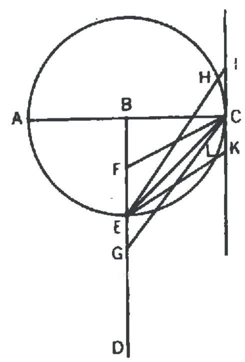
Proposition 30



Drop the vertical BD from a point B in the horizontal line AC, in which take any point C; and in the vertical, take a distance BE equal to the distance BC, drawing CE, I say that of all inclined planes from point C to the vertical [BD], CE is that along which descent will be made to the vertical [BD] in the shortest time of all.

I.e. let α be the angle the plane makes with the horizon – e.g. angle BCF. Let d be the distance along the horizontal from the apex of the inclined plane to the vertical, and s the length of the inclined plane. Then

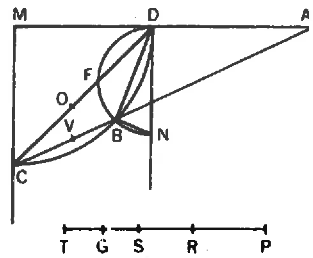
$$s=1/2\,(a\cdot\sin\alpha)\,t^2 \quad \text{and} \quad d=s\cdot\cos\alpha$$

so that

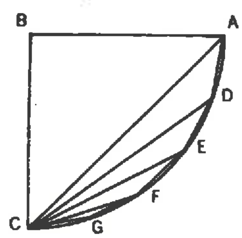
$$d = 1/2\,(a\cdot\sin\alpha\cdot\cos\alpha)t^2$$

Given d, therefore t is least when the expression in parenthesis is a maximum, which is when α is 45 degrees.

Proposition 36 and Scholium



Let the circumference CBD be no more than one quadrant of the verticle circle with its lowest point at C, to which is raised the plane CD; and let two planes be deflected from the ends D and C to some point B taken on the circumference; I say that the time of descent through both the planes DB and BC is briefer than the time of descent through DC alone.



From the things demonstrated, it appears that one can deduce that the swiftest movement of all from one terminus to the other is not through the shortest line of all, which is the straightest line AC, but through the circular arc.... Therefore descent is made in still shorter time through the five AD-DE-EF-FG-GC than through the four AD-DE-EF-FC. Hence motion between two selected points, A and C, is finished more quickly, the more closely we approach the circumference through inscribed polygons.

What has been explained for the quadrant happens also in arcs less than the quadrant.

Sagredo's Assessment of Day 3

It appears to me that we may grant that our Academician was not boasting when, at the beginning of this treatise, he credited himself with bringing to us a new science concerning a most ancient subject. When I see with what ease and clarity, from a single simple postulate, he deduces the demonstrations of so many propositions, I marvel not a little that this kind of material was left untouched by Archimedes, Apollonius, and Euclid, and so many other illustrious mathematicians and philosophers; especially seeing that many and thick volumes have been written on motion.

What exactly has the mathematical theory of "natural" motion in Day Three accomplished?

> ***"Predictive" power: so much***
>
> ***"Explanatory" power: from so little***
>
> ***"Question-answering" power***