Planning and Communicating Risk for Nonstationary Natural Hazards

A dissertation submitted by

Laura K. Read

in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

in

Civil and Environmental Engineering

TUFTS UNIVERSITY

February, 2016

Adviser: Richard M. Vogel

ABSTRACT

This work investigates the probabilistic behavior of the time to occurrence of natural hazards that exhibit nonstationarity through time with special attention to floods. Chapter one combines existing theoretical and empirical results from the literature to provide the first general, comprehensive description of the probabilistic behavior of the return period and reliability under nonstationarity for the case of floods. Findings indicate that under nonstationarity, the underlying distribution of the return period exhibits a more complex shape than the exponential distribution under stationary conditions. Chapter two provides an introduction to the field of hazard function analysis (HFA) for flood events under nonstationary conditions, and demonstrates how HFA can be used to characterize the probability distribution of the return period and the reliability – two primary metrics in hydrologic design. This is the first paper to explicitly link the probabilistic properties of a flood series (*X*) with failure times (*T*) associated with a particular infrastructure design. This work shows that HFA is a relevant and useful approach for characterizing nonstationary flood series, and can provide engineers with tools to support hydrologic design decisions under nonstationary conditions. Chapter three investigates the suitability of HFA to characterize a wide class of nonstationary natural hazards whose peaks over threshold (POT) magnitudes are assumed to follow the widely applied Generalized Pareto (GP) model. Such natural hazards might include: wind speeds, landslides, wildfires, precipitation, streamflow, sea levels, and earthquakes. The hazard function equations are derived for a natural hazard event series (*X*) whose POT follows the 2-parameter GP distribution. The derived model and HFA are used to compute reliabilities and average return periods associated with nonstationary behavior of the original hazard series. These generalized results for a wide class of natural hazards are consistent with the results in Chapters 1 and 2 for floods: nonstationarity adds complexity to computation of traditional design metrics and changes the shape of the probability distribution of the return period. General implications for planning and design of nonstationary natural hazards are discussed.

ACKNOWLEDGMENTS

I must give credit to all of the people who have academically and personally supported me before and during graduate school. Each friend, family member, and mentor in my life has contributed a piece in my dissertation experience, one that is now complete.

Deciding to attend Tufts University for my PhD was a tough choice that meant leaving behind a great network of peers, advisors, and friends at the University of Texas – Austin, where I had completed my Master's degree in Environmental and Water Resources Engineering. In particular, I must acknowledge my adviser and mentor at UT-Austin, Dr. Daene McKinney, who taught me how to conduct research, introduced me to the Cordillera Blanca, Peru, and encouraged me to pursue a doctoral degree.

The reason I selected Tufts University was to work with Dr. Richard Vogel through a National Science Foundation Interdisciplinary Graduate Education Research Traineeship (IGERT) program. This program had a network of advisers and students across several disciplines at Tufts, and promoted interdisciplinary research and growth. I credit the IGERT program for providing the opportunity to broaden the scope and implications of my dissertation research, from straight engineering to include social dimensions of risk communication. I would like to acknowledge Dr. Shafiq Islam and Dr. Michael Reed, with the support of Dr. Antje Danielson at Tufts Institute of the Environment, for making the IGERT learning space collaborative, interesting, and relevant.

The most important person that contributed to my academic research was Dr. Vogel. The transformation that I experienced as a graduate student was only possible through the guidance and advising he provided. From Dr. Vogel, I learned how to write precisely, to present a deeply technical topic to a wide audience, and to think as a researcher – a problem solver. The expertise and advice from my committee members, Dr. Casey Brown, Dr. Kaveh Madani, Dr. Daene McKinney, and Dr. Laurie Baise, also contributed to this transformation. I feel very fortunate to have had their support and time through my experience. My peers in the Civil/Environmental Engineering Department provided invaluable feedback and philosophical discussions that deeply added to my academic experience. In particular, I would like to thank the Vogel Research Group, William Farmer, Brent Boehlert, and Stephanie Galaitsi.

While graduate school has been a rewarding experience, I often relied on friends and family for support when times were tough (and when they were good). My happiness and balance as a person is what enabled me to reach this point, and is largely due to the endless support from my mom, Mary Cooper Wallace, and my dad, Bill Read. My stepdad, Bill Wallace, has also been a rock and a great source of support. In addition to my family, I have the best friends on the planet, who have never let me down, been loyal, honest, and given me the courage to be myself. I cannot list them, but without the support of Drew Gemmer, Katy Collins, the Twist family, the Betz-Brown family, Elizabeth Ballard, and my two Sals – Fernando Salas and Elliott Gall, I would not be the person I am today, graduating with my PhD.

A funny thing happened in the middle of graduate school: I met my life partner and we began our life together. Though I cannot be sure which transformation came first – I am not even convinced they were not around the same time – I know that the presence of this person gave me the confidence and assurance that I needed to transform myself into the student and researcher that a PhD requires. Thank you Kelly for never giving up on me, never just telling me it will be all okay, and always listening when I needed to talk. I love you and hope I can do the same as you've done for me someday.

Table of Contents

List of Tables and Figures

Tables

Figures

FOREWORD

In literature of many disciplines, hydrologists, climate scientists, engineers, and others concerned with natural hazards are discussing the existence, attribution, and relevance of trends, and underlying concepts of stationarity and nonstationarity. On one extreme some argue that stationarity is dead! (Milly et al., 2008), while others proclaim its immortality (Montanari and Koutsoyiannis, 2014). Rich literature exists in between these two extremes, addressing the notion of whether nature is *trendy* (Cohn and Lins, 2005) and the implications of assuming stationarity or nonstationarity for planning and design of infrastructure. Although the future is uncertain, the scientific community generally agrees that certain elements of climate-to-come will be different from the past – e.g. atmospheric carbon dioxide levels from greenhouse gas emissions will increase. While hydrologists do not know precisely how changes in climate will manifest to impact the hydrologic system, we have been able to measure certain impacts, such as how urbanization affects hydrology (Villarini et al., 2009; Prosdocimi et al., 2015 and others).

The risks imposed by adverse climate and corresponding changes in the hydrologic system are potentially devastating to populations around the world, and thus engineers have a unique responsibility to understand these processes and develop ways to communicate associated risks. Vogel et al. (2015) argues that hydrology is inextricably linked with society and human behavior, addressing points raised in previous papers that continuing to assume stationarity and not consider human influences when designing hydrologic (and other) infrastructure may not be a strategy we can afford. For example Rosner et al. (2014) introduce a risk-based approach for comparing no-action to action strategies for hydrologic infrastructure design. The question at the center of these discussions is how to proceed with design and communication of risk given unresolvable uncertainties in the system, recognizing that underdesign may expose many to high risks, and overdesign may be wasteful.

The three chapters of this dissertation intend to contribute to the aforementioned ongoing discussion in the literature, specifically to the point on how nonstationarity, if present, may impact the risk of failure and reliability of natural hazard systems. Briefly, this work uses concepts from probability theory to show how the distributions of floods and natural hazards change – and grow in complexity – when trends are present in the historic record; and, the implications on hydrologic design and risk communication for trends that continue into the future. Though this work does not advocate for trend extrapolation into the future nor attempt to attribute trends to anthropogenic or natural sources, it intends to push readers to consider that if a trend has indeed been detected, engineers and managers need a plan for moving forward with design and communication to the public. Currently there is no cohesive standard for implementing nonstationarity into hydrologic or natural hazard infrastructure design, though many studies require that research include climate change in their scopes. This lag in adoption of new standards at a higher level is slowly dissipating, as organizations like the American Society of Civil Engineers have recently written on the need to update principles and practice to include climate change and adaptation (Olsen, 2015).

While this is progress, it does not address the issues associated with misperception of risk and need to modify communication to appropriately inform the public on risks associated with hazards they may encounter. This technical and social challenge requires that engineering and social science align, recognizing the interdisciplinary nature of hydrology to find solutions (Vogel et al., 2015). Indeed this alignment is perhaps one of the most exciting, yet greatest challenges facing water resource engineers in the coming years.

The three chapters presented in this dissertation seek to document the impact of nonstationarity on the probability distribution of floods and natural hazards in general; and based on these findings, suggest innovative ways to communicate this much more complicated story to those responsible for design and planning. Chapter 1 presents an argument for replacing the concept of average return period with reliability for communicating risk of flooding and for designing infrastructure; Chapter 2 provides an introduction to the theory of hazard function analysis for characterizing floods in a nonstationary context; and Chapter 3 extends the application of hazard function theory to nonstationary natural hazards in general.

Chapter 1: Reliability, Return Periods, and Risk under Nonstationarity

Journal reference: Read, L. K., and R. M. Vogel (2015), Reliability, return periods, and risk
under nonstationarity, Water Resour. Res., 51, doi:10.1002/2015WR017089

ABSTRACT

Water resources design has widely used the average return period as a concept to inform management and communication of the risk of experiencing an exceedance event within a planning horizon. Even though nonstationarity is often apparent, in practice hydrologic design often mistakenly assumes that the probability of exceedance, *p*, is constant from year to year which leads to an average return period T_o equal to 1/*p*; this expression is far more complex under nonstationarity. Even for stationary processes the common application of an average return period is problematic: it does not account for planning horizon, is an average value that may not be representative of the time to the next flood, and is generally not applied in other areas of water planning. We combine existing theoretical and empirical results from the literature to provide the first general, comprehensive description of the probabilistic behavior of the return period and reliability under nonstationarity. We show that under nonstationarity, the underlying distribution of the return period exhibits a more complex shape than the exponential distribution under stationary conditions. Using a nonstationary lognormal model, we document the increased complexity and challenges associated with planning for future flood events over a planning horizon. We compare application of the average return period with the more common concept of reliability and recommend replacing the average return period with reliability as a more practical way to communicate event likelihood in both stationary and nonstationary contexts.

1. Introduction

Traditional probabilistic approaches for defining risk, reliability, and return periods under stationary hydrologic conditions assume that extreme events arise from serially independent time series with a probability distribution whose moments and parameters are fixed. *Gumbel* [1941] and *Thomas* [1948] defined a return period of a flood as the interval between flood events, where an event is any streamflow discharge exceeding a known threshold. In some studies, the return period has been defined as the (conditional) interval between two flood events [*Lloyd*, 1970; *Haan*, 1977; *Mays*, 2005], whereas the more common definition of a return period in practice is the unconditional waiting time until an exceedance event [*Fuller*, 1914; *Gumbel*, 1941; *Fernández and Salas*, 1999]. The unconditional return period does not assume a flood has occurred in year one. Though the two definitions are equivalent for stationary conditions, the conditional return period is not sensitive to hydrologic persistence [*Lloyd*, 1970; *Douglas et al.*, 2002], an attractive feature in drought planning because droughts tend to exhibit persistence. The more commonly used unconditional definition of a return period is useful for describing the recurrence of hydrologic events because it does not depend on knowledge of a previous event, yet its value is sensitive to hydrologic persistence.

Consider the case of planning for a random future annual maximum extreme event X, where the design quantile X_p is the threshold of exceedance, and determines whether a flood event with exceedance probability p, occurs in a given year. Assume the hydrologic event X is defined as the annual maximum streamflow which has a stationary probability distribution function (pdf) denoted by $f_X(x)$ and cumulative distribution function (cdf) denoted by $F_X(x)$. In the case where a structure is built to protect against an event with an annual nonexceedance probability, $1-p = F_X(x)$, the design event for such a structure is computed as simply the inverse of the cdf and equal to the quantile X_p. Under stationary conditions, if we assume that the exceedance probability p, of annual floods is constant and that flood events are independent and identically distributed, then the return period, T, follows a geometric distribution with probability mass function (pmf) given by:

$$f(t) = P(T = t) = (1 - p)^{t-1} p \qquad (1)$$

where p is the annual exceedance probability. Similarly, for the continuous case, the random variable T follows an exponential pdf. In either case, the average return period is:

$$E[T] = T_o = \sum_{t=1}^{\infty} t(1 - p)^{t-1} p = \frac{1}{p} \qquad (2)$$

Similarly the variance of T is

$$Var[T] = E[T^2] - E[T]^2 = \sum_{t=1}^{\infty} t^2 \cdot P[T = t] - \frac{1}{p^2} = \frac{1-p}{p^2} \qquad (3)$$

In the design of hydrologic infrastructure, the probability of failure over its lifetime or its associated system reliability over a project lifetime is perhaps the most important piece of information an engineer can communicate to planners and the public. Prior to the 1983 Principles and Guidelines [*WRC*, 1983], standard practice in the United States for designing hydraulic infrastructure had been to select a design event, compute its average return period, and build the lowest cost structure. Such an approach does not consider the risk of failure over the planning horizon as a decision variable, and instead reported this risk of failure as more of a posterior calculation [*Yen*, 1970]. Since then, hydrologists and the Army Corps of Engineers have adopted a probabilistic approach, or risk-based design [*WRC*, 1983], where a level of infrastructure (i.e. protection) is selected based on minimizing the expected annual damage costs from a hazard, i.e. a flood [*U.S. Army Corps of Engineers*, 1996]. More recently, Risk-Based Decision Making (RBDM) has become a well-established methodology that determines appropriate levels of infrastructure based on the expected damages avoided vs. the cost of the infrastructure required [*Tung*, 2005; *National Research Council*, 2000]. RBDM can be used in place of the traditional design storm approach to first select a particular design event (a distinct T_o-year event usually specified by regulation), and then select the necessary infrastructure to protect against the flood event with that specified return period. *Rosner et al.* [2014] document how RBDM can be applied in a nonstationary setting.

We define risk of failure over a planning period ($Risk_n$) here as in most introductory hydrology textbooks [*Bras*, 1990; *Viessman and Lewis*, 2003; *Mays*, 2005] and hydrology handbooks [*Tung*, 1999; *Stedinger et al.*, 1993; *IACWD*, 1982; *Chow*, 1964], as the likelihood of experiencing at least one event exceeding the design event over a given project life of *n* years:

$$Risk_n = 1 - (1 - p)^n \quad (4)$$

Note that the risk of failure can be directly computed from the stationary average return period by replacing p=1/T_o from (2) into (4). *Yen* [1970] points out that when the project lifetime (*n* years) is equal to the average return period (T_o) in equation (4), the project risk approaches a value of 0.63 as the project life nears infinity. This result emphasizes an important link that exists between project life and its risk of failure under stationary conditions.

