THE NEWTONIAN REVOLUTION – Part One

Philosophy 167: Science Before Newton's *Principia*

Class 12

Newton's Early Unpublished Work in Mechanics

November 25, 2014

Philosophy 167: Science Before Newton's Principia

Assignment for November 25

Newton's Early Unpublished Work in Mechanics

Reading:

Newton, Isaac, "On Circular Motion," from Herivel, The Background to Newton's Principia, pp. 195-198.

-------, "On Motion in a Cycloid," ibid., pp. 203-207.

-------, "The Laws of Motion," ibid., pp. 208-218.

-------, Excerpts from the manuscript "De Gravitatione et aequipondio fluidorum," ibid., pp. 226-235.

Wilson, "From Kepler's Laws, So-Called, to Universal Gravitation: Empirical Factors," pp. 136-147.

Questions to Focus On:

1. How does Newton's treatment of circular motion differ from Huygens's? To what extent do the differences reflect their being preoccupied with different problems?

2. What physical principles does Newton take to be axiomatic in his proof that motion along a cycloid is isochronous? What is it about the cycloid that makes it more readily amenable to a solution than the large arc pendulum?

3. Which "laws of motion" does Newton take to be fundamental -- i.e. axiomatic -- in his paper on impact? How do these differ from Descartes' laws?

4. How does Newton's theory of the motion of bodies under impact differ from Huygens's? What are the differences attributable to?

5. What was Newton getting at in the so-called Moon test of the late 1660's? In particular, how would he have then proceeded if the test had been successful -- i.e. if he had used the more accurate measure of the radius of the earth that Picard obtained during the 1670's?

6. Exactly what objections is Newton lodging against Descartes' "imaginings" in "De Gravitatione et aequipondio fluidorum" ("On the Gravity and Equilibrium of Fluids")? To what extent are these objections based on empirical considerations?

Newton's Early Unpublished Work in Mechanics

I. Background: Newton in the 1660's and 1670's

A. Huygens and the Science of Mechanics

1. Huygens's efforts in mechanics from the mid-1650's until publication of *Horologium Oscillatorium* in 1673 yielded a substantial core of our modern science of mechanics

a. Much of the core that is taught in the first semester of introductory physics courses -- viz. results that do not depend on inverse-square gravity, instead taking g to be constant and parallel

b. Huygens not only put forward a wide range of results within this subdomain, all of which still stand, but he also provided evidence in support of them that meets modern standards

2. Huygens did more than just extend Galileo's fragment on "natural" motion to such topics as pendulums; his results extended the domain of the field beyond just motion governed by gravitational fall

a. Uniform circular motion and motion under perfectly elastic impact are not as such related to motion governed by gravitational fall at all

b. But Huygens ended up relating them to it by pursuing evidence for them that was tied to -- indeed, parasitic on -- Galileo's fragment of a theory of motion governed by gravitational fall

c. Two examples of this: the use of his version of Toricelli's principle in the treatment of impact, and the use of the conical pendulum as a way of getting evidence on centrifugal forces

3. In the process Huygens also extended the prevailing conception of empirical evidence to such an extent that one can argue not only that he met modern standards of evidence, but that he went a large way toward "discovering" and establishing them in these efforts

a. First, he made clear the extent to which theory is a vehicle for evidence, extending the range of predictions that can be derived from hypotheses and allowing much stronger inferences to be drawn from observations

(1) What allows him to achieve higher quality evidence than Galileo is, more than anything else, his having a larger fragment of a theory than Galileo had

(2) The extension of Galileo's theory to circular motion and the pendulums opened far more tractable ways of developing evidence

b. Second, he displayed the advantages of having a growing network of interrelated theories

(1) Doing so allows increasingly diverse data to be brought to bear on all parts of the network

(2) Evidence can then start accruing to older parts of the network while it is being extended to new topics and areas

c. The clearest illustration of this is the way in which Huygens's account of circular motion and the conical pendulum supplied the first real evidence for the principle of inertia

(1) The difficult evidential problem posed by this principle is that one must have some way of showing that an impediment or force is always required for a body not to continue moving uniformly in a straight line (the contrapose of the principle of inertia)

(2) Solution of this evidential problem: derive a theoretical measure of the force required for departure from such motion, and then confirm this measure experimentally

(3) Precisely how Huygens proceeds in his derivation of v^2/r: a measure of the departure from what the motion would be if it were to continue unimpeded

(4) This is the general form that the evidence for the law of inertia continued to take with Newton and thereafter: develop a theoretical measure of forces required for departure from such motion and then confirm this measure empirically

(5) Inertial motion not confirmed by direct experiment, but by serving as a sort of "null hypothesis" in the development of theories of force

d. Third, Huygens brought out, perhaps unintentionally, how measures of fundamental quantities that occur pervasively in a theory -- e.g. *g* -- can provide extremely high quality evidence bearing on that theory

(1) Convergent measured values provide stronger support for a theory than simple hypothesis testing does

(2) And small discrepancies in the measured values bring out the limiting bounds of accuracy of a theory and provide a basis for new discoveries, as the variation of g with latitude did

4. Using Kuhn's terms, Huygens established a paradigm for a tradition of normal science in mechanics, in the process transforming it from an immature into a mature science

a. The approach involved defining problems in a way that would make them amenable to mathematical solution -- "the mathematization of nature", to use Yoder's sub-title

b. But the distinctive feature of the approach was to tie these problems into previous theoretical efforts on other problems, in the process constraining solutions to them and opening avenues for empirical evidence

c. The resulting tradition, known as rational mechanics, might be better called theoretical mechanics insofar as its successful emphasis on theorizing went a long way toward making theoretical physics a distinct subfield

5. Huygens's efforts, for all their accomplishment, left a great deal of work to be done within this tradition:

a. Problems to be solved: e.g. the large arc circular pendulum, a fully general theory of elastic impact, vibration on a spring

b. Further mathematical development: e.g. improved methods with infinitesimals, instantaneously varying *conatus*, quadratures

c. Cleaner foundations: e.g. more general principles governing conceptualization of motion, preferred axioms, a unified treatment of forces

d. New and extended generalizations: as illustrated by the conservation of *vis viva* (ultimately leading to modern conservation of momentum, angular momentum, and energy)

e. New and refined experiments: e.g. other measures of *g*, exposing the contrast between rolling and falling, and improved ballistic pendulum measurements

6. That said, the state of the sciences of mechanics and orbital astronomy had progressed a good deal -- with regard to both what had become known and the capacity to marshall evidence -- from where Kepler and Galileo (and Descartes as well) had left matters
   a. Both of those sciences had come to look much more like modern science
   b. In mathematical astronomy it had become clear that Keplerian theory holds to high enough approximation to set the standard, yet the proliferation of alternatives to it had forcefully raised questions about whether it holds exactly or even essentially exactly, and not just approximately
   c. And in celestial physics, Cartesian inertia as developed in Huygens's treatment of curvilinear motion had focused increasing interest on inverse-square centrifugal forces
7. Questions now: what was happening with Newton while all this was going on? In particular, how did the changes that were taking place affect his intellectual development?

B. Issue: What Difference Did Newton Make?

1. If Huygens's efforts before 1675 established the modern, maturing science of mechanics, and Hooke was already proposing interactive inverse-square central forces in celestial physics, what difference did Newton make?
   a. We have spent all this semester preparing to read Newton, only to find the usual answer to this question undercut -- Newton did not transform the immature science of mechanics into our modern mature science, nor was he the first to propose inverse-square central forces governing orbits
   b. Given that the course is called the "Newtonian revolution," the appropriate question now is, exactly what revolution?
2. This question is really best posed as two distinct questions which we will be answering from here on, beginning next week
   a. How, if at all, would the sciences of mechanics and mathematical astronomy have progressed differently if Newton had published nothing at all on the topic?
   b. And how would they have progressed differently if he had stopped with the tract "De Motu Corporum in Gyrum," and not gone on to the *Principia*?
3. As initial food for thought, consider Descartes' critique of *Two New Sciences*, a critique he would surely have extended to Huygens's work in mechanics: all of this is without foundation
   a. One complaint is that Huygens's theories do not proceed from fundamental, universal axioms -- e.g. Torricelli's principle is a glaringly parochial claim tied to the surface of the earth
   b. Another complaint is that the entire science is predicated on the assumption that gravitational fall and resistance are two distinct mechanisms and hence can be treated separately -- an assumption Cartesians rejected

