b. As such, completely replaced *Astronomia Nova* as the fundamental work of Kepler, as well as replacing *De Revolutionibus* (and as Kepler announces, Aristotle's *De Caelo*)
c. Because the reasoning in it proceeds from physics to orbits, the more predominately astronomical reasoning of *Astronomia Nova* -- i.e. reasoning from observations to orbital motions -- becomes lost from view, as does some of the continuity with earlier astronomy

4. Includes a treatment of the moon covering Ptolemy's inequality; the inequality called the "variation," discovered by Tycho; and a new inequality, the annual equation, discovered independently by Tycho and Kepler
   a. All depend on the position of the Sun vis-a-vis the Earth and Moon
   b. "Kepler realized that any physical theory must involve a double interplay of the Earth and Sun." -- i.e. Moon, driven by emanations from both Earth and Sun -- (Gingerich, p. 75)
   c. This was Kepler's second published cinematic model of the moon; an earlier one in the ephimerides of 1617 stayed faithful to the area rule
   d. Whatever else may be said for them, both models fall far short of the level of accuracy in predicting longitude and latitude of the Moon achieved by the model for Mars
5. In a way, a revolution in astronomy textbook writing, for includes physics and mathematical astronomy together, unlike e.g. Ptolemy and Regiomantanus's *Epitome of the Almagest*
   a. As Kepler himself says, "You might doubt whether you were doing a part of physics or astronomy, unless you recognized that speculative astronomy is one whole part of physics." (p. 5)
   b. Even though most key readers ended up discarding the physics early on, they did not discard the need for a physics to the same extent

G. *Tabulae Rudolphinae*: The Culmination (1627)

1. The closest Keplerian counterpart to Ptolemy's *Almagest* insofar as it includes no physics, but only mathematical astronomy; really though, Kepler's counterpart to Ptolemy's *Handy Tables*
   a. 275 pages explaining how to use the Tables (including how to use logarithms) and in some places giving some background on the orbital elements
   b. Followed by 104 pages of tables and then a star catalogue
2. The frontispiece summarizes Kepler's own view of where he fit into the process of reforming mathematical astronomy
   a. Main pillars for Copernicus and Tycho, but pillars to Ptolemy, Hipparchus, etc.
   b. Emphasis on Tycho, Hven, etc. because of the critical role of Tycho's observational efforts, which Kepler never belittled, and the project of the Tables having originated with Tycho
   c. Kepler in basement toiling on calculations by candlelight, with a few coins on the table
3. Kepler's orbits are simpler than they first appear, involving one set of elements pertaining to heliocentric longitude, another pertaining to heliocentric latitude, and the third pertaining to location of the planet at some epochal time

a. The longitude of aphelion, the eccentricity, and the mean distance (or its double, the length of the line of apsides) -- quantities that basically carry over from Ptolemy -- determine the ellipse

b. For, given any "eccentric anomaly," need only simple trigonometry to determine the sun-planet distance and the angular location of P, angle ASP (see figure), leaving only the time at P to be determined, knowing the period and e.g. the time T when at aphelion (see below)

c. Kepler's orbital scheme points to certain preferred observations for determining aphelion, eccentricity, and mean distance for a planet, adding to those from Ptolemy forward

d. As with the preferred observations for determining location of nodes (longitude of the ascending node) and inclination, the two elements dictating latitudes

4. Kepler could not let the area rule determine the shape of the other orbits in the way he had done for Mars, for, given the area rule, Tycho's data could not distinguish their ellipses from eccentric circles

   a. Not able to do a vicarious theory for the inner planets, for they are never in opposition, where heliocentric longitudes match geocentric longitudes

   b. And, as the table in the Appendix shows, the low eccentricity of the orbits of Jupiter and Saturn make their trajectories too close to circles for the area rule to distinguish the ellipse

   c. Indeed, even in the case of Mars, as shown last week, a bisected eccentricity equant in a circular orbit produces heliocentric longitude errors less than 10 min of arc