A more modern definition of risk used in environmental and water resource planning involves both the magnitude and frequency of the event [*Krimsky and Golding*, 1996], whereas the definition in (4) is only indicative of the probability of failure over an *n* year period. For this reason, we recommend no longer using the term risk when discussing the concept defined in equation (4). Instead, we recommend the term 'reliability' over a planning period ($Reliability_n$), which is defined as the probability that a system will remain in a satisfactory state [*Hashimoto et al*., 1982; *Salas and Obeysekera*, 2014] during its lifetime, i.e. that an exceedance event will not occur within a project life of *n* years:

$$\mathrm{Re}liability_n = (1 - p)^n \tag{5}$$

The concept of reliability is not new to hydrology and is widely used in water supply planning [*Hirsch*, 1979; *Vogel*, 1987; *Harberg*, 1997; *Loucks et al*., 2005] and many other engineering fields [*Kottegoda and Rosso*, 2008; *Tung et al*., 2006]. For stationary systems, the relationship between the reliability and average return periods for *n* = 25, 50, and 100 year planning horizons is illustrated in Figure 1-1 by simply substituting p=1/T_o into (5):

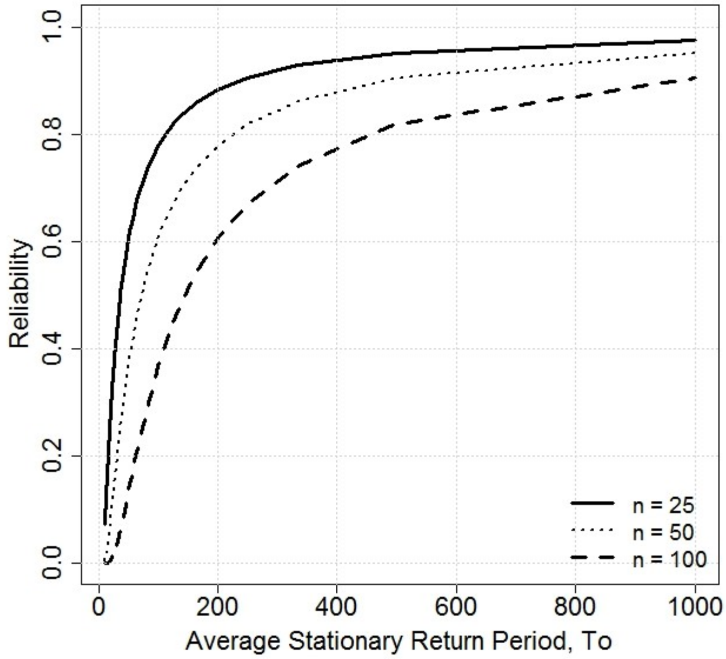


FIGURE 1-1. Reliability of a stationary system which is designed on the basis of an average return period T_o, corresponding to n = 25, 50, and 100 years

Figure 1-1 illustrates that to achieve a reliability commensurate with other areas of design (i.e. Reliability > 0.9), over typical project lifetimes, the average return period of the design event must be in the hundreds of years. Most importantly, Figure 1-1 illustrates that knowledge of the average return period alone gives little guidance regarding the likelihood that a given project will perform as expected. Under nonstationary conditions, the exceedance probability associated with the design event is likely to change over time, creating additional challenges in selecting a suitable design event. Furthermore, a prerequisite to the use of RBDM under nonstationary conditions is that we develop a complete understanding of the impact of nonstationarity on traditional design metrics such as the expected return period as well as the reliability of a system over its planning horizon.

Overall, our goal is to initiate a discussion as to how engineers can effectively communicate the risk of failure and reliability of hydrologic design over planning horizons for a range of possible conditions. This goal is in line with a number of recent papers in the hydrologic science literature which discuss the existence and mortality/immortality of stationarity as it relates to hydrologic design and extreme events [*Cohn and Lins,* 2005*; Milly et al.,* 2008*; Montanari and Koutsoyiannis,* 2014*; Koutsoyiannis and Montanari,* 2014; *Serinaldi,* 2015*; Serinaldi and Kilsby,* 2015*; Condon, et al.,* 2015]. In the following sections we provide detailed reasoning for the need to replace the use of the average return period with the concept of reliability over a planning horizon as a metric for design and more efficient means for communicating risk of failure. We show this to be the case under stationary conditions, (Figure 1-1), and even more dramatically under nonstationary conditions. In addition, we document the general impact of nonstationarity on statements of risk of failure, reliability, and the average return period using a simple, realistic and representative two-parameter nonstationary lognormal (LN2) flood model. We begin by reviewing the past and current approaches to hydrologic design under stationary and nonstationary conditions. Then we introduce the nonstationary LN2 flood model and apply it to develop general relationships among risk of failure, reliability, and average return periods introduced by previous investigators, and demonstrate its use for hydrologists and planners. This investigation is presented in the context of the growing societal interest in the impact of future nonstationarities and the need for guidance in selecting a representative design event over a future planning horizon. Finally, we conclude with recommendations concerning suitable statements of risk of failure, reliability, and average return periods in both stationary and nonstationary settings.

2. Hydrologic design under stationary conditions

In the water resources literature the concept of an average return period has many applications, with each definition based on the type of hydrologic event described [see *Fernandez and Salas*, 1999 and references therein]. For example several investigators have considered the behavior of average return periods for hydrologic processes that exhibit persistence (long-range dependence) such as droughts and water supply failures [*Lloyd*, 1970; *Vogel*, 1987; *Fernandez and Salas*, 1999; *Douglas et al.*, 2002], and taken steps to create stochastic models that can reproduce such persistence [*Efstratiadis et al.*, 2014]. We focus our attention on flood events which do not tend to exhibit significant interannual persistence as do droughts and water supply failure.

The risk of failure defined by *Yen* [1970] and our preferred definition of reliability are essential metrics for communicating the likelihood associated with a system failure during a project lifetime. Since society and planners are concerned with knowing whether a system will remain undamaged within a given design period, reliability is a more effective tool than the average return period for directly communicating the likelihood that a flood exceeding the design event will occur over a planning horizon. *Serinaldi* [2015] and *Serinaldi and Kilsby* [2015] relay a similar message, pointing out that risk of failure better describes this likelihood over a planning horizon because it summarizes the joint probability instead of the average probability, and does not require computations involving sums to infinity (i.e. to times beyond the design life).

Figure 1 shows that systems designed on the basis of typical average return periods are not nearly as reliable as one might anticipate over a typical project life. According to Figure 1-1, system reliability is only 78% when the design event is based on the 1% exceedance event (100-year flood) for n = 25 years. Considering that the design life of some public structures can be much longer than 25 years, and that reliability decreases as project life increases for a given average return period, careful attention should be given to how the system reliability of such structures is impacted by the planning horizon for structures which have been designed on the basis of an average return period.

Other fields concerned with risk and hazard planning ensure a much higher reliability over typical planning horizons than corresponding reliabilities associated with the 100-year flood so commonly used in hydrologic planning. Table 1-1 provides several examples, highlighting the differences between average return periods and reliabilities associated with various disciplines concerned with hazard planning. For example earthquake design regulations, suggest protection against a "less than 2% chance of failure (collapse) occur[ring] in a 50-year project life" [*NEHRP*, 2010]. This level of protection corresponds to an earthquake magnitude with an average return period of 2,475 years [obtained by combining equations (2) and (5)]. By comparison, traditional flood frequency analysis which often bases designs on an average return period of T_o = 100 years corresponds to reliabilities of 78%, 61%, and 37% over a range of *n* = 25, 50, and 100 years, respectively.

TABLE 1-1. Reliabilities and average return periods associated with important events associated with several disciplines which seek to protect against extreme events

Discipline	**Reliability for typical *n*-year planning period**	**Average Return Period (Years)**	**Citation**
Earthquake (shaking)	98%, *n* = 50	2475	*NEHRP* [2010]
Retirement portfolio planning	95%, *n* = 30	585	*Ameriks et al*. [2001]; *Stout & Mitchell* [2006]
Nuclear power plant accident	95%, *n* = 21	409	*U.S. NRC* [1975]
National Flood Insurance	75%, *n* = 30	105	*FEMA*
Flood design (infrastructure)	61%, *n* = 50	100	*Yen* [1970]

In communicating flood risk, the “100-year flood” has a long history as a regulatory concept endorsed in the 1970s by the National Flood Insurance Program (NFIP) and FEMA in an effort to standardize flood risk. See *Pielke* [1999] for a complete discussion of the many misconceptions regarding the “100-year flood” and its influence on flood risk perception. Today the NFIP communicates flood risk to the public on their website (available at https://www.floodsmart.gov/floodsmart/pages/flooding_flood_risks/defining_flood_risks.jsp) by relating the likelihood of flood damage over a typical 30-year mortgage, and categorizing locations as “high-risk” (residences within the 100-

year floodplain) and “moderate-to-low” risk (outside the 100-year floodplain). Interestingly the “moderate-to-low risk” areas, which are not required to purchase flood insurance, receive over 1/3 the payouts of disaster flood assistance or 20% of claims, begging the question of how the category “low risk” was defined. As *Salas* [2013] notes, the omission of uncertainty from estimating the floodplain is in itself a compounding issue.

2.1 Should we consider replacing the average return period with concept of reliability?

We are not the first researchers to question the use of the average return period as a design metric for hydrologic purposes and for communication of extreme events [*Fernandez and Salas*, 1999*; Pielke,* 1999*; Douglas et al.*, 2002*; Cooley*, 2013*; Serinaldi*, 2015*; Serinaldi and Kilsby*, 2015]. A summary of others’ points and our own thoughts suggest that even under stationary conditions, using the average return period as a design metric for hydrologic infrastructure and as a conceptual tool for communicating risk of failure is problematic because: (1) the shape of the geometric (discrete) and exponential (continuous) distribution associated with the time to failure under stationary conditions have a very long right tail, thus the mean time to failure (average return period) is a poor representation of the most likely time to failure; (2) the average return period is not explicitly tied to a planning horizon and thus is unable to characterize the likelihood of an event occurring during a project lifetime. When hydrologic processes exhibit nonstationarity, the probability of exceedance associated with the design event is changing over time, and thus the traditional formulae in equations (1)-(4) no longer hold. In this situation the average return period may become even less representative of the future system reliability than under stationary conditions as is shown in the following sections.

3. Hydrologic design under nonstationary conditions

This work is in part motivated by the studies of *Olsen* [1998], *Cooley* [2009], *Parey et al.* [2007, 2010], and *Salas and Obeysekera* [2014], who introduced much of the mathematics needed to describe the risk of failure, reliability, and average return periods in a nonstationary context. Interestingly, most of the key developments extending hydrologic design indices to nonstationary conditions have appeared primarily in the statistics and climate change literature, likely as a result of the attention and resources devoted to understanding and characterizing climate change [*Wigley*, 1988; *Katz*, 1992, 2010; *Olsen et al.,* 1998; *Parey et al.*, 2007, 2010; *Cooley*, 2009, 2013].

When a trend exists in annual maximum streamflow, the expressions for Risk_n, Reliability_n, and average return period T_o presented in Equations (1)-(4) are no longer correct because the probability of experiencing a flood which exceeds a fixed design threshold is increasing (positive trend)/decreasing (negative trend). Under stationary conditions the return period follows a homogeneous geometric distribution as described in Equations (1-2), whereas, under nonstationary conditions the exceedance probability p_t associated with a particular annual maximum flood discharge changes every year. Under nonstationary conditions, the average return period is no longer a sufficient statistic of the distribution of return periods. For example, p, or the average return period $1/p$, are both sufficient statistics for the geometric distribution in (1) because each one is all that is needed (sufficient) to describe the pdf of return periods in a stationary context. However, under nonstationary conditions, knowledge of the average return period is no longer the only piece of information needed to specify the complete distribution of the return period [*Rootzén and Katz*, 2013]. *Olsen et al.* [1998] and *Salas and Obeysekera* [2014] introduce expressions for the average return period for the case where the exceedance probabilities of extreme events are increasing, such that *p* continuously increases until reaching unity at some future time, $t = t_{max}$. From *Cooley* [2013] and others, the pmf of a return period or waiting time distribution, for a nonstationary process, is given by

$$f(t) = p_t \prod_{i=1}^{t-1} (1 - p_i) \qquad t = 1, 2, \ldots t_{max} \quad (6)$$

where *t* is the time until the first flood that exceeds the design event. An expression for the probability of failure in year *t*, p_t, can be derived for any distribution depending on the random variable of interest *x*. Note that as expected for a stationary hydrologic process, equation (6) reduces to equation (1) because p_t is constant in every year. *Cooley* [2013] and *Salas and Obeysekera* [2014] show that the expected value of the return period, T_1 = E[*T*] under nonstationary conditions is given by:

$$T_1 = E[T] = \sum_{t=1}^{t_{\max}} t f(t) = \sum_{t=1}^{t_{\max}} t p_t \prod_{i=1}^{t-1} (1 - p_i) \qquad (7)$$

Here we denote the average return period under nonstationary conditions using the notation T_1 to distinguish it from the average return period T_o under stationary conditions given in equation (2). Equation 7 is a general form to compute T_1; in cases with increasing exceedance probabilities, one hopes that the maximum time (t_{max}) is very far into the future, as it corresponds to experiencing annual floods in excess of some important design threshold, with certainty (i.e. exceedance probability of unity). In the case of decreasing exceedance probabilities (downward trends), t_{max} and the expected return period itself may be both infinite. These are additional challenges for the practical application of applying a nonstationary return period for future planning purposes. An equivalent numerically simplified formula from *Cooley* [2013] is:

$$T_1 = E[T] = 1 + \sum_{t=1}^{t_{\max}} \prod_{i=1}^{t} (1 - p_i) \quad (8)$$

Parey et al. [2007, 2010] define a non-stationary return period as the number of years, T_2, for which the expected number of exceedance events is equal to one, which can be solved numerically by computing the upper limit of the summation in:

$$1 = \sum_{t=1}^{T_2} (1 - p_t) \quad (9)$$

Cooley [2013] shows why (9) can be interpreted as the number of years in which the number of expected exceedances is equal to unity. It is interesting to note that under stationary conditions all three measures of the expected time to failure are equal so that, $T_o = T_1 = T_2$; however, this is not the case under nonstationary conditions. Equations (6-9) can be applied to any variable or process which leads to a system failure defined as the occurrence of an annual maximum streamflow (flood) above some level. All that is needed is the nonstationary cdf of the variable of interest to compute its average return period using (6)-(9). In the sections which follow we provide examples of the application of (6)-(9) for the nonstationary lognormal model summarized by *Vogel et al.* [2011], *Prosdocimi et al.* [2014] and others. We begin by using this model because it offers the possibility to generalize the probabilistic behavior of floods under nonstationary conditions, and because it is a parsimonious nonstationary model that reproduces the behavior of floods for a large percentage of U.S. and U.K. river systems as shown by *Vogel et al.*, [2011] and *Prosdocimi et al.* [2014], respectively.

