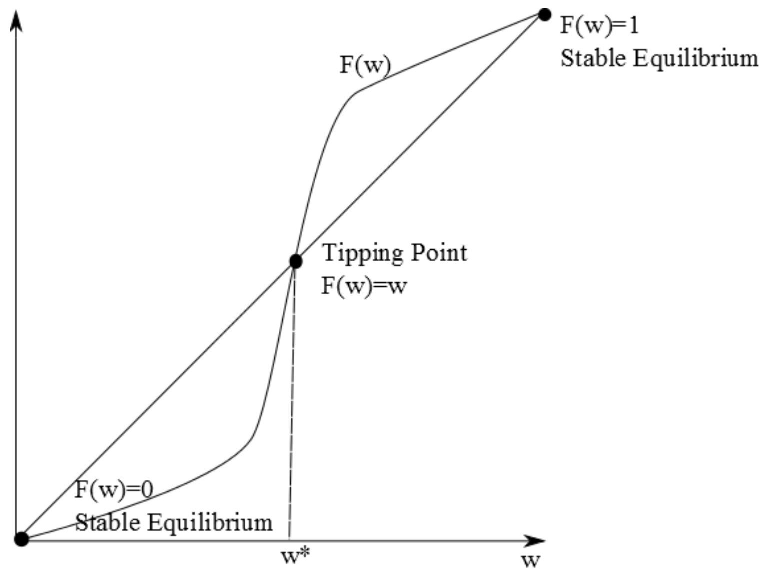
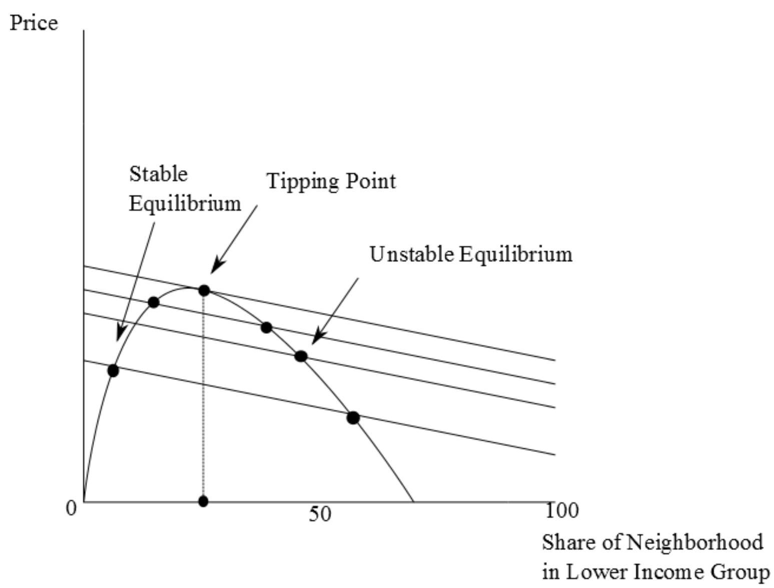


Figure 2: Visual Description of Theorectical Framwork



Appendix B: Tables

Table 1-Summary Statistics

	1970	1980	1990	2000
Tract Population	2.262	2.767	3.800	4.300
	(2.243)	(2.107)	(1.768)	(2.142)
Average Family Income	8.356	19.004	42.415	61.932
	(6.529)	(12.665)	(21.817)	(32.342)
Percent between 0-4	6.069	6.731	8.690	6.579
	(4.515)	(4.427)	(2.953)	(2.363)
Percent between 5-17	18.519	15.317	16.535	18.498
	(13.323)	(9.355)	(5.211)	(5.506)
Percent between 18-64	39.750	46.281	61.137	61.429
	(26.082)	(27.848)	(9.195)	(8.688)
Percent 65+	6.463	8.569	12.795	12.941
	(6.805)	(7.884)	(7.414)	(7.343)
Male-Female Ratio	0.987	1.023	0.976	1.114
	(2.658)	(2.956)	(3.125)	(3.406)
Percent with High School Diploma and no higher	17.235	20.244	18.200	17.127
	(18.803)	(23.409)	(22.700)	(20.733)
Percent with Bachelors Degree or higher	6.546	10.681	14.564	17.484
	(12.482)	(21.859)	(30.793)	(34.126)
Percent People under poverty line	7.777	9.265	13.803	13.385
	(9.687)	(10.833)	(12.671)	(11.671)
N	65443	65443	65443	65443

*Means of variables are presented for the corresponding census. Standard deviations are in Parentheses. Population and Income are in thousands. Income is in nominal terms.