4. In the spirit of such complaints, notice how little information was available in 1675 on whether Galileo's and Huygens's results would hold exactly in the absence of air resistance, and if not, what sort of approximations they were
   a. I.e. do they hold skewed or in the mean, and -- air resistance aside -- are they idealizations or mere approximations
   b. Air resistance effects make this question hard to get at, although some progress was made -- e.g. on the physical pendulum
5. Huygens himself developed a mathematically correct theory for motion under resistance proportional to velocity in the 1660s
   a. Horizontal, vertical, and projectile motion
   b. Experiments he conducted then showed that resistance appears to vary with v^2, and he was unable to handle this case mathematically
   c. Published his work only after Newton's *Principia*, preferring his mathematical approach
6. Similarly, little information was available on the ranges over which the various results hold and the *ceteris paribus* conditions -- beyond no air resistance -- under which they hold
   a. E.g. does g vary systematically at extreme high altitudes or at great depths below the surface of the earth
   b. Given any observed departures from the results -- e.g. Richer's -- the confounding effects of air resistance and other factors make it difficult to determine what to attribute them to
7. By comparison, a fair amount of information was available to support the claim that the results should be taken to be nomological, both from their derivation from a unified theory and the diverse evidence accruing to this theory, including high quality evidence
   a. Even so, a Cartesian could challenge their nomologicality on the grounds that the split between resistance mechanisms and the mechanisms treated in the results is spurious
   b. More information on underlying mechanisms -- e.g. from a theory grounded on universal rather than parochial axioms -- would strengthen the claim to nomologicality
   c. Still, arguably more claim to nomologicality than Kepler's rules as of late 1670s

C. Newton: A Biographical Sketch (to 1679)

1. Newton's father died two months before he was born, which was on Christmas day, 1642 (old calendar); and after his mother left Woolsthorpe to remarry three years later, he was raised by his maternal grandmother until 1653, when his step-father died
   a. He rejoined a family with three younger children, but two years later left for grammar school in Grantham, where he was most remembered for "his strange inventions and extraordinary inclination for mechanical works" (Westfall, p. 60)
   b. After a year away from Grantham managing his farm, with marked lack of success, his mother was persuaded in 1659 that he should return to school in preparation for university

2. Newton entered Cambridge -- specifically, Trinity College -- in 1661, one of roughly 300 students entering what had become somewhat of a degree-mill for the well-to-do
   a. Newton entered as a "subsizar", a student earning his keep by performing tasks for the fellows
   b. His education was classical -- including Aristotle -- until roughly 1664, when he started branching out on his own, reading extensively and beginning an intense study of mathematics
   c. Newton was elected to a scholarship in 1664, to a fellowship at Trinity in 1667, and he was appointed Lucasian Professor of Mathematics, succeeding Barrow, in 1669
3. Even though he published virtually nothing at the time, Newton was extraordinarily productive in the decade from 1665 to 1675
   a. By 1666 he had invented the calculus, and was de facto probably the leading mathematician in the world
   b. He followed this up with further work in mathematics, especially further development of the calculus, in the late 1660's and early 1670's: *A Treatise of the Method of Fluxions and Infinite Series, with its Application to the Geometry of Curved Lines* (1671, published in 1730s)
   c. He also devoted time to optics in particular, but to mechanics too and, to a lesser extent, theology, and he began his interest in alchemy during these years, when in his own words he was "in the prime of my age for invention"
4. Cambridge closed twice for periods during the plague years of 1665-66, and Newton returned home to Lincolnshire, where he had his "Annus Mirabilis"
   a. In addition to developing the calculus during that time, he developed his theory of colors and did various work in mechanics, including the first "Moon test"
   b. The attached accounts of this year in the Appendix, including Newton's own, attest to the extraordinary productivity that the year away from Cambridge generated, even after allowances are made for embellishments
5. The unpublished material on mechanics assigned this week presumably derives from work he did in the period 1665 to 1675, with the possible exception of "De Gravitatione" (controversial, but on my view at most a few parts of it date from around 1684)
   a. Newton was a pack rat, so that we now have an enormous body of notebooks, manuscripts, annotated books that he read, etc. on every topic: see the “Newton Project”
   b. But he did not generally date this material, so that we have to surmise when various pieces were written on the basis of his handwriting, the content, and ancillary information
   c. Newton's own later remembrances of this period add to our confusion in dating, for his recollections are not entirely accurate, often in ways that seem disturbingly suited to help him defend various claims to priority
6. The work in mechanics assigned tonight is fully representative of Newton's efforts in this field before 1679 -- indeed, before 1684, when for the first time he began to do more than just dabble

a. An earlier notebook -- "The Waste Book" -- and a brief manuscript -- the "Vellum" manuscript -- contain precursors of some of the papers assigned for tonight, as noted below

b. The only other work was a brief, unsuccessful foray into projectile motion under air resistance -- this following publication of James Gregory's work on this topic and pendulums (1673)

D. Newton's Work in Mathematics: 1664-1680

1. Newton seems to have begun educating himself in mathematics in 1663, from an elementary text on arithmetic and algebra by Oughtred (1631) and a more advanced text by van Schooten (1646)

   a. The work that appears to have brought him to the then-current forefront of the field was van Schooten's second Latin edition (1659) of Descartes' *Géométrie*

   b. From there he turned to numerous sources, including Wallis's work on indivisibles and infinite series and Barrow's works and (presumably) lectures at Cambridge

2. He discovered the fundamentals of what we now call the calculus over a two-year period from 1664 to 1666 (see chart in Appendix), culminating in his first tract, "To resolve problems by motion"

   a. General algorithms for solving problems concerning infinite sums, maxima and minima, tangents (see Appendix), quadratures, being worked on by Fermat, Pascal, and Huygens (among others) in France and by Wallis, Barrow, and Gregory (among others) in England

   b. Employing a Barrow device of a curve described by a moving point, and taking what we would now call derivatives with respect to time, which Newton called "fluxions" of "fluents"

3. This was followed by a tract in Latin, "De Analysi per Aequationes Infinitas," in 1669, which Barrow circulated, gaining Newton recognition as the leading figure in mathematics in England, and then a full-fledged treatise, *De Methodis Serium et Fluxionum*, in 1671, for which Newton was unable to find a publisher; in all of these he continued with fluents unfolding over "time"

   a. The range of the problems addressed in the latter is spectacular (see Appendix, where examples on curvature and a table of integrals are included as well): Newton had full control of the algorithmic methods that came to be known as the calculus by 1671

   b. The history of mathematics would have been quite different if that book had been published then (rather than finally in two different English editions during the 1730s)

   c. Leibniz's initial work on the calculus began in the mid-1670's and came to fruition in the mid-1680's, but unlike Newton's it was published in the leading journals and led to a tradition of research involving the Bernoullis, l'Hôpital, Varignon, and later Euler and several others

4. One shortcoming of Newton's early work on the calculus was a lack of perspicuous notation; only after the *Principia* in 1687 did he invent the dot notation that came to be associated with him

   a. For example, in the early work he often used lower-case letters to represent the fluxions (time-derivatives) of fluents represented by the corresponding upper-case letters

   b. By contrast Leibniz had a more perspicuous notation from the outset

   c. So even if Newton's early works had been published, they might not have caught on quickly

5. Over the next decade Newton studied classic geometry in more detail than he had before and in lectures rigorously developed foundations of algebra, later published as *Arithmetica Universalis*
   a. These efforts appear to have convinced him that limits could never be done with proper rigor in symbolic methods, but only by extending geometry to incorporate them
   b. That view is reflected in the mathematical method used throughout most of his *Principia*, and subsequently in his unfinished treatise *Geometria*, intended to carry out that project on limits
6. Newton's first publication of work in the calculus was as an appendix to the 1704 first edition of his *Opticks*, and comparatively little more made it into print until posthumously
   a. In 1710 the priority controversy over the calculus began, growing into something that then colored all publication of his work in mathematics after that (see Hall, *Philosophers at War*)
   b. Meanwhile, calculus as we know it continued to be developed in the tradition stemming from Leibniz, with a real explosion in the hands of Euler from 1730 to 1780
   c. View now is that they discovered the calculus indepedently, but calculus as we know it, including the word itself and much of the notation, derives from Leibniz