From *Salas and Obeysekera* [2014], the system reliability over a planning period (n) under nonstationary conditions is given by

$$\mathrm{Re}liability_n = \prod_{i=1}^{n} (1 - p_i) \tag{10}$$

Another metric defined by *Stedinger and Crainiceanu* [2000] is the average annual risk of failure, which is simply the average of the annual exceedance probabilities over a planning period (n years):

$$AAR(n) = \frac{1}{n}\sum_{i=1}^{n} p_i \tag{11}$$

Stedinger and Crainiceanu [2000] use (11) along with a damage function and discount rate, to compute the discounted equivalent risks for four flood forecast models. Following *Stedinger and Crainiceanu* [2000], we elect to introduce another measure of reliability which we term the annual average reliability:

$$\text{Re}\,liability_a = \frac{1}{n}\sum_{i=1}^{n}\left(1 - p_i\right) \qquad (12)$$

The above review of average return periods, risk of failure, reliability over a planning horizon and average reliability over a planning horizon under both stationary and nonstationary conditions led us to question which among the various indices would be most useful for communicating risk of failure under both stationary and nonstationary conditions. The following sections present our investigation of the average return period and reliability under nonstationarity in the context of design, planning, and communicating risk of floods.

4. Generalized Probabilistic Behavior of LN2 Model of Flood Hazard

4.1 LN2 Stationary Flood Hazard Model

Vogel et al. [1996] reviewed the pdfs which were commonly used in flood frequency analysis prior to that date. While the generalized extreme value (GEV), log Pearson type III (LP3) and three-parameter lognormal (LN3) are perhaps the most commonly used pdfs used for flood frequency analysis, the two-parameter lognormal (LN2) model was found by *Beard* [1974] and others to provide an excellent parsimonious alternative to those models and has since been commonly applied in flood studies [*Stedinger and Crainiceanu*, 2000; *Lund*, 2002]. Note also that an LN2 model is a special case of the log Pearson type III distribution, the mandated distribution for use in U.S. federal flood studies by Bulletin 17B [*IAWCD*, 1982]. *Vogel et al.* [2011] and *Prosdocimi et al.* [2014] document that a simple nonstationary LN2 model can provide an approximate yet general representation of the behavior of floods for rivers across the United States and the

United Kingdom, respectively, thus we employ that model here. Since our goal is to explore the behavior of flood risk under nonstationary conditions we begin with a two-parameter pdf, which enables more general comparisons. Future studies should consider extending our results to nonstationary models corresponding to other two parameter pdfs such as the Gumbel distribution as well as some of the more common three parameter models such as the GEV, LN3 and LP3 distributions.

Consider a stationary series of annual minima or maxima, X, which follows a two-parameter lognormal (LN2) distribution where $Y = \ln(X)$, and the mean and standard deviation of y are given by:

$$\mu_y = \ln\left(\frac{\mu_X}{\sqrt{1 + C_X{}^2}}\right) \tag{13a}$$

$$\sigma_y = \sqrt{\ln(1 + C_X^2)} \tag{13b}$$

where μ_x is the mean of X, C_x is the coefficient of variation in real space $C_X = \sigma_X / \mu_X$, and σ_X is the standard deviation of X. The design event corresponding to an LN2 pdf is given by the quantile function

$$X_p = \exp(\mu_y + Z_p \sigma_y) \tag{14}$$

where Z_p is a standard normal variable with a corresponding exceedance probability p.

Under stationary conditions, the quantile function X_p can be evaluated for the event which has an average return period T_o, by computing the fixed exceedance probability using (2) to obtain $p_o = 1/T_o$. Under stationary conditions, all moments of both *X* and *Y* are assumed to be fixed over the planning horizon *n*.

4.2 Nonstationary LN2 Flood Model

We follow *Vogel et al.* [2011] and *Prosdocimi et al.* [2014] who show that a simple exponential model of *X* versus time t captures the behavior of annual maximum floods at rivers in the United States and the United Kingdom, respectively. Such a model is easily fit using ordinary least squares (OLS) regression to fit the simple log linear trend model to describe the relationship between *y* over time *t*

$$y_t = \ln(x_t) = \alpha + \beta t + \varepsilon_t \tag{15}$$

Note that we do not advocate the use of a trend model using time as a covariate, this model is only used here for illustrative purposes. We do advocate use of meaningful covariates to reflect future changes in the mean annual flood levels, such as covariates which reflect changes due to urbanization and/or climate. Regardless of whether or not a significant trend was detected in the observed flood series at thousands of rivers across the conterminous U.S., *Vogel et al.* [2011] found that the residuals in (15) were homoscedastic and well approximated by a normal distribution, both important assumptions needed to perform further statistical inference. Although their evaluations included detailed hypothesis tests concerning the normality of regression model residuals and t-tests on model slope coefficients as well as graphical evaluations of homoscedasticity, their evaluations did not include a comprehensive assessment of the stochastic independence of the model residuals or the flow series. *Vogel et al.* [2011] (see their appendix) concentrated their analysis on the approximately 11% of rivers in the study which exhibited (obvious) positive trends. When applying ordinary least squares (OLS) linear regression, the resulting fitted model yields the conditional mean of the dependent variable so that the expectation of (15) yields an estimate of the conditional mean of *Y*, denoted here as $\mu_{y|t} = \alpha + \beta t$, because the residual term is assumed to have zero mean. Importantly, this nonstationary trend model implies a reduction in $\sigma_y^{\,2}$ as compared with stationary conditions, with that reduction proportional to the degree of the trend. As shown in Appendix A-1, the coefficient of variation of the nonstationary flood series is:

$$C_{x|t} = \sqrt{(C_x{}^2 + 1)^{(1-\rho^2)} - 1} \qquad (16)$$

Note the two extreme cases of no trend in which case (16) reduces to $C_{x|t} = C_x$ and a perfect trend model with ρ=1, which leads to $C_{x|t} = 0$. Figure A1 (in Appendix A-1) illustrates the relationship in (16) and documents the small reduction in the coefficient of variation of the flood series *X* under typical nonstationary conditions. Since values of ρ are likely to be quite small in actual situations (i.e. ρ<0.5), we elected to assume $C_{x|t} = C_x$ because we found this effect to be of minor importance to our overall findings.

Now if a hydrologist has a historic streamflow record of length *N* represented by x_i for $i = t_1, \ldots, t_N$ years. The trend model $\mu_{y|t} = \alpha + \beta t$ becomes

$$\mu_{y|t} = \bar{y} + \beta\left(t - \left(\frac{t_1 + t_N}{2}\right)\right) \qquad (17)$$

where β is the slope of the trend that extends from t_1 to t_N, , $\bar{y}$ is simply the mean $\bar{y} = \frac{1}{N}\sum_{t=t_1}^{N} \ln(x_t)$, and the annual maximum floods x_t are measured for N years starting at t_1.

The trend model in (17) can be used to calculate the conditional mean of Y_t=ln(X_t) for any year in the future, $t > t_N$. We do not advocate the use of (17) for trend extrapolation, unless the user considers the use of physically meaningful covariates in the regression model and/or includes prediction intervals associated with such extrapolations which are known to widen considerably when a model is used in 'extrapolation mode.' [See *Serinaldi and Kilsby* [2015] for examples and a discussion on the possible implications of such extrapolations for a different model.] The use of OLS regression is quite powerful for trend extrapolation, because when the residuals are approximately independent, homoscedastic, and normally distributed, analytical expressions are readily available for computing both prediction intervals and the likelihood of type I and II errors. Such type I and II probabilities may be readily integrated into a risk-based decision framework [*Rosner et al.* 2014] as probabilities of over- and under-design, respectively, and have been shown to be particularly important for understanding hydroclimatic change [see *Vogel et al.*, 2013; and *Prosdocimi et al.* 2014].

Since exceedance and nonexceedance values are no longer fixed under nonstationary conditions, the quantile function under nonstationary conditions is obtained by substitution of the nonstationary mean $\mu_{y|t}$, and standard deviation

$\sigma_{y|t}$ in (17) and (A2), respectively, into the nonstationary quantile function:

$X_{p|t} = \exp(\mu_{y|t} + Z_p \sigma_{y|t})$ which results in:

$$X_{p|t} = \exp\left[\bar{y} + \beta(t - \bar{t}) + Z_{p_t} \sigma_y \sqrt{1 - \rho^2}\right] \quad (18)$$

See Appendix A-1 for a derivation of the reduction in the variance of *Y* which is implied by the nonstationary LN2 trend model.

To provide a more physically intuitive understanding of the impact of trends on flood quantiles under nonstationary conditions, we employ the idea of magnification factor introduced by *Vogel et al.* [2011] and also tested by *Prosdocimi et al.* [2014]. *Vogel et al.* [2011] define the magnification factor *M* as the ratio of the *T*-year flood at some future $t+\Delta t$ period to the T_o-year flood at time *t*. They further show that

$$M = \frac{x_p(t + \Delta t)}{x_p(t)} = \exp[\beta \Delta t] \quad (19)$$

for the nonstationary LN2 model given here.

For a stationary LN2 variable the *fixed* exceedance probability $p = P(X \le x)$ associated with a design event X_p, is given as

$$p = 1 - \Phi\left[\frac{\ln(X_p) - \mu_y}{\sigma_y}\right] \quad (20)$$

where the function Φ is the cdf for a standard normal variable. This is easily adapted under nonstationary conditions, to compute the changing values of p_t, the annual exceedance probability associated with experiencing an event greater than the design event $X*$:

$$p_t = 1 - \Phi\left[\frac{\ln(X^*) - \mu_{y|t}}{\sigma_{y|t}}\right] \tag{21}$$

Here we denote the design event as $X*$ to emphasize that it is a fixed value which must be chosen by the design engineer under nonstationary conditions. Under stationary conditions, the design event is denoted as X_p and is given by (14), so that only under stationary conditions is $X_p = X*$. Note that for the stationary case (21) reduces to (20) and either can be used to compute both the average return period (Eqn. 2) and reliability (Eqn. 5) under stationary conditions. Under nonstationary conditions (21) can be inserted into (7)-(9), and (10) to compute nonstationary average return periods and reliabilities corresponding to a particular design event x_p. Again, the nonstationary mean $\mu_{y|t}$, and standard deviation $\sigma_{y|t}$ in (21) are given in (17) and (A3), respectively, We emphasize that under stationary or nonstationary conditions, the choice of a design event is fixed, denoted here by $X*$. Selection and computation of $X*$ is quite straightforward under stationary conditions, but under nonstationary conditions, with values of p_t changing in every year, its definition and computation is much more complex, as is discussed later in this paper.

5. Results

In the following sections we explore the general behavior of the nonstationary LN2 model with the goal of improving our understanding of the likelihood of future floods (increasing exceedance probabilities and upward trends) associated with a particular design event X_*, under nonstationary conditions. To enable general comparisons, analyses, and conclusions, we make a number of simplifying assumptions including the following: (1) the stationary coefficient of variation C_x is defined as that value at the end of the period of streamflow record, ($t = t_N$), (2) C_x is assumed to be fixed throughout each planning horizon and equal to the nonstationary coefficient of variation, so that $C_{x|t} = C_x$, and (3) the trend in the mean of the annual maximum streamflow series is increasing over time. Now the behavior of floods under nonstationary conditions can be generalized using the nonstationary LN2 model in (18) and (21) along with the magnification factor in (19) and the general relationships between the moments in real and log space given in equation (13).

5.1 *Investigation of Return Period Distribution under Nonstationarity*

If the field of flood planning and management is to continue to use the average return period as a tool for communicating risk and informing infrastructure planning and design, it is important to understand the behavior of the probability distribution of the return period under nonstationary conditions. In this section we examine the behavior of the pdf of the return period under nonstationary conditions for increasing flood magnitudes. We assume a decadal magnification factor ($\Delta t = 10$), where $M = \exp(10\beta)$, and thus a value of $M = 1.02$

implies a 2% increase in flow magnitude every 10 years for all design events, regardless of their probability of exceedance.

We begin by exploring how nonstationarity affects the return period or the waiting time until experiencing an event which exceeds the design event. The pdf of the return period under nonstationary conditions is given in (6) with exceedance probability p_t computed from (21). We assume that a design engineer has chosen a design protection level based on the stationary LN2 flood frequency model using the quantile function in (14) with $p = 0.01$, corresponding to a traditional '100-year' flood.

Of interest here is how a trend in the annual maximum floods impacts the pdf of the return period associated with the traditional 100-year design event. Figure 1-2 illustrates the pdf of the return period associated with a 100-year ($p_o = 0.01$) design event for the case when $C_x = 1$ for several levels of nonstationarity, as described by increasing values of the magnification factor M. Figure 1-2 illustrates that what was once an exponential distribution for the return period associated with the 100-year flood under stationary conditions becomes a very different probability distribution as the degree of nonstationarity increases.