Table 2-'Vanilla' Income Divisions

1970	1980	1990	2000
$ 1,000.00	$ 2,500.00	$ 5,000.00	$ 10,000.00
$ 2,000.00	$ 5,000.00	$ 10,000.00	$ 15,000.00
$ 4,000.00	$ 7,500.00	$ 12,500.00	$ 20,000.00
$ 6,000.00	$ 10,000.00	$ 15,000.00	$ 25,000.00
$ 7,000.00	$ 12,500.00	$ 17,500.00	$ 30,000.00
$ 9,000.00	$ 15,000.00	$ 20,000.00	$ 35,000.00
$ 12,000.00	$ 17,500.00	$ 22,500.00	$ 40,000.00
$ 50,000.00	$ 20,000.00	$ 25,000.00	$ 45,000.00
	$ 22,500.00	$ 27,500.00	$ 50,000.00
	$ 25,000.00	$ 30,000.00	$ 60,000.00
	$ 27,500.00	$ 35,000.00	$ 75,000.00
	$ 30,000.00	$ 40,000.00	$ 100,000.00
	$ 35,000.00	$ 50,000.00	$ 125,000.00
	$ 40,000.00	$ 60,000.00	$ 150,000.00
	$ 50,000.00	$ 75,000.00	$ 200,000.00
	$ 75,000.00	$ 100,000.00	
		$ 125,000.00	
		$ 150,000.00	

*For each of listed levels the variable in question is the proportion of people in the census tract who had an annual income below that level in the last year.

Table 3-Summary of Tipping Points

		1970-80			1980-90			1990-2000		
		Fixed Point Method	Search Method	Jump Method	Fixed Point method	Search Method	Jump Method	Fixed Point Method	Search Method	Jump Method
Poverty	Mean	12.467	11.117	8.706	10.014	9.336	10.476	10.845	8.273	10.735
	SD	10.663	7.424	6.327	9.456	6.091	10.505	11.969	7.921	10.565
	# of MSAs where no tipping was indentified	0	--	--	0	--	--	0	--	--
10th Percentile	Mean	9.819	7.371	6.452	7.971	7.987	7.065	13.315	13.300	14.435
	SD	8.212	4.550	4.250	7.595	6.553	6.683	16.279	16.008	16.113
	# of MSAs where no tipping was indentified	1	--	--	2	--	--	3	--	--
25th Percentile	Mean	12.734	16.927	14.239	15.410	16.674	17.975	27.696	32.573	30.838
	SD	10.792	7.976	8.137	12.056	11.502	13.813	19.985	26.486	22.314
	# of MSAs where no tipping was indentified	0	--	--	0	--	--	0	--	--
50th Percentile	Mean	29.354	38.156	34.215	33.522	40.534	36.199	41.558	60.551	51.742
	SD	19.354	29.356	21.356	16.504	24.087	21.660	20.971	26.962	23.849
	# of MSAs where no tipping was indentified	3	--	--	0	--	--	1	--	--
	# of MSAs in sample	97	97	97	105	105	105	113	113	113