E. Newton's Work in Optics: 1665-1680

1. Much of Newton's initial work on optics and the theory of light occurred during this period, undoubtedly stimulated by Descartes' *Optics* and *Principia*, as well as by Barrow's lectures in geometric optics
   a. Extraordinary experiments showing that white light is composed of light of different colors, along with a theory of refraction explaining chromatic aberration as a consequence of differing refraction indices of different color light -- Lucasian lectures, 1670-72
   b. Papers in the form of letters published by the Royal Society in 1671 and 1672 on this work and the reflecting telescope (and then in reply to objections until 1675) made Newton famous and respected throughout the scientific world
2. The controversy they initiated -- in particular, the insistence by Hooke and others that the experiments were predicated on a particle theory of light -- then led Newton to shun further publication
   a. One source of this insistence was Newton treating rays of light in the abstract, concluding that rays of different color are differently refrangible, and then suggesting that rays are the paths of light particles without in any way using this (see his reply to Pardies in Appendix)
   b. To some extent the controversy also stemmed from the difficulty of replicating his experiments -- though Hooke managed to do so; for, as the key pages (see Appendix) describing his so-called *experimentum crucis* make clear, the experiments were elaborate and required great care
3. But it stemmed even more from differing conceptions of science, where Newton was outspokenly negative toward the "method of hypotheses," especially those concerning underlying processes
   a. Newton already insistent on a strict distinction between experimentally established theoretical claims, on the one hand, and hypothetical conjectures, on the other

b. Others, including Huygens, thought this a mere matter of degree: all theoretical claims remain hypothetical, with evidence conveying on them only degrees of probability

c. This difference between Newton and almost everyone else remained a source of confusion throughout the rest of his life

4. To underscore Newton's conception, a sequence of quotations on method in optics from this period is included in the Appendix

   a. The first is a passage which Oldenburg elected to cut from the first published paper

   b. The second a translated passage from the Latin in reply to Pardies

   c. But the most important is from his Lucasian Lectures on optics from the early 1670s, in which he puts forward the idea of combining mathematical methods with those of natural philosophy to "finally achieve a natural science supported by the greatest evidence"

   > Thus although colors may belong to physics , the science of them must nevertheless be considered mathematical, insofar as they are treated by mathematical reasoning. Indeed, since an exact science [*accurata scientia*] of them seems to be one of the most difficult that philosophy is in need of, I hope to show -- as it were, by my example -- how valuable mathematics is in natural philosophy. I therefore urge geometers to investigate nature more rigorously [*strictius*], and those devoted to natural science to learn geometry first. Hence the former shall not spend their time in speculations of no value to human life, nor shall the latter, while working assiduously with an absurd method, perpetually fail to reach their goal. But truly with the help of philosophical geometers and geometrical philosophers , instead of the conjectures and probabilities that are being blazoned about everywhere, we shall finally achieve a science of nature [*scientiam Naturae*] supported by the greatest evidence [*summis evidentiis*].

5. Newton's *Opticks* published in 1704 (in English; second Latin edition in 1706; third edition, in English, in 1717); it includes not just his work on refraction and reflection, but on diffraction too, conducted largely after his *Principia* was published

   a. The book does not begin to reveal the number of enabling, supporting, and cross-checking experiments he carried out for each experiment he then published

   b. But his Optical Lectures, first published in 1729, do reveal just how extraordinarily careful he was as an experimentalist (available in Latin and English as Volume 1 of *Newton's Optical Papers*, edited by Alan Shapiro)

F. Newton's Intellectual Style, versus Huygens's

1. My standard way of explaining Newton's uniqueness: he was one of the two or three greatest mathematicians, one of a handful of the greatest experimental physicists, and one of three or four of the greatest theoretical physicists -- this all in one person

   a. Huygens was in the same rank as an experimental physicist and not far behind as a theoretical physicist, but, good as he was as a mathematician, he was not in the league of Newton or Gauss

   b. While this description goes some way toward explaining Newton, it does not begin to address the uniqueness of his style

   c. Perhaps the most telling remark is by J. M. Keynes, *Essays in Biography*

> I believe that the clue to his mind is to be found in his unusual powers of continuous concentrated introspection.... Anyone who has ever attempted pure scientific or philosophical thought knows how one can hold a problem momentarily in one's mind and apply all one's powers of concentration to piercing through it, and how it will dissolve and escape and you find that you are surveying a blank. I believe that Newton could hold a problem in his mind for hours and days and weeks until it surrendered to him its secret. (p. 312)

2. Newton had a peculiar mathematical talent, enabling him to zero in on the essence of problems, finding elegant solutions to them and brief proofs (that were generally harder to understand at first); combined with this was a capacity to take so many threads of thought into consideration
    a. Proofs seemingly based on tricks, but the tricks then turn out to capture the fundamental features of the problem, sometimes by bringing together ideas others had seen as disparate
    b. While others would struggle to identify and clarify the principal obstacle in a mathematical problem, Newton would see it almost immediately and put all his effort into resolving it
    c. And his work on the calculus gave him a range of methods well beyond those of Huygens
3. Newton had this talent to a far greater degree than any of his contemporaries, even including Leibniz (who had complementary talents)
    a. Newton's proofs stand in striking contrast with, for example, those of Huygens -- e.g. less than three pages on the isochronism of the cycloid, where Huygens has more than 20 pages using heights to infer velocities, combined with Wren's solution for lengths along the cycloid
    b. Why it is not surprising that Newton had pulled together the rudiments of the calculus (out of Descartes, Wallis, Barrow and others) years before anyone else did
4. This peculiar talent enabled Newton to work by himself, without discussions or contact with others working on similar problems
    a. Indeed, given his personality, he was probably better off working alone; the point is that he did not pay the usual price for doing so
    b. 'Alone' here does not mean in complete isolation from the work of others, for Newton read voluminously -- e.g. the Latin edition of Descartes' *Géometrie*, Wallis, Gregory, etc.
    c. But without the daily give-and-take exchanges of ideas that Huygens enjoyed in his years in Paris (and Leibniz continued in his voluminous correspondence)
5. One consequence of Newton's style was that criticisms of his work often seemed poorly motivated or wrong-headed to him
    a. He had a deeper understanding of what he was doing than anyone around him did -- something of which he was constantly aware
    b. Hooke's criticism of his experiments on colors was a typical example, for the whole point of the experiments was to avoid begging questions of theory
    c. Newton thus had a distinct tendency to be contemptuous of his colleagues, with a few notable exceptions (Huygens, Wren, Wallis and later Halley and Gregory's young nephew, David)

6. Another consequence of Newton's style was that he often thought of problems in rather different ways, not always being informed of the most current way of thinking about them in, say, London
   a. We will see this tonight in work paralleling that of Huygens, in which Newton had his own distinctive approach
   b. Being out of the mainstream put Newton in a position to alter the course of the history of physics

II. Newton's Early Laws of Motion (vs. Descartes, Huygens)

A. Newton's Conceptualization of the Problem

1. Material in the "Lawes of Motion" paper appears to be in response to his reading Descartes' *Principia* in 1664; but the amaneunsis handwriting (John Wickens) in the first half points to its dating between 1672 and 1675, perhaps in response to the *Phil. Trans.* 1669 papers
   a. Descartes' rules of impact not only difficult to comprehend, but glaringly in conflict with observation, putting the problem at the forefront in both Paris and London in the 1660's
   c. But, given Newton's distinctive approach, we have no clear evidence of how the "Lawes" relates to work by others
2. His first efforts on impact in the Waste Book are restricted to spherical objects colliding head-on
   a. He adopts Descartes' Bulk∗velocity measure of force, but restricts the conservation of motion to one direction at a time
   b. And he conceptualizes impact itself as involving elastic deformation of the spheres, with the force tied to the distortion, yielding the principle, separation v = approach v
3. The most striking feature of the way in which he conceptualizes the problem in the "Lawes of Motion" paper is its generality: arbitarily shaped objects at arbitrary angles of attack with respect to one another, not just spheres head-on
   a. Given Descartes' three kinds of matter, this is the appropriate form in which to pose the problem (though no one else did so)
   b. The obvious disadvantage is that it makes the problem mathematically much more complicated
4. Newton also conceptualizes the problem within the context of Descartes' three laws of motion, though with some modifications
   a. Presupposes straight line, constant velocity motion in the absence of forces -- in the form of motion in absolute space
   b. The force to persevere in a given motion is equivalent to the motion that it can create or destroy in another body -- expressed in terms of $\Delta(B*v)$, just as in Descartes' third law
5. Newton, however, distinguishes components of motion, and adds a parallelogram rule for compounding these components
   a. No hint of Galileo's parallelogram rule in this addition to Descartes
   b. Presumably prompted by demands on the problem when he allowed impact other than head-on