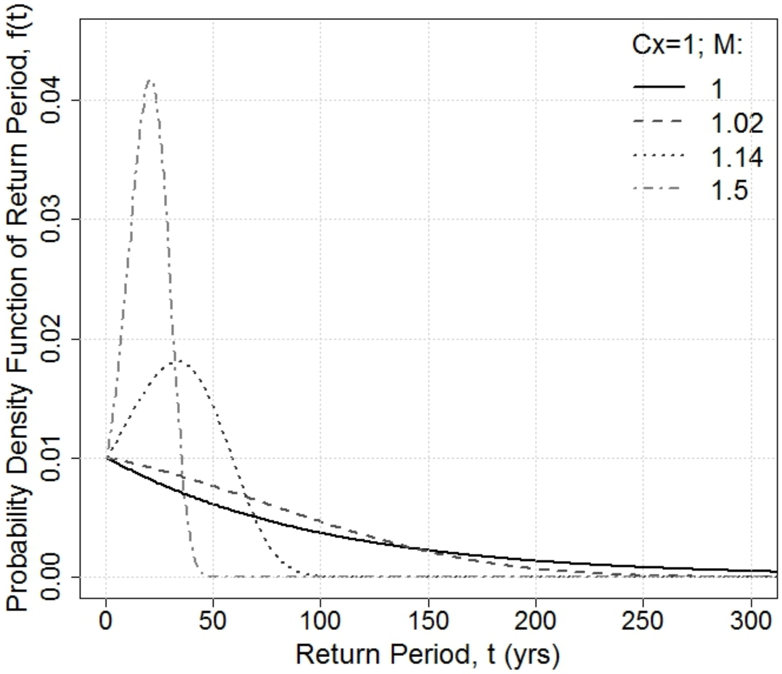


FIGURE 1-2. Probability distribution function (pdf) of the return period associated with a traditional 100-year flood for C_x = 1 and a range of increasing trends (*M* = 1, 1.02, 1.14, 1.5). Note *M*=1 corresponds to stationary conditions

Several conclusions can be drawn from Figure 2 concerning our experience of floods which exceed the traditional 100-year flood, under nonstationary conditions. Our experience of the likelihood of the return period until a flood exceeds the design event changes dramatically both in terms of the shape of the pdf of the return period and its expectation. Importantly, the distribution of the return period is no longer exponential as is the case under stationary conditions (*M* =1). One implication is that if a structure is built for today's p_o = 0.01 event and the future is not known with certainty (always the case), we do not know how the return period distribution will evolve, i.e. the shape it will take. We also note from Figure 1-2 that regardless of the magnitude of a future trend, one can expect the p_o = 0.01 event to occur much sooner than 100 years as the magnitude of the trend increases, as evidenced by *M*.

Of interest is the impact of nonstationarity on our experience of return period associated with design floods of various magnitudes. Figure 1-3 illustrates the distribution of the return period associated with the traditional T_o = 10, 100, and 1000 year design events (p = 0.1, 0.01, and 0.001) under nonstationarity conditions described by M=1.14 and C_x=1. Under these nonstationary conditions the average return period of the 100 year event is shifted to ~32 years, and remarkably, the average return period of the 1,000 year flood is reduced to approximately 66 years. Interestingly, the expected return period associated with the 10-year flood is essentially unchanged (~ 9 years), suggesting that rarer events may be more impacted by nonstationarity than more common floods. We conclude that the evolution of the return period distribution for different levels of nonstationarity (Figure 1-2) and from common to rare events (Figure 1-3) exhibits highly non-linear behavior and is likely to be impractically complicated for purposes of design, planning, management, and risk communication. These findings are consistent with those of *Serinaldi and Kilsby* [2015].

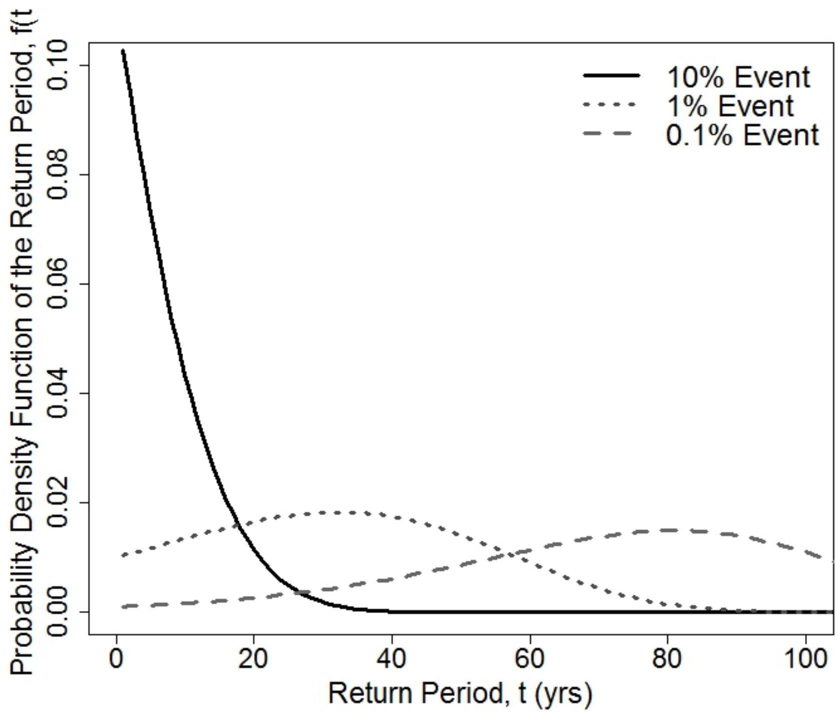


FIGURE 1-3. The probability distribution function of the return period for p_o = 0.1, 0.01, 0.001 events; C_x = 1, M = 1.14

5.2 The Behavior and Choice of the Design Flood under Nonstationary Conditions

Figures 1-1 to 1-3 assume that the design flood is chosen under the assumption of stationary conditions. Under nonstationary conditions, the design flood should be chosen in such a way as to account for the likelihood of future floods. For example, if we are to estimate a design flood under nonstationary conditions, we must employ equation (7) or (8) to describe the average return period T_1, along with an appropriate nonstationary flood frequency model. To determine the design event X^* which will ensure a value of average return period

T_1 under nonstationary conditions defined by M and C_x, one can combine equations (8) and (21) and solve numerically for X^* in the resulting expression:

$$T_1 = E[T] = 1 + \sum_{t=1}^{t_{\max}} \prod_{i=1}^{t} \left(\Phi \left[\frac{\ln(X^*) - \mu_{y|i}}{\sigma_{y|i}} \right] \right) \qquad (22)$$

Equation (22) leads to an estimate of the design event under nonstationary conditions which will have an average return period equal to T_1; this is a useful equation for design engineers who need to size infrastructure based on the "new" 100-year flood under nonstationary conditions. Clearly, solving for the design event under nonstationary conditions using (22) is far more complex than under stationary conditions, and it should be noted that in the case of decreasing trends, if t_{max} is infinite, a numerical solution is not even possible. This is clearly problematic for practical use of such a metric for planning. Figure 1-4 illustrates the behavior of the probability distribution of the return period associated with such a design event chosen so that average return period T_1=100 years in all cases, regardless of the increasing degree of nonstationarity as described by increasing values of M. Figure 4 illustrates how the probability distribution of the return period associated with a design event having average return period of T_1=100 years changes shape from exponential in the stationary case (M =1) to a more normal or symmetrically shaped (M = 1.5) distribution. Interestingly, we find that in the presence of an increasing trend, the mean return period is actually more representative of the waiting time distribution, and may be more representative of when a failure will occur than under stationary conditions.

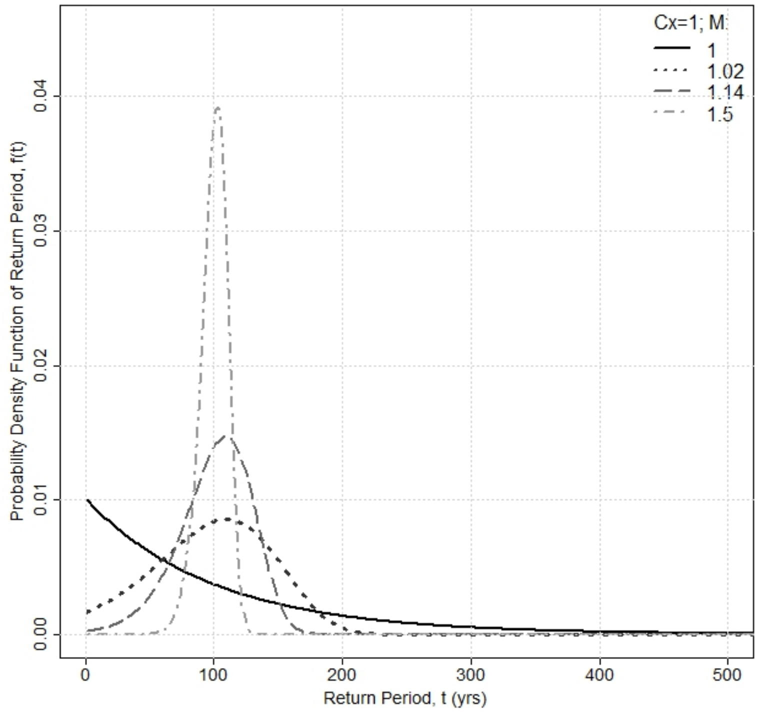


FIGURE 1-4. The pdf of the return period associated with a design flood, chosen in such a way as to ensure that the average return period is always 100 years, regardless of the degree of nonstationarity, considering a range of trends (M=1 to M=1.5) given that C_x = 1.

For comparison purposes, we examine whether this same behavior in the return period distribution is observed when flood flows arise from a nonstationary Gumbel distribution. Using the same log-linear trend model as described in the previous section, we derive p_t for the Gumbel distribution allowing the location parameter to vary with time with fixed C_x (as was described with the LN2 model). As in Figure 1-4, Figure 1-5 fixes the average return period at 100 years to illustrate the impact on the return period distribution due to a 5% increase in magnitude of floods over 10 years (M = 1.05), C_x = 0.25, for both Gumbel and LN2 flood flows. This investigation reveals that regardless of whether the flood flows are LN2 or Gumbel, there is a similar and striking evolution from an exponential to a more symmetric distribution when an exponential trend in the annual maximum flood series is present. Further, Figure 1-5 indicates that the expected waiting time distribution may differ significantly depending on which probability distribution is selected to represent flood flows, yet another element of complexity added under nonstationarity.

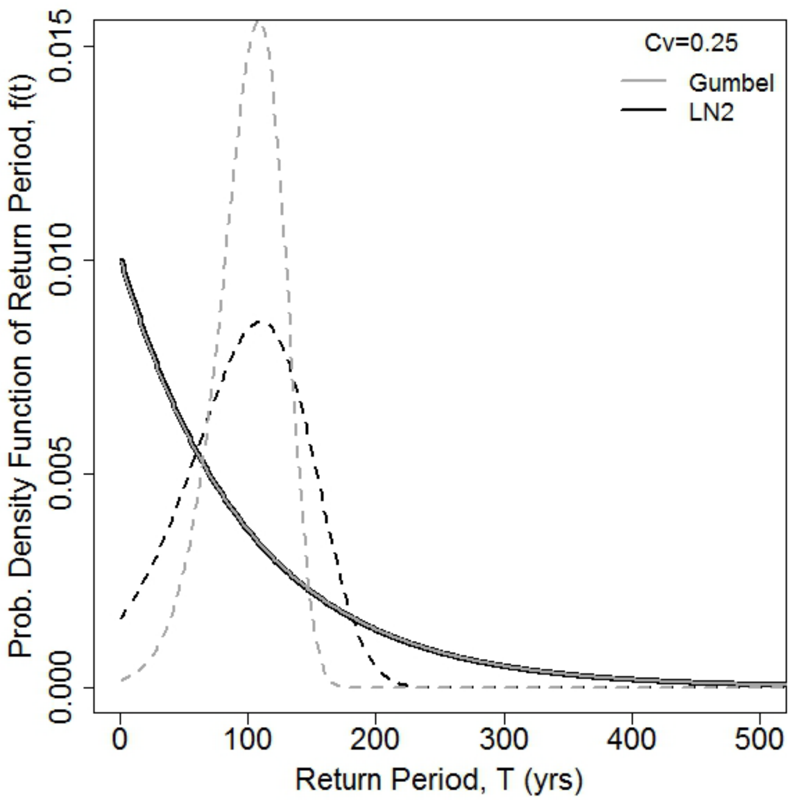


FIGURE 1-5. Comparison of the pdf of the return period associated with a design flood, which has a recurrence interval of 100 years, for $M = 1$ stationary (solid lines) and $M = 1.05$ (dashed lines), $C_x = 0.25$ assuming Gumbel flows (grey) and LN2 (black)

We are also interested in how physical hydrology of the river system affects the probability distribution of the return period under nonstationary conditions. Here we use C_x as a proxy for hydrologic variability, where $C_x < 1$ represents a system with relatively low variability as compared with $C_x > 1$. Since rivers with $C_x < 1$ are dominated by changes in the mean, they are most influenced by increasing trends as shown in Figure 1-6 by the evolution from C_x = 0.25 to C_x = 1.5 in each panel figure, moving from upper left (no trend M = 1) to bottom right (large trend M = 1.5).