Table 4-Correlations between Tipping Points

Poverty								
1970-1980, fixed point	1							
1970-1980, search	0.689	1						
1970-1980, jump	0.530	0.554	1					
1980-1990, fixed point	0.023	0.010	0.039	1				
1980-1990, search	0.115	0.083	0.082	0.024	1			
1980-1990, jump	0.064	0.062	0.125	0.124	0.176	1		
1990-2000, fixed point	0.152	0.229	0.287	0.247	0.014	0.178	1	
1990-2000, search	0.041	0.080	0.097	0.115	0.045	0.192	0.402	1
1990-2000, jump	0.090	0.290	0.358	0.291	0.029	0.187	0.454	0.406
10th Percentile								
1970-1980, fixed point	1							
1970-1980, search	0.033	1						
1970-1980, jump	0.171	0.410	1					
1980-1990, fixed point	0.169	0.054	0.034	1				
1980-1990, search	0.013	0.030	0.031	0.062	1			
1980-1990, jump	0.009	0.018	0.112	0.110	0.032	1		
1990-2000, fixed point	0.130	0.049	0.088	0.267	0.018	0.221	1	
1990-2000, search	0.173	0.032	0.009	0.240	0.046	0.086	0.308	1
1990-2000, jump	0.021	0.140	0.026	0.193	0.177	0.299	0.301	0.446
25th Percentile								
1970-1980, fixed point	1							
1970-1980, search	0.067	1						
1970-1980, jump	0.244	0.445	1					
1980-1990, fixed point	0.046	0.024	0.204	1				
1980-1990, search	0.143	0.111	0.162	0.074	1			
1980-1990, jump	0.089	0.335	0.127	0.290	0.078	1		
1990-2000, fixed point	0.003	0.000	0.080	0.072	0.134	0.048	1	
1990-2000, search	0.036	0.127	0.099	0.077	0.176	0.055	0.201	1
1990-2000, jump	0.050	0.009	0.017	0.038	0.135	0.125	0.300	0.357
50th Percentile								
, fixed point	1							
1970-1980, search	0.013	1						
1970-1980, jump	0.325	0.619	1					
1980-1990, fixed point	0.051	0.035	0.005	1				
1980-1990, search	0.238	0.124	0.156	0.032	1			
1980-1990, jump	0.515	0.351	0.254	0.182	0.172	1		
1990-2000, fixed point	0.066	0.035	0.0156	0.040	0.069	0.043	1	
1990-2000, search	0.028	0.045	0.332	0.172	0.193	0.094	0.012	1
1990-2000, jump	0.168	0.131	0.035	0.119	0.112	0.048	0.006	0.372

Table 5-Size of Tipping Points

		1970-1980			1980-1990			1990-2000		
		Fixed Point	Search	Jump	Fixed Point	Search	Jump	Fixed Point	Search	Jump
Pov	Size of Tip	13.53	17.74	5.21	11.34	11.52	13.60	14.61	8.24	11.47
		(2.41)	(2.78)	(2.53)	(1.74)	(1.83)	(1.71)	(2.13)	(2.29)	(2.14)
	N	10541	10541	10541	11522	11522	11522	11730	11927	11923
	R-Squared	0.34	0.35	0.35	0.37	0.36	0.36	0.15	0.15	0.15
10%	Size of Tip	1.67	14.45	0.41	20.75	17.52	0.74	1.32	7.69	3.52
		(3.30)	(3.24)	(3.06)	(1.68)	(1.70)	(1.62)	(5.28)	(4.98)	(4.80)
	N	9698	10506	10506	10609	11141	11515	10393	11916	11912
	R-Squared	0.38	0.36	0.36	0.40	0.35	0.35	0.09	0.07	0.09
25%	Size of Tip	22.07	18.42	4.20	6.01	8.34	1.05	19.42	17.67	8.96
		(3.35)	(3.67)	(3.54)	(1.57)	(1.72)	(1.74)	(2.85)	(3.13)	(2.98)
	N	10240	10249	10249	11513	11513	11513	11906	11906	11902
	R-Squared	0.35	0.35	0.35	0.37	0.37	0.36	0.17	0.17	0.16
50%	Size of Tip	12.56	1.23	5.63	7.41	2.18	4.39	19.06	6.96	2.37
		(2.68)	(3.56)	(2.95)	(1.84)	(1.98)	(2.12)	(2.95)	(3.35)	(2.35)
	N	10240	10249	10249	11366	11495	11495	11081	11866	11886
	R-Squared	0.29	0.28	0.29	0.35	0.32	0.33	0.24	0.22	0.22

*The coefficient on the regression in equation (1) is presented. To better represent the ‘size of the tip’, I present absolute values, even though the coefficient represents the percentage decrease in amount of people in the tract in the upper income group over the ten year window. Standard errors are in parentheses and are clustered by MSA.