B. The Principles Governing Angular Motion

1. Newton proceeds to distinguish progressive (translational) and angular (rotational) motion, in the process indicating how to conceptualize the full motion of a rigid body
   a. Each body has a center of motion -- a point about which it can revolve with centrifugal tendencies counterpoised
      (1) Notice here the influence of Descartes again: use centrifugal tendencies as basis, counterpoised to explain why no net effect
      (2) Center of motion of course identical with center of gravity, as suggested in Paragraph 4
   b. Progressive motion is that of the center of motion
   c. What we now call inertia holds for both the translational, and also for the motion of the common centers of gravity of any system of interacting bodies (as per Huygens)!
2. Newton offers two measures of angular motion, one applicable to a single body in isolation, and the other allowing for comparisons among bodies
   a. For a single body: angular velocity
   b. More generally: quantity of motion, to be determined experimentally in terms of the amount of translational motion needed to produce it
   c. (An experimental measure only here because Newton not in a position to propose the $I*\omega$ counterpart to $B*v$ (just as Huygens wasn't in his work on the physical pendulum))
3. This measure, without a mathematical specification, introduces an obvious limitation on the theory by restricting the possibilities of mathematical proof
   a. A sharp contrast between this paper and Huygens's on the same topic is lack of mathematical proofs in this one
   b. In part because Newton was pursuing different objectives, but also because his approach interfered with such proofs
4. Newton then extends first the parallelogram rule to angular motions, employing an axis of compound angular motion, and then the principle of inertia
   a. I.e. constant angular motion about axis if counterpoised
   b. If not counterpoised, then axis changes in body, resulting in a spiral motion of the axis when viewed from outside the body
5. All these claims about angular motion presumably from "physical intuition" since he offers no proofs or experiments to support them
   a. No worse than Descartes in this regard, and hence not retreating to a lower standard as far as he could see
   b. (This is evidence of his being unaware of Galileo's *Two New Sciences* and the higher standard it exhibits, as illustrated in Huygens's efforts on impact)

C. The General Laws of Reflection on Impact

1. As already remarked, Newton poses the problem of perfectly elastic impact in a general form, in contrast to Descartes and Huygens
   a. What are the resulting rotational and translational motions after two arbitrarily shaped bodies, both moving rotationally as well as translationally, impact one another at a single point
   b. This is the problem Descartes really needs a solution for in his rules of impact, given the irregular shapes of particles of the first and third kinds of matter
2. Newton bases his solution on two principles, one of which is derivative from Descartes, and the other of which came from his Waste Book
   a. The velocity of separation of the points of contact = the velocity of approach -- a principle which Huygens derives from more fundamental hypotheses
   b. The total change of motion is distributed among the four components proportionately to the easiness with which each component of velocity changes -- akin to Descartes' least modal mutation principle announced in the letter to Clerselier
3. The algebraic solution (see page 212) is readily obtained from these two
   a. Herivel indicates that the solution is fully correct, though I am not sure that he isn't filling in lacunae in a generous fashion in reaching this conclusion
   b. Whether it is correct or not is unimportant for our purposes
   c. (We should notice, however, that Newton is in a position to see the difference between rolling and falling, for in transferring motion to a body, part can go into translation and part into rotation
      (1) Had anyone ever asked him and he had remembered this paper, he would surely have distinguished between rolling and falling
      (2) But he never says so anywhere that I know of)
4. The procedure for determining a solution, unlike Huygens's, requires an experimental step in lieu of specifications of moments of inertia for the impacting bodies
   a. E.g. the radii G and γ for angular motion have to be determined experimentally
   b. The solution is therefore not of great theoretical interest
5. But Newton has provided a correction of Descartes' rules while generalizing them to the problem he needed to be addressing
   a. I.e. provides rules for impact that are compatible with observation and with (Galilean) principle of relativity
   b. Like Descartes, Newton is reasoning from physical intuitions, but from "correct" physical intuitions -- i.e. intuitions that do not become warped by the effort to come up with tractable rules
   c. This reliance on physical intuition is an impressive feature of the "Lawes of Motion" paper, showing how the young Newton was able to devise a non-metaphysical solution to Descartes' problem, in the process remaining within the context of Descartes' fundamental principles

D. Newton's Results Contrasted with Huygens's

1. If Newton's X is taken to be the change in translational velocity, then his algebraic solution is in full accord with Huygens's for the simple case of a moving sphere impacting a sphere head-on
   a. Thus Newton has solved the problem Huygens set himself, which is also the problem Descartes originally set
   b. He has even derived this solution from principles, though not the principles Huygens had used
2. But Newton's rules are much more general than Huygens's, for Newton not focusing an a theoretically basic, mathematically tractable problem, but on a physically realistic problem
   a. Allowing for rotational motion and oblique impact a major complication from a mathematical and theoretical point of view
   b. Huygens is much more clearly in the style of Galileo, working out the theoretically basic, ideal case with mathematical rigor, plus a striking prediction to serve as a test
   c. Newton, by contrast, is in the style of Descartes, with concern for mathematical niceties giving way to a preoccupation with physical concerns
3. Much the same point can be made about the principles that Newton assumes as axiomatic: they are stronger and more empirically loaded than Huygens's
   a. No Torricelli principle, but in its place a least resistance principle that makes a Cartesian claim about changes in motion generally
   b. Not even a relative motion principle, motivating the whole approach, but instead a strong physical claim that Huygens takes the trouble to derive from seemingly weaker principles -- though again a principle that makes a direct claim about changes in motion, albeit a claim in direct contradiction to Descartes
4. Like Huygens, Newton takes the trouble to show that his rules of impact entail that Descartes' conservation of total motion is false
   a. Total motion, ignoring direction, can be greater or less than it was before impact
   b. Newton's reasoning employs oblique encounters, and hence are different from -- indeed, much simpler, though more dependent on physical intuition, than Huygens's
5. Newton does not draw the conservation of $B{*}v^2$ conclusion that Huygens does, but he does draw another important conclusion in line with Huygens: "the common center of two or more bodies changeth not its state of motion or rest by the reflection of those bodies one amongst another"
   a. No proof offered, but a proof for "hard" bodies should not be that hard from the algebraic solution, and a proof from Huygens's solution for the limited problem is straightforward
   b. In effect, then, no created motion of the overall system, so that Newton's results are in accord with Torricelli's principle
   c. This inferred result will prove important later on

6. As noted in the class on Huygens, later in the 1670s Newton gave an algebraic solution for the head-on impact of spheres (see Appendix)
   a. An isolated problem among several other problems in his Lucasian lectures on algebra
   b. Important because, in addition to using the speed of separation principle, he there uses a version of his third law of motion -- the approach he ultimately generalizes in the *Principia*

E. Newton's Reservations about the Potential for Evidence

1. Still a further contrast with Huygens is Newton's lack of interest in pursuing a Galilean type of experimental confirmation for his account of impact
   a. No attempt to tie the theory to Galileo's account of free fall -- e.g. via Torricelli's principle -- nor any hint of developing evidence for it via the ballistic pendulum
   b. And no results for intervening bodies -- indeed, no "salient" predictions derived from the theory at all -- that could be the basis for a qualitative experimental test of the theory
2. The section of "Observations" at the end is present in one version of the paper, and not in another, suggesting that it was an add-on following some disappointing experiments
   a. As Newton's work in optics at the time indicates, he was not at all reluctant to turn to experiments
   b. And his theory of impact calls for some experimentation if only because it presupposes some empirically determined quantities
3. The principal problem in experimenting with impact that Newton calls attention to is the failure to realize the conditions of impact assumed in the theory
   a. Bodies are not perfectly hard, but deform under impact, with some loss in recoil, and also with a consequent sliding of the interfaces
   b. Experiments also confounded by problems in restricting contact to a single point of tangency
4. The other key problem concerns the effects of resisting media, not only in slowing the bodies down, but also in affecting what happens at the point of impact
   a. (Interest in the effects of resisting media was growing in England during the 1660's, though it is unclear whether Newton was aware of this until the early 1670s)
   b. Paragraph 1 of "Observations" lists all the confounding items that need to be taken into account in any experiments on impact (see Appendix)
5. The point to notice is that Newton here has a very different conception of the role of idealizations in the development of a theory and consequently of the approach to be taken in developing empirical evidence for the theory -- i.e. different from Galileo's and Huygens's
   a. Idealizations are not being adopted in order to permit the development of empirical evidence, but only to allow a mathematical solution for a first approximation
   b. Hence, no interest in devising experiments in which confounding effects are minimized or controlled -- i.e. experiments in the style of Galileo and Huygens

c. Instead an insistence on dealing with the physically real case, and disappointment when this proves unmanageable