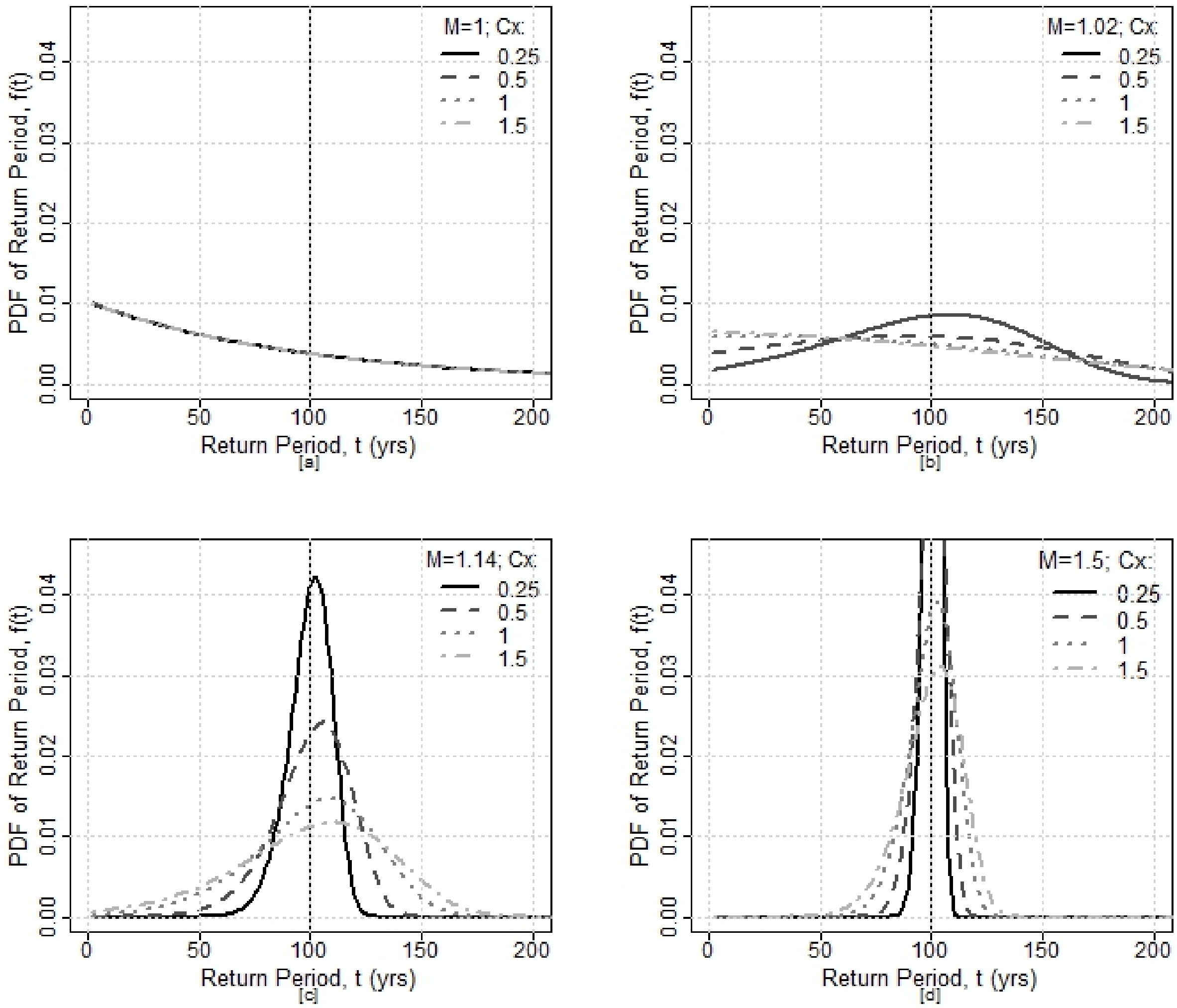


FIGURE 1-6. The pdf of the return period associated with a design flood which is chosen to ensure that the average return period is always 100 years, regardless of the degree of nonstationarity or coefficient of variation. Curves show a range of C_x values (0.25 – 1.5); panel shows trends increasing from M = 1 (top left) to M = 1.02, 1.14, 1.5 (bottom right)

Figures 1-4 and 1-5 illustrate that the mean return period is *more* representative of the time to the event under nonstationary conditions than under stationary conditions. Figure 1-6 illustrates that the shape of the pdf of the return period also depends on the hydrologic variability as described by the value of C_x. We conclude from all of these investigations, that the pdf of the return period exhibits extremely complex behavior under nonstationary conditions, with its shape depending on both hydrologic variability, the level of nonstationarity, and the magnitude of the design event of interest. Further examination of other commonly used pdfs such as the LP3 and GEV models would likely reveal the same patterns as shown here with LN2 and Gumbel.

5.3 *Comparisons of Summary Measures of Average Return Period under Nonstationarity*

Recall that there are two different summary measures of the average return period which have been advanced in the literature. The traditional definition is simply the average of the distribution of the return period T_1 given in (7) and (8). In addition, *Parey et al.* [2007, 2010] and *Cooley* [2013] introduced the return period as the number of years, T_2, for which the expected number of exceedance events is equal to one, which can be solved numerically using eqn. (9). In this section we compare the behavior of these two different summary measures of the time to failure, keeping in mind that under stationary conditions they are equivalent. To accomplish this comparison, we use (14) to compute the design discharge X_p corresponding to stationary conditions for p values corresponding to T_o = 10, 100 and 1000 year events. Now each of those X_p values are assumed to be the fixed design event X^* in eqn (21), which is then used for a particular C_x and M to determine the corresponding set of p_t values for each design discharge. These values of p_t are in turn used to compute average return periods T_1 using (8) and the return level T_2 from (9). Figure 1-7 compares estimates of T_1 (black lines) and T_2 (grey lines) to the values of T_o corresponding to stationary conditions and shows how the average return periods reduce as M increases from stationary (M = 1) to an extreme case (M = 3) for C_x = 0.5 (left) and C_x = 2 (right). Each plot in Figure 1-7 illustrates average return periods under stationary conditions: T_o = 10-year (solid), 100-year (dot-dash), and 1000-year (dashed) events in the figure legend along with the average return periods T_1 and T_2. The average return periods associated with the T_o year event undergo a dramatic reduction even with a small trend; for example the original T_o = 100-year event, when C_x= 0.5 (left panel) and M = 1.1, becomes a T_1 = 30 year event. From this comparison we also note that though T_1 and T_2 differ in their assumptions, most significantly that the shift from $T_o \rightarrow T_2$ is more pronounced than $T_o \rightarrow T_1$, the two measures behave similarly for the LN2 nonstationary model over a range of magnification factors and design events characterized by T_o. Thus our remaining results only concentrate on use of the more common metric, the average return period T_1.

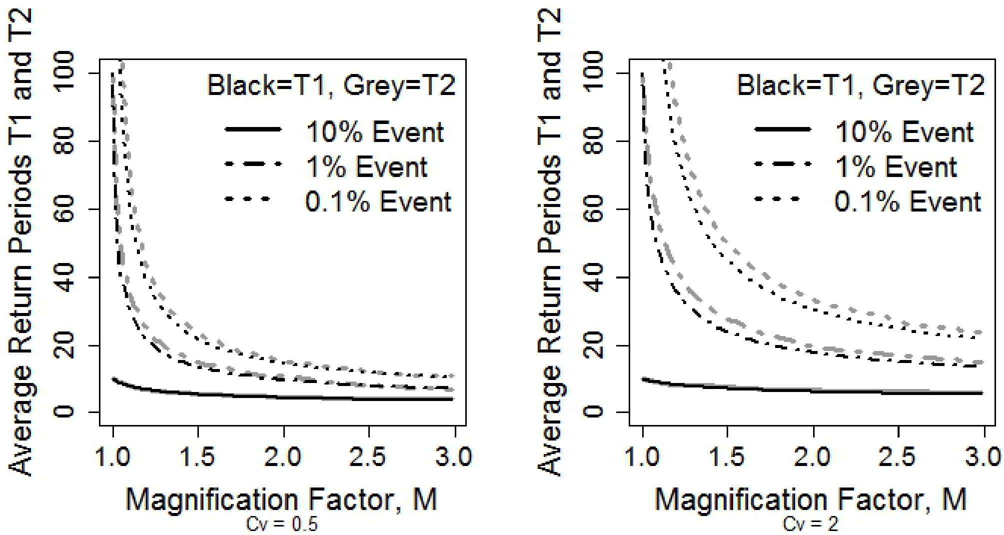


FIGURE 1-7. The average return periods T_1 (black) and T_2 (grey) vs. the decadal flood magnification factor M, for C_x = 0.5 (left) and C_x = 2.0 (right); lines are shown for the T_o = 10- (solid), 100- (dot-dash), and 1000- year (dashed) design events selected today, t_n

5.4 The Relationship between Reliability and Return Periods under Nonstationary Conditions

Recall that under stationary conditions, there is a unique relationship between reliability, average return period and planning horizon as was illustrated in Figure 1-1. Of importance is that the results illustrated in Figure 1-1 for stationary conditions are invariant to characteristics of the flood frequency model and/or hydrologic characteristics of the river under consideration. In this section we explore the same relationships shown earlier in Figure 1-1 under nonstationary conditions by combining the theoretical expressions for average return period and reliability introduced in earlier sections with the nonstationary LN2 model.

We begin by investigating the behavior of the reliability index under nonstationary conditions in Figure 1-8. Figure 1-8 illustrates reliabilities over a realistic range of planning horizons for a fixed average return period of T_1=100 years associated with the design event. Holding the average return period constant at T_1=100, under nonstationary conditions, is similar to the earlier results shown in Figures 1-4 and 1-5, and contrasts with previous results where the design event was chosen using the stationary T_o=100 year event. Each panel in Figure 1-8 represents a fixed trend (M = 1, 1.02, 1.14, 1.5) and uses different lines to represent a range of C_x values (0.25, 0.5, 1, 1.5). We choose to illustrate the impact of nonstationarity due to increasing trends on reliability by fixing the average return period at 100 years in order to highlight that a structure must be built larger (often unrealistically so) to achieve a particular reliability under nonstationary conditions. Such large design events would involve considerable increases in design costs. As expected, regardless of whether conditions are stationary or nonstationary, reliability decreases with planning horizon; and, when a trend is present, in systems with less variability (low C_x), reliability is higher since the time to failure is more predictable (as was shown in Figure 1-6).

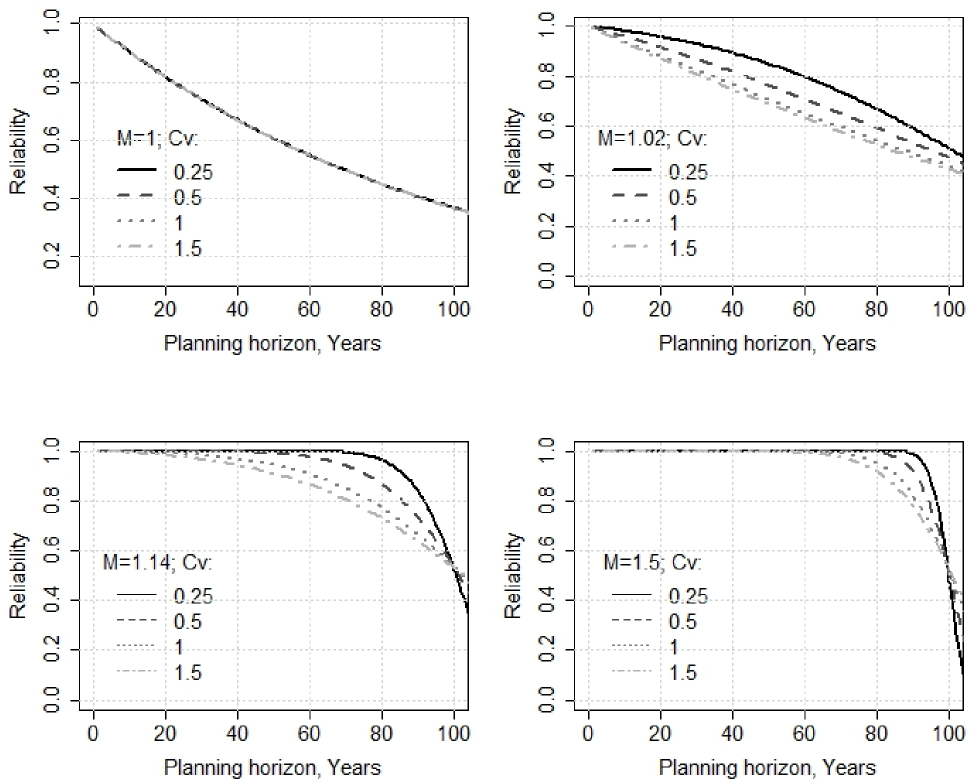
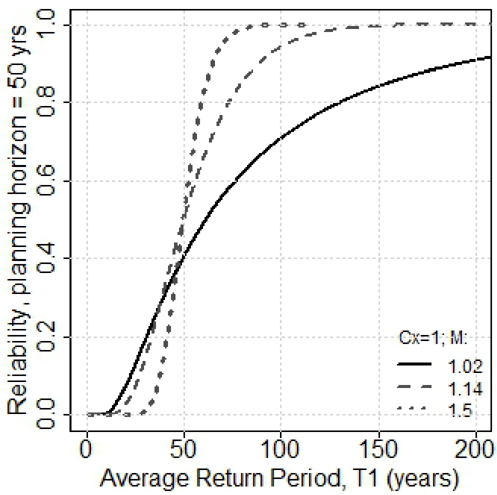
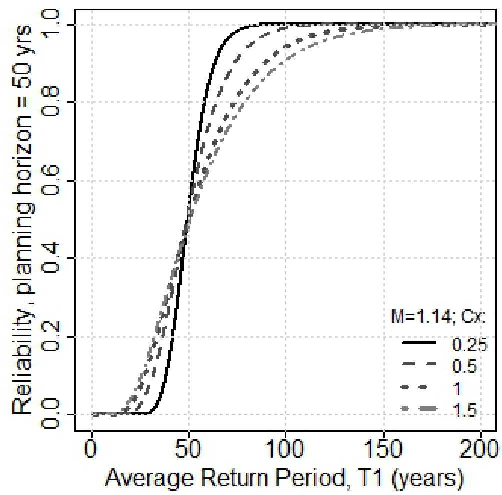


FIGURE 1-8. Reliability over planning horizon for a fixed average recurrence interval T_1 of 100 years; curves show a range of C_x= 0.25, 0.5, 1, 1.5; each plot considers a fixed trend (*M* = 1, 1.02, 1.14, 1.5)

Of considerable interest are the large increases in reliability associated with design events which are chosen based on the average return period T_1 under nonstationary conditions. One observes that in order to protect against a 100-year flood in a nonstationary setting, the chosen design event X^* results in much higher reliabilities than we are normally accustomed to under stationary conditions. The reason, as we observed earlier, is that under nonstationary conditions the distribution of the time to the event of interest becomes much more symmetric and peaked as *M* increases, i.e. the average return period becomes a better indicator of the time to the next event as *M* becomes large. This results in a much higher system reliability for planning horizons less than the design average return period T_1.