5. In one critical respect the calculation system required a bothersome form of calculation that contrasted it sharply with all prior systems

   a. Because of the area rule and the ellipse, going from a given time to the determination of heliocentric longitude required an indirect, iterative solution of a transcendental equation, now called Kepler's equation (see figure):

   $$n * (t - T) = M = E - e * \sin E$$

   where n is the mean motion, M the "mean anomaly", E the eccentric anomaly locating the projection of the planet on its circumscribed auxiliary circle, and T the time at aphelion

   b. This feature, more than any other, impeded the use of the Tables, leading others to try to by-pass the problem -- e.g. by replacing the area rule -- instead of using the tables Kepler provided giving approximate solutions for his equation

6. By the time the *Rudolphine Tables* were published, Kepler knew better than anyone that they left problems open

   a. For example, in the discussion of the moon, he remarks that the motions of the sun and moon and the diurnal motion of the earth are not equable, but are

   "subject to small intensions and remissions extra ordinem; these perturbations being perhaps due to physical influences from the planets. The physics that Kepler had introduced into the skies thus brought with it some of the complexity of the terrestrial physical realm, where many causes are concurrent in a single event" (Wilson, "Horrocks ...," p. 241) -- see X, p. 44, ln 27-30 and p. 90, ln 7-11

b. And in a letter of 1625 he indicates that Jupiter, Saturn, and Mars are subject to an inequality that will require centuries of observation before it will become amenable to analysis -- a point he alludes to in the Preface to the Tables (Wilson, ibid. p. 240) -- see XVIII, p. 237, (letter to Bernegger) and X, p. 44, ln 21-25, which cites observations by Regiomantanus and Walther

c. His view seems to be one of using the Tables to expose and then characterize these discrepancies in order to refine the theory to handle them, as he had begun to do with the moon; and he recognized that his values for the elements depended on Tycho's parallax corrections for the sun

d. Nevertheless, he had clearly become concerned that astronomy might not be "perfectible"

7. Note that Kepler raised the question, is planetary astronomy perfectible? -- i.e. can the (undisturbed) motion of the planets be predicted into the indefinite future to within observable accuracy?

   a. A question receiving its first reasonably conclusive answer only in the last decade of the twentieth century

   b. For Kepler, not a question about whether e.g. ellipse and area rule exact, but about whether any variation over time in the orbital elements can be specified in a way that makes physical sense

   c. If some alternative to Keplerian trajectories can do this better, then for Kepler it would have had claim to being superior to his

   d. The possibility that no account of the motion could do this he found threatening

8. In addition to such effects that Kepler found beyond his reach, the Tables have shortcomings in orbital elements that they need not have had (see table in Appendix)

   a. Primarily from Tycho's theory of the sun, which had a far too large correction for parallax, because of a far too large horizontal parallax of the Sun -- i.e. Tycho had the sun much too near the earth; this error affects everything else because observing from the earth

   b. This resulted in an excess eccentricity of the earth-sun orbit; this in turn contributed to Kepler's eccentricity for Mars being a little too small (0.09253 versus 0.09304, or 430 parts ingress vs. 433.8) and the aphelion being a little advanced (148° 59′ 54″ versus 148° 41′ 58″)

   c. Kepler reduces Tycho's 3′ to 1′ after concluding parallax of Mars is less than 1¼′, but still does not change earth-sun eccentricity

   d. Nevertheless, as Gauss was to remark in 1809, the problem for post-Keplerian astronomers "was no longer to deduce elements wholly unknown, but only slightly to correct those already known, and to define them within narrower limits." (Wilson, "Derivation", p. 25)

II. Some Philosophic Issues Concerning Kepler's "Laws"

A. Kepler's Substantive Legacy: the Generalizations

1. His most obvious legacy was a comparatively simple, yet extraordinarily accurate version of the Copernican system -- the sort of simplicity and accuracy that Copernicus had yearned for

   a. The Copernican system with Keplerian orbital motion