III. Newton on Circular Motion (vs. Descartes, Huygens)

A. Basic Theoretical Results, versus Huygens's

1. The "vellum manuscript" (Add. 3958, f. 45) that anticipates this one is a torn legal parchment with a lease on the front and calculations on the back

a. That manuscript thus likely dates from when Newton was home during the plague years, in 1665 or shortly thereafter

b. If so, then the manuscript on circular motion, written in Latin as if for publication, dates from after that

2. Unlike Huygens, Newton does not formulate the question about the *conatus a centro* in uniform circular motion as one about the (instantaneous) tension in a string

a. Through what space would the *conatus* impel a body in the time of one full revolution if it were not constrained

b. Want the total sum of the incremental distances by which the body would have departed from the circle had it not been constrained during the time of one revolution -- i.e. the s corresponding to the uniform acceleration ($r*\omega^2$) over one period of revolution

3. The answer: $D*\pi^2$ (in full agreement with Huygens since $v^2/r = r*\omega^2 = 2\ D*\pi^2/P^2$ and $s = a*t^2/2$)

a. Newton's formulation amounts to: x/BD = $\text{ADEA}^2/\text{AD}^2$

b. From the end of Book 3 of Euclid, BE/BA = BA/BD (Euclid's Proposition 36: the rectangle formed by BE and BD is equal to the square formed on BA)

c. But for an infinitesimal increment, BE approximates DE, and BA, DA, so that BD approximates AD^2/DE, and answer results by plugging in for BD above

4. Newton's interest in the problem not a mechanical one, like Huygens's, but one concerning the degree of *conatus*, just as Descartes had called for in *Principia* III, 59

a. Notion of total endeavor in a revolution strange to us, though does give a measure of acceleration in terms of distance covered versus time (squared)

b. Simply assumes uniform acceleration (and Galileo's rule for distance in uniform acceleration, taken from the *Dialogue*), whereas Huygens feels required to argue for it! -- that is, to argue that the induced tension in the string is commensurable with the tension in a hanging string

5. Newton does not develop the result in the manner of Huygens, for the issue that concerns him is not posed within the framework of Galilean mechanics

a. Huygens ties theory of centrifugal force to Galilean fall in order initially to obtain a measure of g, and subsequently to provide evidence for theory via measure of g

b. Earlier, in Waste Book Newton had devised solution for circular motion via impact (Appendix);

and in Vellum manuscript had rejected Galileo's value of the fall in 1 second (roughly 5 ft), and then proposed a 45 degree conical pendulum as a means for measuring the correct value

c. Here he does mention the conical pendulum at the end, but he does not follow up the mention with a full mechanical theory in the way Huygens does

6. Nor does Newton derive any easily testable results, like Huygens's result for the intercepted pendulum, for he is pursuing different objectives

a. That is, he is working outside the tradition of Galileo's *Two New Sciences*

b. And he is being provoked by astronomical concerns

B. Four Immediate Applications of the Theory

1. Newton proceeds to derive one primary result about the centrifugal tendency at the surface of the Earth from the basic theoretical result, and then draws conclusions on three other topics

a. With the exception of the acceleration of gravity, Newton is largely using values taken from Galileo's *Dialogue*

b. (Interesting that he would reject Galileo's value for the acceleration of gravity and accept other values from the book)

2. Compare the centrifugal tendency at the surface of the earth to gravity, in the process confirming Galileo's explanation for why we do not feel the centrifugal tendency, much less fall off the earth

a. Assume radius of earth = 3500 (Italian) miles, 5000 ft per mile

b. Then since acceleration of gravity is 32 ft/sec/sec (corresponding roughly to Huygens's announced value), the force (*vis*) of gravity is 350 times stronger than the centrifugal *conatus* at the equator

c. The correct value -- using a correct radius of the earth instead of the value Newton has taken from Galileo's *Dialogue* -- is 288

3. Compare the centrifugal tendency of the moon with that on the surface of the earth, in effect exploiting the fact that r/P^2 gives this proportionality

a. Moon 60 earth radii away, so that centrifugal tendency on the surface of the earth is 12.5 times stronger than that of the moon

b. Therefore, gravitational force at the surface of the earth is around 4000 (4375) times the centrifugal tendency of the moon -- versus 3600 times if $1/r^2$

4. Infer (incorrectly) a lower bound of the horizontal solar parallax (19 sec) from the assumption that the centrifugal tendency of the moon is responsible for our always seeing the same face

a. For then the centrifugal tendency of the moon with respect to the earth would have to be greater than that of the moon with respect to the sun, for otherwise the moon would always show the same face to the sun, and not to the earth

b. Therefore centrifugal *conatus* on surface of earth 132,408 times greater than centrifugal *conatus* with respect to the sun (assuming -- incorrectly -- that horizontal solar parallax is 24 sec)

5. Compare the centrifugal tendencies of the planets from the sun, using fact that this proportionality must be $1/r^2$ since the planets conform with Kepler's 3/2 power rule
    a. Centrifugal tendencies as 614 to 173 to 91 to 39 to 3.33 to 1, from Saturn to Mercury
    b. No reason given or suggested for taking the trouble to obtain this result
6. Final points concerning relation between conical pendulum and circular pendulum probably reflect his earlier idea of using a 45 deg conical pendulum to obtain a more accurate value for strength of gravity than Galileo had proposed
    a. Newton himself carried out a conical pendulum measurement of gravity somewhere around this time, stopping short of full precision once he got within 1 percent of Huygens's value
    b. Perhaps failure to obtain an exact $1/r^2$ for the Moon occasioned him to check Huygens's value

C. Newton's Preoccupation: Copernican Concerns

1. The common element in the three main applications of this manuscript, which appears to have been written for publication judging from how clean it is (see Appendix), is a concern with issues attendant to Copernicanism, more specifically with issues arising within Galileo's *Dialogue*
    a. Even the horizontal solar parallax is pertinent to issues raised in the *Dialogue*
    b. For, the Aristotelian can argue that, if the earth is orbiting the sun, then observed acceleration toward the center of the earth ought to be greater at high noon on the equator than at midnight, since centrifugal conatus additive at noon
2. By contrast, Huygens showed no interest at all in such Copernican issues in his treatment of centrifugal force
    a. Newton prompted by Descartes' discussion of the centrifugal tendency in planetary motion, Huygens prompted by his discussion of the sling
    b. Huygens did not even show interest in the $1/r^2$ implications of Kepler's third "law," something noticed by several others after the *Horologium Oscillatorium* was published in 1673
3. Newton's efforts more akin to those of Kepler's calculations in the *Epitome*, where Kepler was fishing for information about mechanisms underlying the planetary orbits
    a. Newton too appears to be somewhat on a fishing expedition, trying to draw inferences from the theoretical characterization of the *conatus a centro* and various other available information
    b. Almost as if his sole point in deriving the theoretical result on the *conatus a centro* was in the hope of learning more about celestial motion
4. Notice, however, Newton's lack of concern here for exact orbits, which he knew perfectly well from Streete were not circles
    a. In this respect, his effort is less akin to Kepler than to Descartes, for he is showing no interest at all in what might produce trajectories other than circles
    b. Furthermore, the values generated are pertinent to Descartes' vortex model, in which gravity at the surface of the earth is related to vortex pressures

5. The contrast between the moon and the planets is, of course, a Copernican issue discussed in the *Dialogue*, but it is even more so an issue in Descartes' *Principia*
   a. My guess then is that Newton was looking for some way to take Descartes' basic insight concerning the *conatus a centro* associated with curvilinear motion and begin turning it into an argument for Copernicanism
   b. Or at least to bolster, via a few specific numbers, some of the arguments in support of Copernicanism in the *Dialogue*