Table 6-Summary of Tipping Points for Permanent Income

		1970-80			1980-90			1990-2000		
		Fixed Point Method	Search Method	Jump Method	Fixed Point method	Search Method	Jump Method	Fixed Point Method	Search Method	Jump Method
10th Percentile	Mean	6.092	5.912	5.169	8.744	8.110	7.739	9.904	10.180	10.302
	SD	4.118	2.762	2.294	6.681	4.017	5.265	7.269	7.109	6.948
	# of MSAs where no tipping was indentified	0	--	--	0	--	--	0	--	--
25th Percentile	Mean	10.515	14.321	12.735	23.857	21.277	21.277	25.454	25.864	23.545
	SD	11.080	7.110	5.610	12.251	12.892	12.892	12.251	13.146	11.288
	# of MSAs where no tipping was indentified	0	--	--	0	--	--	0	--	--
50th Percentile	Mean	23.565	21.561	19.354	32.530	53.293	44.963	44.126	51.923	45.902
	SD	14.354	12.534	15.658	34.437	25.594	22.759	12.088	20.645	18.634
	# of MSAs where no tipping was indentified	0	--	--	0	--	--	0	--	--
	# of MSAs in sample	97	97	97	105	105	105	113	113	113

Table 7-Correlations between Tipping Point for Permanent Income

10th Percentile								
1970-1980, fixed point	1							
1970-1980, search	0.402	1						
1970-1980, jump	0.117	0.242	1					
1980-1990, fixed point	0.182	0.281	0.236	1				
1980-1990, search	0.032	0.161	-0.004	0.160	1			
1980-1990, jump	0.281	0.299	0.151	0.154	0.415	1		
1990-2000, fixed point	0.012	-0.029	0.006	-0.052	0.339	0.171	1	
1990-2000, search	0.035	0.090	0.201	0.029	0.315	0.171	0.241	1
1990-2000, jump	0.008	0.082	0.230	0.051	0.364	0.303	0.353	0.409
25th Percentile								
1970-1980, fixed point	1							
1970-1980, search	0.355	1						
1970-1980, jump	0.120	0.540	1					
1980-1990, fixed point	0.150	0.024	0.054	1				
1980-1990, search	0.131	0.106	0.089	0.353	1			
1980-1990, jump	0.014	0.083	0.201	0.347	0.412	1		
1990-2000, fixed point	0.191	0.082	0.128	0.082	0.026	0.106	1	
1990-2000, search	0.009	0.137	0.100	0.262	0.112	0.090	0.024	1
1990-2000, jump	0.079	0.154	0.306	0.070	0.051	0.109	0.133	0.523
50th Percentile								
1970-1980, fixed point	1.000							
1970-1980, search	0.358	1.000						
1970-1980, jump	0.251	0.351	1					
1980-1990, fixed point	0.615	0.212	0.325	1				
1980-1990, search	0.212	0.021	0.123	0.239	1			
1980-1990, jump	0.055	0.035	0.231	0.318	0.521	1		
1990-2000, fixed point	0.154	0.045	0.321	0.108	0.045	0.172	1	
1990-2000, search	0.654	0.356	0.115	0.231	0.103	0.222	0.266	1
1990-2000, jump	0.325	0.354	0.152	0.201	0.333	0.362	0.067	0.070

Table 8-Size of Tipping Points for Permanent Income Divisions

		1970-1980			1980-1990			1990-2000		
		Fixed Point	Search	Jump	Fixed Point	Search	Jump	Fixed Point	Search	Jump
10%	Size of Tip	5.00	2.29	3.26	6.31	3.80	2.91	7.98	10.39	10.82
		(1.40)	(1.16)	(1.20)	(3.54)	(3.52)	(3.95)	(3.32)	(3.25)	(3.30)
	N	7063	7063	7063	9354	9354	9354	10026	10026	10026
	R-Squared	0.28	0.28	0.28	0.26	0.27	0.25	0.16	0.16	0.16
25%	Size of Tip	29.40	39.19	51.25	20.15	25.26	10.71	14.88	17.24	14.16
		(11.41)	(11.44)	(11.58)	(3.92)	(3.72)	(3.87)	(3.48)	(3.46)	(3.56)
	N	7063	7063	7063	9354	9354	9354	10026	10026	10026
	R-Squared	0.29	0.29	0.29	0.27	0.28	0.27	0.16	0.15	0.16
50%	Size of Tip	21.32	9.35	15.65	16.71	6.31	0.45	7.25	15.49	8.95
		(9.65)	(4.21)	(5.32)	(6.00)	(5.28)	(5.07)	(4.14)	(3.92)	(4.13)
	N	7063	7063	7063	9354	9354	9354	10026	10026	10026
	R-Squared	0.27	0.27	0.27	0.18	0.19	0.19	0.12	0.12	0.12

*The coefficient on the regression in equation (1) is presented. To better represent the ‘size of the tip’, I present absolute values, even though the coefficient represents the percentage decrease in amount of people in the tract in the upper income group over the ten year window. Standard errors are in parentheses and are clustered by MSA.