In Figure 1-9a,b we compare reliability to the average return period T_1, as we did in Figure 1, this time considering trends (M = 1.02, 1.14, 1.5). Several points arise from Figure 1-9, namely that the relationship between reliability and average return period is now extremely complicated and can no longer be defined by a single curve for a given n. The relationship between reliability and average return period under stationary conditions is invariant to the flood frequency model, whereas as is shown in Figure 1-9, the relationship between T_1 and reliability depends critically on the values of both M and C_x. Figure 1-9 also reiterates that for a given average return period T_1, reliability is higher for larger trends because the design event is larger. For example, in the theoretical case illustrated in Figure 1-9a, design flows for the T_1 = 100 year event are x = 0.20 m^3/s for M = 1 (stationary), and increase to x = 0.23 m^3/s for M = 1.02, x = 0.52 m^3/s for M = 1.14, and x = 5.45 m^3/s for M = 1.5; thus to design for a structure with a 100-yr average return period under nonstationary conditions, the required infrastructure will be much larger than under stationary conditions.





[a] [b]

FIGURE 1-9. Reliability versus average return period T_1, for n = 50, [a] $C_x = 1$; and the set of curves compare the stationary (M=1) case to a set of increasing trends M = 1.02, 1.14, 1.5; [b] trend is fixed at M = 1.14; set of curves represent different values of C_x = 0.25, 0.5, 1, 1.5.

Note also in Figure 9 that knowledge of T_1 alone is insufficient to provide a complete understanding of the likelihood of future flood events, because the reliability associated designs corresponding to a particular value of T_1 vary dramatically, depending upon both M and C_x. Nevertheless, the relationships shown in Figure 1-9 can guide hydrologists and design engineers on the likelihood of failure and the expected return period of a flood which will exceed the design flow under a range of nonstationary conditions. Both Figures 1-8 and 1-9 indicate that in order to secure high average return periods, we must consider much higher reliabilities, which are likely to be unrealistic in practice. If instead we start with reliability as a way to determine the size of the design event we are willing to protect against, then we can better communicate the likelihood of failure under both stationary and nonstationary conditions.

6. Discussion and Summary

In this paper we have drawn on existing theory and empirical results from the recent literature to provide the first general, comprehensive analysis of the probabilistic behavior of the return period and reliability under nonstationary conditions. Our results are a summary of old and new reasons for rethinking the use of an average return period as a design and communication metric for flood hazard planning. Under assumptions of stationarity, the average return period does not adequately communicate the likelihood of experiencing a failure over a given project life, which is precisely the concern for engineers designing infrastructure for flood management. We provide a theoretical example using a nonstationary lognormal (LN2) distribution to demonstrate that when evidence of nonstationarity exists in historic data, the time to failure distribution changes shape, the average return period is dramatically impacted, and the relationship between average return period and reliability becomes more complex since reliability now depends on C_x and the magnitude of the trend (M). From studying the time to failure distribution corresponding to a fixed average return period, we note its evolution from a right-skewed exponential tail to a very peaked and nearly symmetric pdf for larger values of M. A similar investigation with the Gumbel distribution reveals the possible generality of this finding.

The illustrations presented here are general and can also be applied to decreasing trends to examine how the behavior of the return period distribution changes and whether these alterations are consistent or inconsistent with results presented here for the case of increasing trends. Further, the relationships here should also be compared with those from other distributions such as the log

Pearson type III distribution (LP3) and the generalized extreme value (GEV) [*Salas and Obeysekera*, 2014; *Serinaldi and Kilsby*, 2015].

When assuming stationarity, the relationship between reliability, average return period and planning horizon is independent of the properties of the river under consideration and the resulting model of flood frequency. However, under nonstationarity, the relationship is far more complex, so that our experience of the reliability of flood management systems as well as the average return period associated with the next flood exceeding a critical design event depend on a number of additional considerations including: the characteristics of the underlying nonstationarity, the form of the probability distribution of the annual maximum flood discharges, and the planning horizon. The complexity in the shape and behavior of the distribution of the return period is further confounded by our inability to know and describe both the pdf and the trend in future flood series. Our analysis has only considered a single model of nonstationary flood frequency, so that considering other probabilistic models, as well as other models of trends in the mean, variance, and skewness, would lead to considerable additional complications. In other words, even for one of the simplest possible probabilistic models of nonstationarity, we have shown that our experience of the probability distribution of the return period is considerably more complex and unpredictable than its counterpart under stationary conditions. The assumption of an exponential trend model of annual maximum floods combined with the LN2 distribution, while reasonable, may not apply to all cases (nonstationarities) and thus we recognize that the same patterns may not result from different models.

Thus the complexity of the relationship between the model parameters (M, C_x) and the design metrics (reliability and T_1) grows in complexity as one considers additional uncertainties associated with nonstationary behavior of future floods. *Serinaldi and Kilsby* [2015] discuss how these increases in complexity add uncertainty when developing nonstationary models, and thus suggests careful treatment of uncertainty characterization before employing such models for design purposes.

Interestingly, for the case of increasing trends presented here, we showed that such trends actually improve our ability to know the time to failure, because the distribution of return periods becomes much more peaked and symmetric, so that the average return period becomes a better indicator of the time to an (exceedance) event. Correspondingly, large trends tend to produce very large design events and associated infrastructure costs, which are shown to be more reliable over a planning period than we normally experience under stationary conditions. In order to achieve high reliabilities under nonstationary conditions, we must build structures for these larger design flows or accept a greater risk of failure, a decision that is now further complicated by the uncertainty of future nonstationarities. We show that a unique relationship between reliability and average return period exists for a given nonstationary flood frequency model but without knowing the form of the nonstationary model with certainty, drawing inferences about the reliability from the average return period becomes difficult.

Further, given that the average return period is not intrinsically tied to a planning horizon, one might make a statement similar to the following in order to communicate event likelihood over a certain period of time: “we are 80% sure that this structure, built to withstand today’s 1,000-year event, will not experience at least one exceedance event within the next 25 years.” From this confusing statement, it is difficult to discern the meaning of the average return period and much more succinct to simply report the 80% reliability of the design over the future 25 year planning horizon. If instead planners designed for a certain desired level of reliability as is done in other fields concerned with planning under risk, then the statement could read: “we are 80% confident that this structure will not fail in the next 25 years.” And, the associated average return periods (assuming one would desire reliability > 80%) may be orders of magnitude higher as is illustrated in this study; the need to avoid misrepresenting the risk of failure is one reason we recommend replacing the average return period with the notion of reliability over planning horizon.

The consequences of misinterpreting the expected waiting time to an extreme event do not only impact the physical system, they also impact perceptions of individuals interacting with the floodplain. In the literature of flood risk communication, *Lave and Lave* [1991] report that floodplain residents generally expect to be protected for a particular average return period and have little understanding of the true risk of flooding during their lifetimes. Risk communication is itself a complex issue, as empirical research suggests that obstacles to understanding risks from natural hazards often require strategies deeper than presenting facts, such as using tactics to dispel heuristics and preconceived mistaken theories (see *Bier*, 2001; for a review).

7. Conclusions

We have shown how a parsimonious nonstationary lognormal model can be combined with recent research on nonstationary return periods, risk and reliability to obtain a very general understanding of the risk posed by future floods. The probability distribution of the time to failure of a water resource system under nonstationary conditions no longer follows an exponential distribution as is the case under stationary conditions, with a mean return period equal to the inverse of the exceedance probability $T_o=1/p$. Our findings raises numerous questions about our ability to understand and communicate the likelihood of future flood events under nonstationary conditions. We recommend replacing the notion of an average return period with reliability over a planning horizon, a metric used in almost all other realms of water resources planning and many other fields that communicate long term risk. Referring to a system's reliability directly conveys two pieces of information: the likelihood of no failure within a given number of years (i.e. over a planning horizon) and the accepted level of reliability that is implicit in the design. In the context of climate variability and change, and as cities become more urbanized , decisions on how to plan our water resource infrastructure become increasingly complex [*Rootzén and Katz*, 2013; *Obeysekera and Park*, 2013; *Rosner, et al.*, 2014; *Condon et al.*, 2015]. Our evolving perception of design and adaptation is increasingly recognized by those who study "socio-hydrology" in the context of floods and propose models and frameworks for the feedbacks/interactions between society and risks from the hydrologic environment [*Di Baldassarre et al.*, 2013; *Viglione et al.*, 2014].

Future work will investigate how a modern risk-based decision analysis framework can aid selection of an appropriate design event for hydrologic design under nonstationarity and explore new tools for characterizing nonstationary probability distributions.

References

Ameriks, J., R. Veres, and M. J. Warshawsky (2001), Making retirement income last a lifetime, Journal of Financial planning, 14(12), 60-76

Bier, V.M. (2001), On the state of the art: risk communication to the public, Reliability engineering & system safety, 71(2), 139-150, doi: 10.1016/S0951-8320(00)00090-9

Bras, R.L. (1990), Hydrology: An Introduction to Hydrologic Science, Addison-Wesley Publishing Co., Reading

Chow, V.T. (1964), Handbook of Applied Hydrology, McGraw Hill Book Co., N.Y.

Cohn, T. A., and H. F. Lins, (2005), Nature's style: Naturally trendy. Geophysical Research Letters, 32(23): 1-5, doi: 10.1029/2005GL024476

Condon, L. E., S. Gangopadhyay, and T. Pruitt (2015), Climate change and non stationary flood risk for the upper Truckee River basin, Hydrology and Earth System Sciences, 19(1), 159-175, doi:10.5194/hess-19-159-2015

Cooley, D. (2009), Extreme value analysis and the study of climate change, a commentary on Wigley 1988, Climatic Change 97, 77-83, doi: 10.1007/s10584-009-9627-x

Cooley, D. (2013), Return periods and return levels under climate change, in Extremes in a Changing Climate: Detection, Analysis and Uncertainty, edited by A. AghaKouchak etal., Springer Science + Business media, Dordrecht

Douglas, E.M., R.M. Vogel, and C.N. Kroll (2002), Impact of streamflow persistence on hydrologic design, Journal of Hydrologic Engineering, ASCE, 7(3), doi: 10.1061/(ASCE!1084-0699(2002)7:3(220)

Efstratiadis, A., Y. Dialynas, S. Kozanis, and D. Koutsoyiannis (2014), A multivariate stochastic model for the generation of synthetic time series at multiple time scales reproducing long-term persistence, Environmental Modelling and Software, 62, 139–152, doi:10.1016/j.envsoft.2014.08.017

Fernández, B., and J.D. Salas (1999), Return period and risk of hydrologic events. I: mathematical formulation, Journal of Hydrologic Engineering, 4(4), 297-307, doi: 10.1061/(ASCE)1084-0699(1999)4:4(297)

Fuller, W.E. (1914), Flood Flows, Transactions of the American Society of Civil Engineers,Vol. 77

Gumbel, E.J. (1941), The Return Period of Flood Flows, Annals of Mathematical Statistics, 12(2), 163-190

Haan, C. T. (1977), Statistical Methods in Hydrology, Iowa State University Press, Ames, Iowa

Harberg, R. J. (1997), Planning and managing reliable urban water systems, American Water Works Association

Hashimoto, T., J.R. Stedinger and D.P. Loucks (1982), Reliability, Resiliency, and Vulnerability Criteria for Water Resource System Performance Evaluation, Water Resources Research,18(1),114-20; doi: 10.1029/WR018i001p00014

Hirsch, R.M. (1979), Synthetic Hydrology and Water Supply Reliability. Water Resources Research, 15(6): 1603-15, doi: 10.1029/WR015i006p01603

IACWD (Interagency Committee on Water Data) (1982), Guidelines for determining flood flow frequency, Bulletin 17B, Office of Water Data Coordination, U.S. Geological Survey, Reston, VA

Katz, R. W. (2010), Statistics of extremes in climate change, Climatic Change, 100, 71-76

Katz, R. W., and B.G. Brown (1992), Extreme events in a changing climate: variability is more important than averages, Climatic change, 21(3), 289 302

Kottegoda, N. T., and R. Rosso (2008), Statistics, Probability, and Reliability for Civil and Environmental Engineers, McGraw-Hill, New York

Koutsoyiannis, D., and A. Montanari (2014), Negligent killing of scientific concepts: the stationarity case, Hydrological Sciences Journal, doi:10.1080/02626667.2014.959959

Krimsky, S., and D. Golding (1996), Social Theories of Risk, Journal of Policy Analysis and Management, edited by M. V. Rajeev Gowda, 15(3), 461 466, Wiley Publishing

Lave, T. and L. Lave (1991), Public Perception of the Risk of Floods: Impact on Communication, Risk Analysis, 255-267, doi: 10.1111/j.15396924.1991.tb00602.x

Lloyd, E. H., (1970), Occurrence interval in the presence of persistence, J. Hydrol., 10(3), 291-298

Loucks, D. P., E. Van Beek, J. R. Stedinger, J. P. Dijkman and M. T. Villars (2005), Water resources systems planning and management: an introduction to methods, models and applications. Paris: UNESCO.

Lund, J. R. (2002), Floodplain Planning with Risk-Based Optimization, Journal of Water Resources Planning and Management, 202-207, doi: 10.1061/(ASCE)0733 9496(2002)128:3(202)

Mandelbaum, M., M. Hlynka, and P.H. Brill (2007), Nonhomogeneous geometric distributions with relations to birth and death processes, Sociedad de Estadistica e Investigacion Operativa, Springer, 281-296, doi: 10.1007/s11750-007-0018-z

Mays, L. W. (2001), Water Resources Engineering, Wiley.

Milly, P. C. D., J. Betancourt, M. Falkenmark, R.M. Hirsch, Z.W. Kundzewicz, D.P. Lettenmair and R.J. Stouffer (2008), Stationarity is dead: wither water management. *Science, 319*, 73-574.