D. Universal Gravitation: The Historical Issue
1. The issue is whether Newton considered the hypothesis of inverse-square universal gravitation in the 1660s and rejected it because of the substantial empirical discrepancy between 4000 (4375) and 3600
   a. Or even the weaker hypothesis, gravity varies as $1/r^2$ and extends from the earth to the moon
   b. At least the latter hypothesis compatible with ones being entertained at the time and subsequently by Hooke and Wren, although surely this was not then known to Newton
2. The legend that Newton was already looking to gravity to explain planetary motion in 1666 derives from Whiston and Pemberton, and from Newton too, all after 1700 (see accounts in the Appendix)
   a. The legend of the apple is from Stuckley and Conduit, apparently originally owing to Newton sometime after 1715
   b. Part of Newton's embellished account of how much of the *Principia* was developed in 1666, years before Hooke had suggested anything to him
3. The source of the legend was undoubtedly Newton's defense against Hooke's accusation of plagiarism, an accusation that took on more bite following the dispute with Leibniz
   a. Newton's 1686 defense against Hooke's charge pointed out only the derivation of $1/r^2$ from the 3/2 power rule, and added that Wren and Gregory had derived the same thing before 1679
   b. Thus the 1686 defense is compatible with the manuscript on circular motion (dated by Turnbull as ca. 1669), and the later embellished defense is much less so
4. Note that throughout this manuscript Newton is comparing centrifugal tendencies -- *conati a centro* -- with one another and with the gravitational force at the surface of the earth
   a. No mention is made of gravity in the case of the sun and planets at all
   b. And, as Wilson points out, nothing remotely akin to universal gravity is ever suggested
5. Equally, however, one should notice that the comparison between 4000 and 3600 -- a comparison not in the manuscript -- does not make much sense unless Newton is either assuming or hoping to show that the 3/2 power rule holds for the moon
   a. There is of course no evidence for this at all at the time, though one might nevertheless hypothesize it on the basis that the 3/2 power rule is a general property of all celestial orbiting systems
      (1) Horrocks's improvement in Kepler's orbits from using the 3/2 power rule to determine semi-major axis is grounds for saying this

(2) Newton knew of Horrocks's success as well as the 3/2 power rule from Streete

b. And this hypothesis might well lead one to hypothesize a common mechanism for the planets and the moon, which might then lead to the question, does this mechanism have anything to do with terrestrial gravity

c. But this is a far reach from anything in this or the attendant manuscripts

6. Thus, Wilson's conclusions about the discovery of universal gravity, which are now almost universally accepted, continue to stand up under scrutiny

   a. Suppose even that Newton had used the correct earth radius and obtained a value very close to 3600, in contrast to 4375; what would he have done next?

   b. Equally, supposing Hooke had been clever enough to do the same thing in the late 1670's, what would he, having long been committed to celestial gravity as the mechanism governing planet and comet motion, have done next?

7. (One possible answer in Newton's case is that he would have concluded that the 3/2 power rule is a general feature of celestial orbiting bodies -- or at least a feature of bodies orbiting the earth -- and he might then have proceeded to argue for Copernicanism over Tychonism on the grounds that the horizontal solar parallax is quite incompatible with the 3/2 power law applied to the sun as well as the moon orbiting the earth

   a. That is, Newton may well have been trying to devise an argument against the Tychonic system on the basis of the 3/2 power rule

   b. To this end he needed to show that the 3/2 power rule holds around the earth

   c. Since he cannot get this from comparing bodies orbiting the earth, he instead gets the clever idea of trying to show that it holds by showing that there is an inverse-square relation between the *conatus a centro* of the moon and the acceleration of gravity at the surface of the earth

   d. The seemingly radical accompanying thought -- that terrestrial gravity holds the moon in orbit -- he had already encountered in a brief passage in Streete

   e. This view of what he was up to has the added virtue of fitting in perfectly with the preoccupation with Copernicanism of the rest of the manuscript, as well as the interest in the horizontal solar parallax)

E. The Implicit Conception of Empirical Science

1. As Westfall remarks, the story of universal gravitation coming to Newton in a flash during 1666 "vulgarizes universal gravitation by treating it as a bright idea" -- p. 155

   a. It also leads to a vulgarized conception of empirical science -- geniuses somehow in tune with reality who have profound insights into the nature of the universe, insights that the rest of us ultimately catch up to when nothing disproves the insight

   b. A conception under which the evidential side of science plays a negligible role in discovery and hence in the major events in the history of science

2. Given our central preoccupation with the development of increasingly sophisticated conceptions of empirical evidence in 17th century science, appropriate to ask what conception of empirical science is implicit in Newton's short tract on circular motion

3. One thing for certain is that it is far removed from the idea of achieving exceptionally high quality evidence through convergent, precise measures of a fundamental quality like *g*

   a. At the beginning of this class we listed three ways in which Huygens extended the conception of empirical evidence prevailing at the time

   b. All three are clearly present in Huygens's efforts on uniform circular motion

   c. The only one of the three remotely present in Newton's efforts is the idea that theory is a vehicle for evidence, for Newton is adopting various theoretical claims and using them to draw inferences from observations that could not otherwise be drawn

4. Equally, however, the conception of empirical science implicit in Newton's short tract is far removed from the one Descartes exhibits in his *Principia*

   a. Not putting forward bold hypotheses about underlying mechanisms and then drawing out an explanatory account of phenomena from them

   b. Indeed, no real mention of hypotheses about underlying mechanisms in the tract at all, and as it stands it is at least compatible with a vortex model

5. It is not so far removed from the conception found in Galileo's *Dialogue*, however -- a conception on which one is continually looking for empirical information that can be exploited as evidence to settle questions under dispute

   a. Wilson remarks that, partly through the influence of Barrow, Newton had come to maintain a sharp distinction between conjecture and experimentally established results (p. 139)

   b. On this view the fishing expedition Newton is here on amounts to exploiting Descartes' claim about the importance of the *conatus a centro* in all curvilinear motion to look for potentially compelling evidence for Copernicanism

6. If this is correct, then Newton had not yet begun to fashion a highly sophisticated conception of how to marshal empirical evidence in orbital mechanics at the time he wrote this tract

   a. Indeed, he was still more or less at the level of sophistication exhibited in Galileo's *Dialogue*, and far short of the much greater level Huygens had reached by this time

   b. But then the conception of science implicit here is far removed from that in the *Principia*

IV. Newton on Motion along a Cycloid (versus Huygens)

A. Newton's Two Part Proof of Isochronism

1. In his brief tract on the cycloidal pendulum, Newton provides derivations for three basic results announced by Huygens, but published only in 1673 in the *Horologium Oscillatorium*

   a. The isochronism of the cycloidal pendulum, and as a corollary that of the small arc circular pendulum

b. The construction of a cycloidal pendulum via a curved (cycloidal) constraint along the length of the pendulum string

c. The law of the period for cycloidal pendulums -- $2\pi\sqrt{(4a/g)}$ -- and hence a means of measuring g

2. The derivations exploit two basic features of the cycloid, both of which had been established by geometrical means, while remaining free of Newton's criticism of Huygens's displayed in Appendix

   a. The tangent at any point on the cycloid is parallel to the chord from the horizontally corresponding point to the bottom of the generating circle -- i.e. as YC in the attached figure

   b. The length of the remaining arc to the bottom of the cycloid is 2 times the length of this chord -- a generalization of Wren's famous "rectification" of the cycloid announced to the public by Pascal in 1658

3. The first of two steps in the proof of isochronism is to show that the acceleration at any point on the cycloid is proportional to the arc length to the bottom

   a. The fundamental feature of all isochronously vibrating systems! (here discovered by Newton)

   b. a_D/a_δ = BC/YC, since the chords are parallel to the tangents and the accelerations are as the lengths of such chords (as Galileo had shown)

   c. But DC/δC = BC/YC, since the arc lengths are twice the chord lengths

4. The second step is to show that the time from any point like P to C is the same as the time from D to C -- this time using an infinitesimal argument

   a. Since a_D/a_P = DC/PC, spaces described in the small unit of time required for Dd and Pp are in the same ratio: DC/PC = Dd/Pp

   b. But then dC/pC = DC/PC, so that accelerations at d and p are in same ratio -- i.e. DC/PC = a_d/a_p

   c. Therefore the remaining distances are at all times in the same ratio, "until both simultaneously dwindle to nothing"

   d. Notice also the last paragraph: Newton is not assuming that velocity is proportional to the square root of height in his proof (as Huygens did in his), but is instead concluding that this holds for fall along a cycloid from his analysis

   e. In other words, where Huygens's solution is based on the *vis viva* principle, Newton has found a way to avoid this