Table 9-Change in Race at Tipping Point

		1970-1980			1980-1990			1990-2000		
		Fixed Point	Search	Jump	Fixed Point	Search	Jump	Fixed Point	Search	Jump
10%	Size of Tip: Race	-0.69	4.54	2.44	5.63	-1.06	-2.06	-4.22	4.16	5.33
		(5.59)	(4.23)	(5.59)	(2.19)	(2.20)	(3.04)	(3.98)	(4.29)	(4.35)
	N	7063	7063	7063	9354	9354	9354	10026	10026	10026
25%	Size of Tip: Race	11.51	4.85	7.81	-5.23	-1.66	0.19	4.92	0.27	0.93
		(3.80)	(4.08)	(5.21)	(2.45)	(2.52)	(2.90)	(4.29)	(4.30)	(4.41)
	N	7063	7063	7063	9354	9354	9354	10026	10026	10026
50%	Size of Tip: Race	0.36	2.56	-1.33	-5.00	0.86	-3.77	-0.02	-7.24	-2.58
		(1.92)	(3.56)	(3.45)	(2.67)	(2.99)	(2.88)	(4.11)	(3.18)	(3.41)
	N	7063	7063	7063	9354	9354	9354	10026	10026	10026

*Size of tipping for race is the same regressions as in (1) with change in white population are the dependent variable and change in upper income group used as a control variable. Size of tipping for the lowest percentile using the change in the lower percentile in the census tract. Permanent income percentiles are used in the calculation. Standard errors are clustered on the MSA level and are presented in Parentheses.

Table 10-Fuzzy Tipping Estimates

		1970-1980		1980-1990		1990-2000	
10th Percentile	B1	5.156	3.432	0.304	0.036	1.203	0.059
		(6.615)	(3.833)	(0.421)	(0.076)	(0.365)	(0.099)
	B2	-8.561	-6.638	-3.515	-1.114	-2.265	-1.275
		(2.651)	(0.412)	(0.518)	(0.356)	(0.407)	(0.133)
	B3	3.615	2.773	0.122	0.686	0.210	0.280
		(2.156)	(1.433)	(0.003)	(0.678)	(0.022)	(0.771)
	Tipping Point	2.650	5.870	10.325	12.544	9.871	13.950
		(0.615)	(0.277)	(2.236)	(2.381)	(1.398)	(1.087)
	Range of Tipping	2.514	3.545	4.223	7.356	11.326	8.366
	Controls	n	y	n	y	n	y
25th Percentile	B1	2.514	0.150	0.725	0.178	0.835	0.097
		(1.254)	(0.134)	(0.267)	(0.068)	(0.377)	(0.093)
	B2	-4.154	-1.713	-1.417	-1.356	-1.528	-0.904
		(2.328)	(1.506)	(0.288)	(0.235)	(0.498)	(0.088)
	B3	0.156	0.383	0.615	0.567	0.197	0.280
		(0.213)	(0.016)	(0.769)	(0.536)	(0.359)	(0.730)
	Tipping Point	10.561	17.800	16.253	25.614	18.730	28.930
		(1.256)	(1.007)	(2.354)	(1.401)	(2.488)	(1.401)
	Range of Tipping	2.654	6.514	12.164	10.235	13.682	9.627
	Controls	n	y	n	y	n	y
50th Percentile	B1	0.561	-0.466	0.235	0.123	-0.557	0.000
		(1.515)	(0.031)	(0.563)	(0.225)	(0.229)	(0.000)
	B2	-5.564	-3.719	-1.230	-1.468	-30.675	-1.114
		(0.351)	(0.813)	(0.298)	(0.313)	(3.823)	(0.220)
	B3	4.321	5.182	0.687	0.688	0.387	0.600
		(4.156)	(1.377)	(0.113)	(2.532)	(0.107)	(6.712)
	Tipping Point	39.654	44.820	33.564	42.356	85.210	40.000
		(4.651)	(3.271)	(1.650)	(4.351)	(1.353)	(7.127)
	Range of Tipping	1.956	3.234	8.223	6.356	1.091	7.427
	Controls	n	y	n	y	n	y

*Tipping Point is the first breakpoint in the regression. The range of Tipping is the estimate of the second breakpoint minus the estimate of the first breakpoint.

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