Montanari, A. and D. Koutsoyiannis (2014), Modeling and mitigating natural hazards: Stationarity is immortal!, Water Resources Research, 50(12), 9748-9756, doi: 10.1002/2014WR016092

National Research Council (2000), Risk Analysis and Uncertainty in Flood Damage Reduction Studies, The National Academies Press, Washington, DC

National Earthquake Hazard Reduction Program (2010), NEHRP Recommended Seismic Provisions for New Buildings and Other Structures, Federal Emergency Management Agency, 1-406

Obeysekera, J. and J. Park (2013) Scenario-Based Projection of Extreme Sea Levels. Journal of Coastal Research, 286, 1–7, doi:10.2112/JCOASTRES D-12-00127.1

Olsen, J. R., J.H. Lambert, and Y.Y. Haimes, (1998), Risk of Extreme Events Under Nonstationary Conditions, doi: 10.1111/j.1539 6924.1998.tb00364.x

Olsen, J. R. (2015). Adapting Infrastructure and Civil Engineering Practice to a Changing Climate, Committee on Technical Advancement of the American Society of Civil Engineers, Reston, VA

Parey, S., F. Malek, C. Laurent and D. Dacunha-Castelle (2007), Trends and climate evolution: Statistical approach for very high temperatures in France, Climatic Change, 81, 331-352, doi: 10.1007/s10584-006-9116-4

Parey, S., T.T.H. Hoang and D. Dacunha-Castelle (2010), Different ways to compute temperature return levels in the climate change context, Environmetrics, 21, 698-718, doi: 10.1002/env.1060

Pielke Jr, R. A. (1999), Nine fallacies of floods, Climatic Change, 42(2), 413-438, doi: 10.1023/A:1005457318876

Prosdocimi, I., T. R. Kjeldsen, and J. D. Miller (2015), Detection and attribution of urbanization effect on flood extremes using nonstationary flood frequency models, Water Resour.Res., 51, 4244–4262, doi:10.1002/2015WR017065.

Prosdocimi, I., T.R. Kjeldsen and C. Svensson (2014) Non-stationarity in annual and seasonal series of peak flow and precipitation in the UK, Nat. Hazards Earth Syst. Sci.,14, 1125-1144. doi: 10.5194/nhess-14-1125-2014

Rootzén, H. and R. W. Katz, (2013), Design Life Level: Quantifying risk in a changing climate, Water Resour. Res., 49, 5964–5972, doi:10.1002/wrcr.20425

Rosner, A., R.M. Vogel, and P. Kirshen (2014), A risk-based approach to flood management decisions in a nonstationary world, Water Resources Research, 50, 1–15, doi:10.1002/2013WR014561.

Salas, J. D. (2013). Discussion -"Pragmatic Approaches for Water Management Under Climate Change Uncertainty" by Eugene Z. Stakhiv, JAWRA Journal of the American Water Resources Association, 49(2), 475-47, doi: 10.1111/jawr.12026

Salas, J.D. and J. Obeysekera (2014), Revisiting the Concepts of Return Period and Risk for Nonstationary Hydrologic Extreme Events, J. Hydrol. Eng., 19(3), 554–568, doi:10.1061/(ASCE)HE.1943-5584.0000820

Serinaldi, F. (2015), Dismissing return periods!, Stochastic Environmental Research and Risk Assessment, 1-11, doi: 10.1007/s00477-014-0916-1

Serinaldi, F. and C.G. Kilsby (2015), Stationarity is undead: uncertainty dominates the distribution of extremes, Advances in Water Resources, 77, 17-36, doi: 10.1016/j.advwatres.2014.12.013

Stedinger, J.R., R.M. Vogel and E. Foufoula-Georgiou (1993), Frequency Analysis of Extreme Events in The Handbook of Hydrology, edited by David R. Maidment, McGraw-Hill Book Company

Stedinger, J. R., and C.M. Crainiceanu (2000), Climate variability and flood-risk management. Risk-based decision making in water resources IX, 77-86, doi: 10.1061/40577(306)7

Stout, R. G., and J.B. Mitchell (2006), Dynamic retirement withdrawal planning. Financial Services Review, 15(2), 117, Available at SSRN: http://ssrn.com/abstract=2542024

Thomas, H.A. (1948), Frequency of Minor Floods, Journal of the Boston Society of Civil Engineers, 35(1), 425-442

Tung, Y-K (1999), Risk/Reliability-Based Hydraulic Engineering Design in Hydraulic Design Handbook, edited by L. Mays, McGraw Hill Co., N.Y

Tung, Y-K (2005), Flood Defense Systems Design by Risk-Based Approaches, Water International, 30(1), 50-57, doi: 10.1080/02508060508691836

Tung, Y-K, B.C. Yen, and C.S. Melching (2006), Hydrosystems Engineering Reliability Assessment and Risk Analysis, McGraw-Hill: New York.

United States Army Corps of Engineers. (1996), Risk-based analysis for evaluation of hydrologic/hydraulics, geotechnical stability, and economics in flood damage reduction studies, *Engineering Regulation 1105-2-101*,Washington D.C.

United States National Nuclear Regulatory Commission (1975) WASH-1400: Reactor Safety Study, NRC: Washington.

Viessman Jr., W. and G.L. Lewis (2003), Introduction to Hydrology, 5th Edition, Addison Wesley Educational Pub. Inc.

Vogel, R. M. (1987), Reliability indices for water supply systems, Jour. Water Resour. Plan. Manage., 113(4), 563–579, doi: 10.1061/(ASCE)0733 9496(1987)113:4(563)

Vogel, R., and I. Wilson (1996), The Probability Distribution of Annual Maximum, Minimum and Average Streamflow in the United States, Journal of Hydrologic Engineering, 69-76, doi: 10.1061/(ASCE)1084 0699(1996)1:2(69)

Vogel, R.M., C. Yaindl, and M. Walter (2011), Nonstationarity: Flood Magnification and Recurrence Reduction Factors in the United States, Journal of American Water Resources Association, 47(3) 464-474, doi: 10.1111/j.1752-1688.2011.00541.x.

Vogel, R.M., A. Rosner, and P.H. Kirshen, (2013), Likelihood of Societal Preparedness for Global Change – Trend Detection, Natural Hazards and Earth System Science, Brief Communication, 13, 1-6; doi: 10.5194/nhess-13-1-2013

Vogel, R. M., Lall, U., Cai, X., Rajagopalan, B., Weiskel, P., Hooper, R. P., & Matalas, N. C. (2015). Hydrology: The interdisciplinary science of water. *Water Resources Research*.

Water Resources Council (1983), Economic and Environmental Principles and Guidelines for Water and Related Land Resources Implementation Studies, Washington D.C.

Wigley, T. (1988), The effect of changing climate on the frequency of absolute extreme events, Clim Monit 17:44–55. Reprinted in (2009) Clim Chang 97:67–76, doi: 10.1007/s10584 009-9654-7

Yen, B. C. (1970), Risks in Hydrologic Design of Engineering Projects, Journal of the Hydraulics Division, 959-965

Chapter 2: Hazard function analysis for flood planning under nonstationarity

Journal reference: Read, L.K. and R. M. Vogel, (submitted, 2015), Hazard function analysis for flood planning under nonstationarity, *Water Resour. Res.*

ABSTRACT

The field of hazard function analysis (HFA) involves a probabilistic assessment of the ‘time to failure’ or ‘survival time’ of an event of interest and is widely used in epidemiology, manufacturing, medicine, actuarial statistics, reliability engineering, economics and elsewhere. HFA assumes that the probability of exceedance of the event of interest, known as the hazard function, is changing over time, thus this field should be of considerable interest to hydrologists concerned with nonstationary processes. We describe how HFA can be used to characterize the probability distribution of the return period and the reliability, two primary metrics in hydrologic design. We provide a general introduction to HFA with application to water resources management by providing a clear linkage between HFA and flood frequency analysis. Explicit linkage between the flood series (X) and failure times (T) enables computation of corresponding average return periods and reliabilities for a nonstationary 1-parameter exponential (EXP1) hazard model from first principles. We then use Monte-Carlo simulation to extend HFA for the case of a nonstationary 2-parameter lognormal distribution, linking the theory of an accelerated life modeling problem with flood frequency analysis. Our findings suggest that a 2-parameter Weibull distribution is a reasonable model for the survival (reliability) function. We document how HFA is a relevant approach for characterizing nonstationary flood series and provide engineers with tools to support hydrologic design decisions under nonstationary conditions.

1. Introduction

Many disciplines are concerned with the *survival* time of some process before experiencing a certain type of event defined as a *failure*. Such analyses termed “Hazard Function Analysis” (HFA) informs many fields, for example, in the medical field, the onset or relapse of a disease; in economics, the time until a person becomes employed; in reliability engineering, the time until a device fails; and in actuarial science, the time to death. Thus, HFA comprises a well-known set of tools applied in many disciplines for characterizing the probability distribution of the time to failure associated with specific events or processes of interest over the course of a lifetime or some other time period of interest. In general, the ability to represent the time to failure and its distribution are crucial for understanding event likelihood through time, given that year to year the likelihood may change. For example, many design standards in reliability engineering and manufacturing, as well as policies for clinical trials in public health, are based on the survival time of a piece of equipment, or a person’s expected lifetime after diagnosis of a disease, etc.

The field of HFA has a rich and long history as evidenced by the number of textbooks on survival analysis for handling a wide range of hazards [see Klein and Moeschberger, 1997; Cleves, 2008; Wienke, 2010; Lawless, 2011; Finkelstein, 2010]. HFA is also included as a chapter in textbooks on probability [Bean, 2001] and statistics [Kottegoda and Rosso, 2008]. Remarkably, in spite of this fact, to our knowledge we are one of the first studies to offer a bridge between HFA and water resources planning and management. The introduction of HFA methods and theory to the hydrologic community for understanding and characterizing nonstationary flood series is the primary goal of this study.

Critical to HFA is the hazard rate function, h(τ), which describes the likelihood through time, τ, associated with a particular event of interest. Table 1 presents definitions and applications of the hazard rate function in other fields [Klein and Moeschberger, 1997].

TABLE 2-1. Cross-disciplinary examples of hazard function definitions and applications

Field	**Definition**	**Example**
Manufacturing	Conditional failure rate	Parts wearing out in a machine
Economics	Inverse Mills' ratio	Control for selection bias in regression
Epidemiology	Age-specific failure rate	Number of people in specific age group contracting a disease
Actuary statistics	Force of mortality	Likelihood of dying at a particular age
Reliability engineering	Failure rate	Failure of electronic devices

Analogous to the fields outlined in Table 2-1, water managers, flood planners, and hydraulic design engineers are concerned with the expected waiting time (average return period) until a system might experience at least one exceedance event. Interest also focuses on a system's reliability, or the likelihood to *survive* a planning period without experiencing at least one exceedance event. Under stationary conditions, computing the average return period and associated reliability of experiencing a design flood (or larger) is straightforward because the exceedance probability (p) of the event of interest is constant over time, and thus the return period or survival time, T, follows an exponential (continuous case) or geometric (discrete case) distribution with average return period equal to $1/p$. Under nonstationary conditions, p is changing through time, which is exactly the condition dealt with by HFA; thus a second goal of this work is to reevaluate the behavior of traditional design metrics such as the average return period and reliability in the context of HFA.

The presence of nonstationarity in hydrologic systems due to possible changes in climate, land use and water infrastructure has received considerable attention since the work of Milly et al., [2008] in spite of the reservations of Cohn and Lins, [2005], Matalas [2012] and many others. The climatology community has been active in publishing research relating the impact of nonstationarity on frequency analysis of climatic events [Katz and Brown, 1992; Olsen, 1998; Cooley, 2013]. Similarly, there is considerable interest in nonstationary hydrologic processes such as trend detection of numerous indicators including: temperature [Parey et al. 2007; 2010], annual maximum precipitation [Westra et al., 2013], alterations in peak flow series [Vogel et al., 2011; Villarini et al., 2009] and streamflow variability [McCabe and Wolock, 2014]. Bayazit [2015] reviews literature on nonstationarity in hydrologic systems and the impact on design metrics such as average return period and risk of failure; Read and Vogel [2015] examine the implications of nonstationarity on reliability of water resource systems. This recent attention to nonstationarity with regard to water resource planning has catalyzed a need to apply new methods for evaluating extreme event risk and reliability, especially given the trajectory of estimated damage costs from certain hazards, e.g. the National Weather Service projects flood damages will increase by $30 billion from 2013-2043 (www.nws.noaa.gov/hic).

There is now a plethora of recent literature relating the impact of nonstationarity on hydrologic systems with most work concentrated on the development of models of nonstationary flood frequency [see Bayazit, 2015 for a review]. There is a much smaller but growing literature which focuses on the impact of nonstationarity on design indices such as reliability and average return periods [see Read and Vogel, 2015 for review]. Surprisingly, none of those studies which consider nonstationarity in hydrologic systems mention the potential of HFA as a framework for this topic, nor provide an explicit link between HFA and flood frequency analysis as we do in this work.

A primary goal of this paper is to provide a general introduction to the theory and background for relating HFA to flood planning under nonstationarity. Since hydrologic engineers are concerned with metrics such as average return period and reliability, which become more complicated under nonstationary conditions, we see value in applying concepts from HFA, which model the risk of event occurrence over time. We begin with an introduction to the definition and theory behind hazard function analysis, followed by a review of hazard function applications from the water resources literature; section 2 presents an analytical derivation of hazard function analysis for the EXP1 model; section 3 describes the methodology for fitting a hazard function model to one example of a more realistic nonstationary model of flood frequency (nonstationary LN2). Discussion and conclusions in sections 4 and 5 focus on implications for employing hazard function analysis tools for extreme event planning under nonstationarity, and recommendations for future work.

1.1 An introduction to hazard function definitions and theory

The hazard function h(τ) describes how the likelihood of experiencing an event changes through time; it is normally not considered a probability but rather a rate that provides an essential piece of information in studies concerned with processes such as the deterioration of infrastructure, remaining ‘survival’ life of a device or human being, or time of relapse to a disease, etc. Hazard functions are of particular interest in environmental and water resource applications due to the impacts of nonstationarity associated with urbanization, climate change, infrastructure decay and other factors which cause abrupt and/or continual changes in the likelihood associated with various flood and other hydrologic events of interest.

The hazard rate function is defined as the conditional likelihood of experiencing a flood within a small interval of time (τ, $\tau + \Delta\tau$), given that a flood has not yet occurred in (0, τ):

$$h(\tau) = \lim_{\Delta\tau \to 0} \frac{1}{\Delta\tau} P(\tau \le t \le \tau + \Delta\tau \mid t \ge \tau) \qquad (23)$$

where h(τ) is in units (1/*time*) and $\Delta\tau$ is an infinitesimally small time step and the variable τ, denotes the deterministic variable time. Although the hazard rate function is not a formal probability function, it is related to the probability density function (pdf), $f_T(t)$, and the cumulative distribution function (cdf) $F_T(t)$, of the return period, or time to failure *T*, by:

$$h(\tau) = \frac{f_T(t)}{1 - F_T(t)} \tag{24}$$

where the complement of the cdf, $1-F_T(t)= S_T(t)$ is termed the survival function, and represents the exceedance probability $P(T > t)$ that a failure will not occur in a given time period. Here *T* denotes the random variable which we term the time to failure or return period of a particular event and *t* denotes a realization of that random variable, both of which are distinguished from τ which denotes deterministic time.

