   a. Corollary: if trajectory a circle, no point of equiangular motion at all (see Appendix); at best a point oscillating in location along the line of apsides
   b. Obvious question: how can the theory give heliocentric longitudes within observational accuracy
   c. Kepler took the trouble to answer (Chapter 21), showing that the location of the equant is more critical than the location of the center
8. Notice finally that without Tycho's accurate measurements, no Keplerian reform of trajectories, for modified vicarious theory would have been found accurate to within 10 min of arc -- the old observational standard
   a. Names and dates in the history of science would be different
   b. Kepler's view of the relative accuracy of Tycho's observations was also crucial here

E. "Phase 3": Bisect the Eccentricity of the Earth-Sun Orbit

1. Kepler chose first to abandon the equant, not just because it had long been criticized, but also because he saw no hope of giving a physical explanation of equal angular motion about a mere point in space
   a. Note here that two separate elements involved in any planetary theory, the trajectory and the variable rate of motion along it
   b. Really then two separate assumptions being adopted in prior theories
2. From before he had begun with Mars, he had conjectured that the (arc length) velocity of the planets was inversely proportional to their distance from the sun -- in keeping with his conjecture that a magnetic flux from a rotating sun drove all the planets, a flux that diminished inversely with distance
   a. He showed that this variation in velocity is approximately correct, via comparison with planets
   b. Moreover, with Ptolemy's bisected eccentricity, exactly correct at two ends of lines of apsides (on the deferent) for each planet, though not elsewhere
   c. Thus reconcilable with classic approaches to fixing apsides and eccentricity
3. Problem: in prior theories of the sun, including Tycho's superior theory, no bisected eccentricity of earth-sun orbit; instead equiangular motion about the center
   a. Therefore a direct empirical objection to hypothesis that planet velocities vary inversely with distance from the sun (taking earth to be a planet)
   b. Had to dispense with this objection before proceeding
   c. Provided multiple arguments from different observations, considering Ptolemaic, Copernican, and Tychonic theories, as well as his own, and ended with an argument laying out his physical conjecture
   d. (A numerical error made the first of these arguments appear better than it was)
4. In his main effort to show that bisected eccentricity should hold for the earth-sun orbit too, Kepler used combinations of three observations 687 days apart so that Mars (presumably) in the same place each time (or, on Ptolemaic view, center of Mars's epicycle in the same place on its deferent)

a. (Note the tacit assumption, which amounts to counting on nature being regular in a certain way; also the assumption that Mars is in orbit around the sun, for on Ptolemaic theory it is not in the same place every 687 days, only the center of its epicycle is)

b. Could then use triangulation to infer ratios of three earth-sun distances in each case, from which could calculate center of presumed circle

c. Calculated distances subject to errors from observational inaccuracies

d. Not surprising, then, that he obtained varying results, ranging from 0.025 to 0.01653 using his vicarious theory of Mars's heliocentric longitudes, and from 0.01837 to 0.01530 using the preliminary Copernican-type theory Tycho had developed for Mars about the mean sun in the 1590s

e. But all showing near middle between Sun and equant, and definitely not coincident with latter

f. Also compared apogeal and perigeal apparent diameters of sun (30/31), giving a value of 0.0164

5. Transporting this same reasoning to a Ptolemaic system showed that Mars's epicycle requires a point around which equiangular motion occurs that is removed correspondingly from the point at which it is attached to the deferent

a. That is, the vicarious hypothesis and observations of geocentric longitudes of Mar again defined three distinct triangles every 687 days between the earth, the point of attachment of the epicycle on the deferent, and Mars on the epicycle (see figure in Appendix)

b. These three defined a circular trajectory for Mars on its epicycle showing that its point of attachment and point around which motion is equiangular straddle the center of the circle, at least to reasonable approximation

c. This, of course, is a conclusion about Mars's epicycle, not about the earth-sun orbit, but that did not stop Kepler:

> That these characteristics belong to the Ptolemaic epicycle, is properly demonstrated. But that they are carried over from the epicycle to the theory of the sun is shown by a probable argument only, pieced together from Ptolemaic opinions (Chapter 26, p. 145)

d. Arguing that whatever is true of Mars's epicycle should be true of those of Jupiter and Saturn as well, Kepler offers a diatribe against Ptolemaic systems (see Appendix), while nevertheless granting that his bisected eccentricity reform can be incorporated within them

e. Having shown that the reasoning from observations (and the vicarious hypothesis) can be transported from one system to another, Kepler from here on reasons in terms of the Copernican

6. On this basis hypothesized exact bisection for a circular earth (or sun) orbit, contrary to a tradition dating back to Hipparchus: 0.018, half of Tycho's value, 0.03584, rounded

a. (Note: did not use average of calculated values; hence an idealization)

b. Checked by comparing heliocentric longitudes of earth calculated with new model against Tycho's original at the quadrants and octants of anomaly, finding good agreement (9 sec, but correcting for calculation error, 1 min 7 sec)