c. Instead an insistence on dealing with the physically real case, and disappointment when this proves unmanageable

III. Newton on Circular Motion (vs. Descartes, Huygens)

A. Basic Theoretical Results, versus Huygens's

1. The "vellum manuscript" (Add. 3958, f. 45) that anticipates this one is a torn legal parchment with a lease on the front and calculations on the back

a. That manuscript thus likely dates from when Newton was home during the plague years, in 1665 or shortly thereafter

b. If so, then the manuscript on circular motion, written in Latin as if for publication, dates from after that

2. Unlike Huygens, Newton does not formulate the question about the *conatus a centro* in uniform circular motion as one about the (instantaneous) tension in a string

a. Through what space would the *conatus* impel a body in the time of one full revolution if it were not constrained

b. Want the total sum of the incremental distances by which the body would have departed from the circle had it not been constrained during the time of one revolution -- i.e. the s corresponding to the uniform acceleration ($r*\omega^2$) over one period of revolution

3. The answer: $D*\pi^2$ (in full agreement with Huygens since $v^2/r = r*\omega^2 = 2\ D*\pi^2/P^2$ and $s = a*t^2/2$)

a. Newton's formulation amounts to: x/BD = ADEA^2/AD^2

b. From the end of Book 3 of Euclid, BE/BA = BA/BD (Euclid's Proposition 36: the rectangle formed by BE and BD is equal to the square formed on BA)

c. But for an infinitesimal increment, BE approximates DE, and BA, DA, so that BD approximates AD^2/DE, and answer results by plugging in for BD above

4. Newton's interest in the problem not a mechanical one, like Huygens's, but one concerning the degree of *conatus*, just as Descartes had called for in *Principia* III, 59

a. Notion of total endeavor in a revolution strange to us, though does give a measure of acceleration in terms of distance covered versus time (squared)

b. Simply assumes uniform acceleration (and Galileo's rule for distance in uniform acceleration, taken from the *Dialogue*), whereas Huygens feels required to argue for it! -- that is, to argue that the induced tension in the string is commensurable with the tension in a hanging string

5. Newton does not develop the result in the manner of Huygens, for the issue that concerns him is not posed within the framework of Galilean mechanics

a. Huygens ties theory of centrifugal force to Galilean fall in order initially to obtain a measure of g, and subsequently to provide evidence for theory via measure of g

b. Earlier, in Waste Book Newton had devised solution for circular motion via impact (Appendix);

and in Vellum manuscript had rejected Galileo's value of the fall in 1 second (roughly 5 ft), and then proposed a 45 degree conical pendulum as a means for measuring the correct value

   c. Here he does mention the conical pendulum at the end, but he does not follow up the mention with a full mechanical theory in the way Huygens does

6. Nor does Newton derive any easily testable results, like Huygens's result for the intercepted pendulum, for he is pursuing different objectives
   a. That is, he is working outside the tradition of Galileo's *Two New Sciences*
   b. And he is being provoked by astronomical concerns

B. Four Immediate Applications of the Theory

1. Newton proceeds to derive one primary result about the centrifugal tendency at the surface of the Earth from the basic theoretical result, and then draws conclusions on three other topics
   a. With the exception of the acceleration of gravity, Newton is largely using values taken from Galileo's *Dialogue*
   b. (Interesting that he would reject Galileo's value for the acceleration of gravity and accept other values from the book)
2. Compare the centrifugal tendency at the surface of the earth to gravity, in the process confirming Galileo's explanation for why we do not feel the centrifugal tendency, much less fall off the earth
   a. Assume radius of earth = 3500 (Italian) miles, 5000 ft per mile
   b. Then since acceleration of gravity is 32 ft/sec/sec (corresponding roughly to Huygens's announced value), the force (*vis*) of gravity is 350 times stronger than the centrifugal *conatus* at the equator
   c. The correct value -- using a correct radius of the earth instead of the value Newton has taken from Galileo's *Dialogue* -- is 288
3. Compare the centrifugal tendency of the moon with that on the surface of the earth, in effect exploiting the fact that r/P^2 gives this proportionality
   a. Moon 60 earth radii away, so that centrifugal tendency on the surface of the earth is 12.5 times stronger than that of the moon
   b. Therefore, gravitational force at the surface of the earth is around 4000 (4375) times the centrifugal tendency of the moon -- versus 3600 times if $1/r^2$
4. Infer (incorrectly) a lower bound of the horizontal solar parallax (19 sec) from the assumption that the centrifugal tendency of the moon is responsible for our always seeing the same face
   a. For then the centrifugal tendency of the moon with respect to the earth would have to be greater than that of the moon with respect to the sun, for otherwise the moon would always show the same face to the sun, and not to the earth
   b. Therefore centrifugal *conatus* on surface of earth 132,408 times greater than centrifugal *conatus* with respect to the sun (assuming -- incorrectly -- that horizontal solar parallax is 24 sec)