5. Newton then gives a geometrical proof that the same cycloid can serve as an evolute to produce a cycloidal pendular arc

   a. The key to the proof is to choose an arbitrary point Q on the horizontal, construct two parallelograms using this point and then show that the line common to these parallelograms is tangent to the evolute cycloid and perpendicular to the involute

   b. The isochronism of the small circular arc pendulum Newton then infers as a corollary to this construction

B. The Relationship to the Theory of Free Fall

1. Finally, Newton addresses the problem of the time of descent from D to C versus the time of vertical free fall from B to C
   a. The solution in effect gives the period of oscillation in terms of the acceleration of gravity g
   b. Namely, the law of the cycloidal pendulum: $P = 2\pi\sqrt{(4a/g)}$, where a is the radius of the generating circle
2. The key step in the proof is to find an appropriate geometric measure of time, namely the arc lengths along the generating circle
   a. t_{Dp} / t_{DC} = arc Bσ/arc BC
   b. Thus a geometric measure of just the sort Galileo had to find in *Two New Sciences* and Newton will have to find repeatedly in his *Principia*, so that the proof serves to illustrate a general feature of a kind of argument
3. The proof employs a pre-calculus argument
   a. $v \propto \sqrt{(BV)} \propto BT$ since $BT^2 = BV*BC$
   b. Let Pp be a small element so that Pp and Tτ can be taken as straight and parallel to one another
   c. But then Tστ and TKB are similar triangles since their respective sides are mutually perpendicular
   d. Hence BT/TK = τT/Tσ, so that BT∗Tσ = TK∗Tτ
   e. But BK=TK and Tτ can be taken as given (Tτ because it is equal to the given infinitesimal Pp)
   f. Therefore BT $\propto 1/T\sigma$; but velocity and time are also in reciprocal proportion, so that Tσ can represent the time!-- the key enabling result
   g. Since each part in time corresponds to an arc in the generating circle, the time required for fall from D corresponds to the arc of the circle
4. Then time to fall from D $\propto (\pi/2)\sqrt{(2BC/g)}$
   a. Since $BT^2 = BV*BC$, time to fall BV can be represented by BT
   b. At outset, descent from D and vertically from B are the same, and this implies that the two representations of time are commensurate with one another (see p. 206)
      (1) For each individually always bears the correct proportions to the length representing the first increment of time
      (2) And hence each always bears the correct proportions to the other
   c. But then t_{DP}/t_{BV} = arc BT/chord BT
   d. Result follows from arc BC/chord BC
5. Corollary: can measure acceleration of gravity by determining period of oscillation and inferring g
6. Also notice the parting remark about motion in a cycloid resembling the harmonic motion of a point on the wheel -- a remark developed in more detail by Huygens (and later by Newton too)

C. Newton's Analysis Contrasted With Huygens's

1. While Newton thus obtains Huygens's principal results for the cycloidal pendulum, his derivations are distinctly different from those in the *Horologium Oscillatorium*
   a. Newton does not develop a general theory of evolutes, as Huygens does, but develops the result only for the rather special case of the cycloid
   b. His derivations of isochronism and for relating the period to vertical fall in 1 sec are much briefer than Huygens's, and throughout use the true smooth curve, not an approximation to it
   c. Newton manages to give a necessary and sufficient condition for isochronism (viz. that acceleration be proportional to the distance to the point of maximum velocity) -- something Huygens later discovered without publishing
   d. And Huygens shows full appreciation for the need to physically tune the clock -- i.e. to include the results on the physical pendulum -- to measure *g* with high precision
2. Still, Newton's work on the cycloid is the one place in his early work in mechanics, most of which parallels Huygens's work in one way or another, in which Newton displays a scientific style at all like that of Huygens (and that of Galileo in *Two New Sciences*)
   a. Some of the Galilean results employed -- e.g. s varies as t^2 and the inclined plane results -- are given in the *Dialogue*, along with claims about the pendulum that would have made it evident to Newton how Huygens's efforts were tied to Galileo's
   b. But no where else does Newton try to tie his efforts in mechanics to Galileo's in a way that will open avenues of evidence
3. There are three possible datings of the brief tract on the cycloidal pendulum, all based on the assumption that some external stimulus provoked him to look at the topic since he had shown no interest in it or in any related topic earlier other than the geometry of the cycloid
   a. After receiving his copy of the *Horologium Oscillatorium* -- but then to what end, and why different proofs in Latin, unless showing how to get around his critique of Huygens
   b. After receiving James Gregory's "stimulating essay on pendular and projectile motion" -- "Tentamina Quaedam Geometrica De Motu Penduli and Projectorum" -- in 1672, as Whiteside suggests
   c. Or still earlier -- e.g. around 1670, as Newton said to David Gregory -- perhaps after hearing of Huygens's announced results through the grapevine
4. Regardless, the impressive thing about this short tract is the appreciation it displays for Huygens's style of science, and hence for the conception of empirical evidence implied by Huygens's work from late 1650's and 1660's
   a. Either Newton had managed to form such a conception independently for himself, and simply not evinced it in his other work in mechanics
   b. Or Newton was very quick to pick up such a conception from mere hints of it (or from *Horologium Oscillatorium*)

c. Only thing missing is Huygens's refinement for the center of oscillation for the circular pendulum, removing an element of idealization

5. This absence makes the third option above more credible because of the emphasis Newton was always inclined to place on virtually exact agreement between experiment and theory
   a. Did not have the sort of cavalier attitude about the limits of experimental precision displayed by Galileo, Descartes, and Mersenne
   b. Hence he was always looking to achieve high quality evidence through precise measurement
6. We can thus have reason to think that Newton fully appreciated the conception of empirical evidence implicit in Huygens's work after reading the *Horologium Oscillatorium*
   a. Newton could have used this conception as a point of departure when he began developing a still more sophisticated conception while fashioning the *Principia*
   b. And this would explain his seeming expectation that Huygens would appreciate the full thrust of the evidential argument in the *Principia*
7. Nonetheless, the parallels and overlaps between Huygens and early Newton notwithstanding, it is important to see that Newton was not embarked on a project of developing a full mechanics before 1684 in the way Huygens was
   a. Newton was working on isolated problems, generally in response to Descartes and with an eye toward developing a compelling argument for Copernicanism -- to complete the work of Galileo's *Dialogue*
   b. Huygens, by contrast, showed no interest in arguing for Copernicanism at the time; he was developing a general mechanics, extending the work of Galileo's *Two New Sciences*
   c. The question, What started Newton into developing a general mechanics?, will be answered in the next two classes

V. Newton's Fragment on Hydrostatics (vs. Descartes)

A. Background: the Anti-Cartesian Introduction

1. The fragment, "De Gravitatione et Aequipondio Fluidorum," is presumably an introduction to a book on hydrostatics which stops abruptly after making just a few claims about pressure in liquids
   a. 35 pages long (in Hall and Hall, 40 handwritten), the last 4 of which deal with deforming fluids, and the first 31 of which comprise an anti-Cartesian introduction to the rest in which Newton is endeavoring "to dispose of his [Descartes'] imaginings" -- "figmenta"
   b. Date uncertain, though thought for a long time to be around the same time as the "Lawes" if only because it too is clearly a response to having read Descartes' *Principia* and much of the physics in it seems far removed from that of Newton's *Principia*
   c. But Betty Jo Dobbs put forward an argument that it dates from the early 1680's -- probably 1684 -- and as such represents Newton's final break with Descartes; from manuscript evidence Rob DiSalle and I have now concluded that it does not date from 1685