5. Compare the centrifugal tendencies of the planets from the sun, using fact that this proportionality must be $1/r^2$ since the planets conform with Kepler's 3/2 power rule
    a. Centrifugal tendencies as 614 to 173 to 91 to 39 to 3.33 to 1, from Saturn to Mercury
    b. No reason given or suggested for taking the trouble to obtain this result
6. Final points concerning relation between conical pendulum and circular pendulum probably reflect his earlier idea of using a 45 deg conical pendulum to obtain a more accurate value for strength of gravity than Galileo had proposed
    a. Newton himself carried out a conical pendulum measurement of gravity somewhere around this time, stopping short of full precision once he got within 1 percent of Huygens's value
    b. Perhaps failure to obtain an exact $1/r^2$ for the Moon occasioned him to check Huygens's value

C. Newton's Preoccupation: Copernican Concerns

1. The common element in the three main applications of this manuscript, which appears to have been written for publication judging from how clean it is (see Appendix), is a concern with issues attendant to Copernicanism, more specifically with issues arising within Galileo's *Dialogue*
    a. Even the horizontal solar parallax is pertinent to issues raised in the *Dialogue*
    b. For, the Aristotelian can argue that, if the earth is orbiting the sun, then observed acceleration toward the center of the earth ought to be greater at high noon on the equator than at midnight, since centrifugal conatus additive at noon
2. By contrast, Huygens showed no interest at all in such Copernican issues in his treatment of centrifugal force
    a. Newton prompted by Descartes' discussion of the centrifugal tendency in planetary motion, Huygens prompted by his discussion of the sling
    b. Huygens did not even show interest in the $1/r^2$ implications of Kepler's third "law," something noticed by several others after the *Horologium Oscillatorium* was published in 1673
3. Newton's efforts more akin to those of Kepler's calculations in the *Epitome*, where Kepler was fishing for information about mechanisms underlying the planetary orbits
    a. Newton too appears to be somewhat on a fishing expedition, trying to draw inferences from the theoretical characterization of the *conatus a centro* and various other available information
    b. Almost as if his sole point in deriving the theoretical result on the *conatus a centro* was in the hope of learning more about celestial motion
4. Notice, however, Newton's lack of concern here for exact orbits, which he knew perfectly well from Streete were not circles
    a. In this respect, his effort is less akin to Kepler than to Descartes, for he is showing no interest at all in what might produce trajectories other than circles
    b. Furthermore, the values generated are pertinent to Descartes' vortex model, in which gravity at the surface of the earth is related to vortex pressures

5. The contrast between the moon and the planets is, of course, a Copernican issue discussed in the *Dialogue*, but it is even more so an issue in Descartes' *Principia*
   a. My guess then is that Newton was looking for some way to take Descartes' basic insight concerning the *conatus a centro* associated with curvilinear motion and begin turning it into an argument for Copernicanism
   b. Or at least to bolster, via a few specific numbers, some of the arguments in support of Copernicanism in the *Dialogue*

D. Universal Gravitation: The Historical Issue
1. The issue is whether Newton considered the hypothesis of inverse-square universal gravitation in the 1660s and rejected it because of the substantial empirical discrepancy between 4000 (4375) and 3600
   a. Or even the weaker hypothesis, gravity varies as $1/r^2$ and extends from the earth to the moon
   b. At least the latter hypothesis compatible with ones being entertained at the time and subsequently by Hooke and Wren, although surely this was not then known to Newton
2. The legend that Newton was already looking to gravity to explain planetary motion in 1666 derives from Whiston and Pemberton, and from Newton too, all after 1700 (see accounts in the Appendix)
   a. The legend of the apple is from Stuckley and Conduit, apparently originally owing to Newton sometime after 1715
   b. Part of Newton's embellished account of how much of the *Principia* was developed in 1666, years before Hooke had suggested anything to him
3. The source of the legend was undoubtedly Newton's defense against Hooke's accusation of plagiarism, an accusation that took on more bite following the dispute with Leibniz
   a. Newton's 1686 defense against Hooke's charge pointed out only the derivation of $1/r^2$ from the 3/2 power rule, and added that Wren and Gregory had derived the same thing before 1679
   b. Thus the 1686 defense is compatible with the manuscript on circular motion (dated by Turnbull as ca. 1669), and the later embellished defense is much less so
4. Note that throughout this manuscript Newton is comparing centrifugal tendencies -- *conati a centro* -- with one another and with the gravitational force at the surface of the earth
   a. No mention is made of gravity in the case of the sun and planets at all
   b. And, as Wilson points out, nothing remotely akin to universal gravity is ever suggested
5. Equally, however, one should notice that the comparison between 4000 and 3600 -- a comparison not in the manuscript -- does not make much sense unless Newton is either assuming or hoping to show that the 3/2 power rule holds for the moon
   a. There is of course no evidence for this at all at the time, though one might nevertheless hypothesize it on the basis that the 3/2 power rule is a general property of all celestial orbiting systems
      (1) Horrocks's improvement in Kepler's orbits from using the 3/2 power rule to determine semi-major axis is grounds for saying this