    d. Howard Stein regards it as the fundamental expression of Newton's metaphysics, as underlying the *Principia*, and hence as perhaps contemporaneous to the latter, and I have emphasized that it uses the word *inertia* just as Newton first started using it (and *massa*) in late 1685
    e. I am going to treat it as an amalgam, the (unusually clean) manuscript dating from 1684, but with critical thoughts about Descartes dating from late 1660s and early 1670s
2. Herivel's excerpts have dropped a good deal of further anti-Cartesian material, including philosophical discussions of God in relation to physics and discussions of substance and accident; the best full translation is in Janiak's volume, though it too misleadingly translates *moles* as "mass"
3. Herivel's excerpts have also dropped claims about the isotropy of pressure in fluids and the consequent equilibrium of fluids
    a. Claims mathematically proven in the Greek style
    b. With an apology for accommodating "these definitions not to physical things, but to mathematical reasoning, after the manner of Geometers"
4. Given the abrupt ending of the fragment, it is not especially clear what Newton was intending to do, nor what, if any, obstacles prevented him from fulfilling his aims
    a. Pressures in fluids a central concern in Descartes' vortex theory, even though Descartes himself offers no detailed account of pressure, but relies on physical intuition
    b. So perhaps Newton saw himself as providing background physics that would then have telling implications -- either for or against -- the vortex theory
    c. Or this may be an immediate forerunner of those parts of Book II of the *Principia* where Newton develops a systematic argument against the Cartesian vortex theory -- a theory he himself had held through much of the 1670's
5. Our interests in the fragment are primarily with his concepts of motion and force
    a. Want to emphasize the dominant influence of Descartes even when he is being thoroughly anti-Cartesian in his explicit remarks
    b. Spirit of undertaking, as with "Lawes of Motion," one of giving a correct account of a phenomenon Descartes called attention to, for much the reasons that Descartes called attention to it, while rejecting Descartes' account
    c. I am not going to put much weight on the "metaphysics" in it, as several philosophers commenting on Newton have, if only because the dating is too uncertain

B. Newton's Early Views on Space and Motion
1. In contrast to Descartes' account of space in terms of the relative positions of bodies, Newton invokes absolute space, a receptacle parts of which physical bodies fill
    a. The idea of space as a receptacle dates back at least to Plato's *Timaeus*
    b. Space and extension neither substance nor accident, but having a mode of existence all to itself

2. Absolute rest and absolute motion occur within absolute space, for motion is defined to be change from one absolute place to another
    a. In principle (though not necessarily in practice) determinate whether a body is in rest or motion
    b. Motion itself also in principle determinate -- both speed and direction
    c. (Unclear whether Newton thought Descartes denied these claims, given his rules of impact, though perfectly clear that Descartes denies the existence of absolute space as a receptacle)
3. The arguments that Newton offers against Cartesian claims about space and motion being relative are of reductio form; they aim to show that Descartes himself was committed to absolute motion and hence to absolute space
    a. In particular, properties of vortices that are indispensable to Descartes' planetary theory require determinate circular motion insofar as whether an object has a certain centrifugal conatus must have a single determinate answer!
    b. More generally, all talk of motion presupposes such determinate answers to questions about direction and magnitude
4. Newton thus effecting a reform of Descartes' way of conceptualizing space and motion that will allow such things as the principle of inertia and centrifugal conatus in a vortex to remain meaningful
    a. Newton not abandoning Descartes' emphasis on the principle of inertia and its consequent implications for curvilinear motion
    b. Rather, he is doing what he thinks must be done in order to preserve the Cartesian insights he takes to be crucial
    c. (The appeal to curvilinear motion as a basis for distinguishing between absolute and relative motion will show up at the beginning of the *Principia*)
5. Newton uses his conclusions about absolute motion, however, to draw conclusions about absolute space as a receptacle that then permit him to defend the atomistic school of mechanical philosophy
    a. E.g. Descartes' principal argument against vacuums entirely undercut, for what can exist between two bodies can be empty space, even though empty space is not a substance
    b. No need then for plenism, and therefore no obstacle to adopting an atomistic view, yet still incorporating Descartes' key insights about inertia and curvilinear motion
    c. And no need to deny bodies essential properties beyond extension

C. Newton's Early Conception of Forces

1. The conceptualization of force in this fragment is closely akin to Descartes' and far removed from that of Newtonian mechanics, though not so far removed from that of Newton's *Principia*
2. Two kinds of force (*vis*): external force changing motion, and internal force conserving motion and opposing impediments to motion
    a. External, impressed force either creates or destroys motion, and hence supplies answers to questions about changes in motion

b. Internal force supplies answers to questions about why motion continues unless impeded

c. *Inertia* is expressly identified with this internal force

3. *Conatus* -- endeavor or tendency -- is an impeded force, or a force insofar as it is resisted

   a. Pressure is a conatus, for it is a force that would disappear if the fluid could pass through the boundaries on which the pressure is exerted

   b. Centrifugal conatus a real property of bodies arising from the resistance of the internal force of the body

4. Gravity is an intrinsic force imparted to a body to descend, not just to the center of the earth, but generally -- suggesting that centrifugal *conatus* imparted by curvilinear motion is somehow a species of gravity

5. Two measures of force -- intensity and extension -- with the absolute measure the product of these two; intensity a measure of power, extension in terms of quantity of space and time

   a. E.g. absolute quantity of motion (and hence force) compounded from magnitude of the body (intensity) and velocity (extension)

   b. Absolute quantity of force in the case of pressure compounded from the intensity of the pressure and the quantity of the surface pressed

D. Newton's Orientation: Copernican Concerns

1. Given the animus with which Newton attacks Descartes' solution to the problem of reconciling Copernicanism with the Church's views, reasonable to suggest that what is driving Newton here is at least in part his desire to establish Copernicanism

   a. Newton a rabid anti-Catholic (typical of Puritans)

   b. Was clearly displeased with Descartes' suggesting that the earth is not really in motion merely because it is exhibiting no relative motion with respect to matter contiguous to it

2. Newton had read the English translation of Galileo's *Dialogue* at roughly the same time as he had read Descartes' *Principia*

   a. Had formed a goal of proving Copernicus correct somewhere around this time, a goal that remained with him thereafter

   b. Saw Descartes' treatment of space an impediment to doing so, Descartes' arguments against the Ptolemaic and Tychonic systems notwithstanding

3. At the very least, Newton saw a strict relativity of motion view as an impediment to showing that Copernicus is correct and Tycho is wrong, for not clear how to distinguish between the two if no determinate answer to the question whether the earth is moving

4. Perhaps Newton saw Descartes' concept of *conatus a centro* as the key to effecting a proof that the earth is undergoing curvilinear motion

   a. Could be a reason for developing an account of pressure in vortices that would enable him to show that the Earth is moving

    b. E.g. by showing that Kepler's third law must hold, and that it cannot hold for the Moon and the Sun orbiting the Earth

5. This is speculation, but it is supported by a good deal of evidence from the 1660's that Newton wanted to settle the question of the motion of the earth after reading the *Dialogue*
    a. The important thing to notice is that Newton is not fashioning the account of space and force in this fragment in order to develop a general mechanics; indeed, no mention of centripetal force
    b. His interests, which seem far narrower, fit better with the view that Galileo provoked him, and he was now looking for a better argument than Galileo had managed to supply
    c. This would fit with the selectivity Newton shows over which features of Cartesian physics he is prepared not just to retain, but to build around, and which features he is prepared to discard and to rail against
6. When Newton finally finds a way of using centrifugal conatus to argue for the Copernican system and embarks on the *Principia*, he returned to his earlier objections to Descartes, but now formulating them in a philosophically more sophisticated way tied to it
    a. If so, then this is an important tract, just as Stein suggests, for it offers a philosophic response to Descartes' philosophic arguments
    b. Keep this tract in mind as we read the *Principia*

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Credits for Appendix

Slide 1: ?

Slides 2-5: Hall (1992)

Slides 6, 7, 16, 17: Newton (Add. 4000)

Slides 8, 10, 11: Whiteside (1967-1981)

Slide 9: Cohen and Westfall (1995)

Slides 12-15, 26: Whiteside (1964)

Slides 18-20: Cohen and Schofield (1978)

Slide 21: Shapiro (1984)

Slides 22-25, 27-31, 33, 38, 39, 42, 43: Herivel (1965)

Slides 32: Hall (1999), Pemberton (1972)

Slide 34: Streete (1661)

Slide 36: Turnbull (1959-1963)

Slide 37: Huygens (1986)

Slides 40, 41: Newton (Add. 4003)