Most previous applications of HFA assume a functional form for $h(\tau)$, and then derive $f_T(t)$, $F_T(t)$ and $S_T(t)$ using the relation in (24) rather than explicitly linking $h(\tau)$ to the properties of the random variable of interest, which here is the magnitude of a design flood event *Xp*, where *X*p denotes the quantile function of the AMS of floods *X*. Since numerous textbooks provide the governing relationships for relating $h(\tau)$, $S_T(t)$, $F_T(t)$ and the cumulative hazard function $H(\tau)$ [Lawless 2011] we do not derive them here, and only present the following well known results:

$$S_T(t) = 1 - F_T(t) = \exp\left(-\int_0^t h(s)ds\right) \tag{25}$$

$$H(\tau) = -\ln(S_T(t))\big|_{t=\tau} = \int_0^\tau h(s)ds \qquad (26)$$

We use subscripts for the pdf, cdf, and the survival function to highlight that the random variable *T* is equal to return period, in contrast with the hazard functions, h(τ) and H(τ), which are not formal probability functions and thus do not depend on the random variable *T*. In the following sections the above theoretical relations for HFA are applied to the problem of nonstationary flood frequency analysis.

HFA normally begins with the hazard function, h(τ), usually chosen based on empirical evidence or expert judgement. For example, the user assumes that h(τ) takes a certain shape (e.g. 'bathtub', increasing, decreasing, etc.) that reflects ones intuition about the nature of future changes in the exceedance probability of a certain hazard over time (τ). For example hazard rate functions for electronic devices are usually assumed to be constant through time, and for mechanical devices are often "bathtub" shape curves, with a decreasing failure rate initially, leveling off for a duration, and then increasing monotonically as parts eventually wear out. Similar hazard rate functions have been advocated for infrastructure problems [see Figure 9.4.3 in Kottegoda and Rosso, 2008 for an example]. If little or no information about the hazard function is available, a parametric survival function $S_T(t)$ such as the Exponential or Weibull, or non-parametric estimator can be implemented to define the survival function from an empirical data set [Klein and Moeschberger, 1997]. A discussion of the advantages and limitations of using non-parametric methods is given in Wienke [2010]. The survival function $S_T(t)$ is equivalent to the reliability function which represents the probability of no-failure, defined as the exceedance probability for the random variable time, T, for which t is the realization. The survival function is more commonly defined in terms of the cumulative distribution function (cdf) of t, termed $F_T(t)$, so that $S_T(t) = 1 - F_T(t)$. Our challenge is to connect the hazard function $h(\tau)$ and survival function $S_T(t)$ with the probability distribution of the flood event X. Our work differs from previous studies on hazard function theory because it begins with rigorous definition of $h(\tau)$ defined by the exceedance probability (p_o) associated with the design flood event x_o, so that $h(\tau) = p_o$ for a stationary case.

Consider in detail the stationary case where failure rates are constant through time, thus the hazard function $h(\tau) = p_o$ is fixed in which case the time to failure T, follows a one-parameter exponential distribution (continuous case) or geometric distribution (discrete case), regardless of the probability density function (pdf) associated with the design discharge, X_o [Gumbel, 1941; Chow, 1965]. In this case the average return period, or the mean time to failure (MTTF), is simply expected value of an exponential random variable, where $E[T] = 1/p$, the p% exceedance event.

Now consider the nonstationary condition where historic streamflow data show an increasing trend in the mean of the annual maximum peak flows over time. Such trends have found to be quite common in urbanizing areas for annual maximum streamflow series (AMS) [Vogel et al., 2011; Prosdocimi et al. 2014]. A trend in the flood series also indicates that the exceedance probability associated with the design event changes as a function of time, p_τ, and thus the expected time E[T] until a flood occurs is no longer $1/p$, the common assumption in the majority of all hydrologic and water resource investigations to date. HFA is ideally suited to this task because a nonstationary hazard rate function, $h(\tau)$ can be derived function which reflects the change in the exceedance probability over time, for a design given by $x_o(p_o)$ at time $\tau = 0$. Thus in contrast with previous applications of HFA, in this context $h(\tau)$ has a very important and precise definition equivalent to the probability that a flow will exceed the design event conditioned on τ, or precisely, $h(\tau) = p_\tau = 1 - F_X(x_o, \tau)$. This definition in and of itself is useful in planning, because it reflects the increasing exceedance probability associated with the original design if the future flood regime exhibits nonstationary behavior.

1.1 *Review of hydrologic planning metrics from climate and hydrologic literature*

Our primary concern is with modeling the probability distribution of the return period, or time to failure for infrastructure which provides protection against future floods. We begin by reviewing the stationary case, where the hazard rate function or failure rate is constant through time, and an engineer needs to design a structure to withstand a certain design event, e.g. the 1% exceedance event, $p = 0.01$, or the 100-year flood, $E[T] = 1/0.01 = 100$. The probability of this event in each year over a planning period n, $\tau = 1,\ldots n$ is assumed constant (i.e. $h(\tau) =$ constant in eq. 2), and the average return period $E[T]=1/p$, is equal to the expected value of a geometric series (discrete) or an exponential series (continuous) with rate parameter p.

Now consider the nonstationary condition where historic flood data show an increasing trend in the mean of the AMS over time. Such trends are quite common in urbanizing areas [see Vogel et al., 2011; and Prosdocimi et al. 2014]. While a number of recent studies have raised concerns over use of the average return period in the context of nonstationarity [Sivapalan and Samuel, 2009; Cooley, 2013; Salas and Obeysekera, 2014], few examples of practical applications of nonstationary flood frequency models and alternatives to the average return period are available in the hydrologic literature.

Concerned with climate change, Olsen (1998) first introduced a nonhomogeneous form of the geometric distribution to define the average return period (T_1) under nonstationary conditions. That work was later extended by Cooley (2013) and Salas and Obeysekera (2014) to hydrologic systems. Their formulations rely on knowledge of p_τ, defined as the annual probability of exceeding a design flood assumed to be continuously increasing every year; then in general the expected return period is:

$$T_1 = E(T) = \sum_{t=1}^{t_{\max}} t \cdot f(t) = \sum_{t=1}^{t_{\max}} t \cdot p_\tau \prod_{i=1}^{t-1} (1 - p_i) \qquad (27)$$

where T_1 is defined as the average return period, or expected waiting time until the next exceedance event. Another definition of the average return period was defined as the time it takes to experience exactly one exceedance event [Parey et al., 2007, 2010; Cooley, 2013]. Read and Vogel [2015] argue that the concept of an average return period is confusing and misleading for communication of the risk posed by and the likelihood of future floods under both stationary and nonstationary conditions. Read and Vogel [2015] recommend use of the reliability function as an alternative metric to communicate flood risk and likelihood to engineers and decision makers. Considering the increasing societal awareness and associated literature on planning under nonstationary conditions in the water resources field we see great relevance in extending HFA to enable future discussions of hydrologic planning in a nonstationary context.

1.2 *Survival and hazard function analysis in the water resources literature*

Applications of survival and HFA in the water resources literature are sparse and targeted toward specific flood or other case studies rather than a general analysis that attempts to relate HFA to flood frequency analysis as is the goal here. Several researchers have applied proportional hazard (PH) Cox regression models [Cox, 1972] for characterizing flood risk [Futter and Mawdsley, 1991], climate variation [Maia and Meinke, 2010], and to changes in flood behavior by the peaks over threshold method [Smith and Karr, 1986; Villarini et al., 2010], based on covariates. Cox models are classified as PH models, because the hazard for one participant (location) is a fixed proportion of the hazard for another participant (location), and the rate of occurrence process has a functional dependence on a covariate process [Cox, 1972]. Literature on ‘trend attribution’ in hydrology have applied PH models to identify mechanisms for changes in peak flood regimes [Cunderlik and Burn, 2004; Villarini et al., 2010], though much more research is needed in this area [Merz et al. 2012]. Cox type models are popular in the bio-statistics literature [Klein and Moeschberger, 1997], and are considered very useful in determining whether covariates (stationary or time-varying) influence the rate of occurrence of events (i.e. does climate influence the occurrence of floods, droughts, etc. for our application). One advantage is that the Cox model does not require knowledge of the functional form for the hazard function, which as we have discussed is often unknown in practice; one potential issue in the practical application of HFA is the ability to determine a plausible baseline hazard function, a requirement for the Cox model. Our study takes a different approach by deriving the hazard function from our knowledge of the probabilistic properties of the flood series *X*, and the design event *Xp*.

Several disciplines concerned with the cdf or survival function of the time to an event of interest have employed HFA: e.g. in economics [Kiefer, 1988] and transportation [Hensher and Mannering, 1994]. In the hydrology literature, Lee et al. [1986] applied HFA to the problem of multi-year drought durations. After testing several hazard functions for modeling the temporal behavior of exceedance probabilities corresponding to observed droughts, the authors suggested a logistic model for $h(\tau)$ with an exponential correction for short durations.

Despite the obvious applicability of HFA to hydrologic challenges associated with infrastructure design and planning under nonstationarity, only a handful of hydrologic studies even mention HFA [Katz and Brown, 1992; Wang et al., 2010; Zhong & Hunt, 2010], and only one application of the type of accelerated failure time (AFT) hazard modeling, introduced here, could be found in the water resources literature [Lee et al., 1986].

In this initial application of HFA to nonstationary flood frequency analysis we present two realistic cases to show how HFA can be useful as a framework for characterizing reliabilities and average return periods under nonstationary conditions. Naturally, future studies are needed to extend our analysis to other realistic cases. We provide a derivation of the hazard function, survival (reliability) function and cumulative hazard function for two cases which are representative of the behavior of a wide class of flood management systems. Our analysis results in a complete probabilistic description of the return period corresponding to nonstationary flood series arising from: (1) a nonstationary one parameter exponential (EXP1) distribution to demonstrate how the properties random hazard variable X corresponding to a flood series, can be related to the probabilistic properties of the random variable time T; and (2) a more realistic case considering a nonstationary lognormal model which has been shown to provide an excellent approximation to AMS floods for thousands of rivers in the US (Vogel et al. 2011) and the UK (Prosdocemi et al. 2014). In the first case, our entire analysis is analytical resulting in very general closed-form results. For the second, more realistic case, Monte-Carlo simulations are used to obtain general results, because analytical results are unavailable. In this second case we use goodness of fit methods to document that the 2-parameter Weibull (Weibull-2) distribution provides a reasonable model of the reliability (survival) function. We then use multivariate regression to relate the parameters of the Weibull-2 reliability model with those of the nonstationary LN2 flood model and compare estimates of the average return period corresponding to the HFA analysis with

Monte Carlo results. The resulting model can be applied for any case where AMS floods exhibit a log-linear trend in the mean and follow a LN2 distribution.

The analysis presented through these examples provides readers with a methodology for integrating HFA with standard hydrologic frequency analysis and design metrics that hydrologists are familiar with, and also with the language and understanding for accessing the HFA literature. Thus we expect our findings and approach outlined to enable numerous future extensions to the hydrologic community for planning and design under nonstationarity, specifically demonstrating how planning metrics such as reliability and average return periods can be computed for a river system using independent methods from HFA.

2 Hazard Function Analysis for Exponential Flood Peaks

The exponential distribution is widely used in the peaks over threshold (also called partial duration series or PDS) method for characterizing the magnitudes of flood exceedances above some set level [Stedinger et al., 1993]. For example, Stedinger et al. [1993] document that if the number of PDS flood arrivals follow a Poisson process and their magnitudes follow an exponential distribution, then the series of AMS flood magnitudes will follow a Gumbel distribution. Clearly the exponential pdf plays a pivotal role in the theory of extremes. In this section we derive general results corresponding to HFA for the case when flood series follow an EXP1 distribution. This analysis enables us to demonstrate how the probabilistic properties of as a series of POT flood magnitudes X, and an associated design event, are related to their corresponding probabilistic properties of their failure times, T.

To contrast the application of HFA with the nonstationary case, we begin by considering the much simpler and more common stationary case. The pdf and cdf of an EXP1 random variable, *X*, representing the flood magnitude above some threshold are given by

$$f_x(x) = \lambda \exp(-\lambda x) \tag{28}$$

$$F_x(x) = 1 - \exp(-\lambda x) \tag{29}$$

where λ is the rate parameter, with E[X] = μ_x = 1/λ. Consider p_o = 1- $F_X(x)$ the fixed exceedance probability associated with design event X_{p_o} given by the quantile function obtained from (29):

$$X_{p_o} = -\frac{1}{\lambda}\ln(p_o) \tag{30}$$

Recall from eqn. (23) that h(τ)=*p*, so that the survival function is given by substitution of h(τ) = *p* into eqn (25) resulting in

$$S_T(t) = 1 - F_T(t) = \exp\left(-\int_0^t p\,ds\right) = \exp(-p \cdot t) \tag{31}$$

Similarly, since $f_T(t)=dF_T(t)/dt$ we obtain the pdf of the time to failure *T*, as an exponential distribution with parameter *p*, so that the average time to failure E[T]=1/*p* for this simple stationary case.

Now consider the nonstationary case in which the random variable *X* increases with time (τ) due to a trend in the mean, represented by the following exponential model:

$$\mu_x(\tau) = \frac{1}{\lambda}\exp(\beta \cdot \tau) \tag{32}$$

Note that for no trend, the term β=0 and the nonstationary mean reduces to the stationary mean $\mu_x = 1/\lambda$. Although the trend parameter β denotes the magnitude of the flood trend, this parameter is difficult to physically interpret. Instead, Vogel et al. [2011] and Prosdocimi et al. [2014] define the more easily understood flood magnification factor *M* as the ratio of the flood magnitude in year ($\tau + \Delta\tau$) to the flood magnitude in year τ. Combining the quantile function in (30) under stationary conditions with the model for the nonstationary mean in (32) leads to an expression for *M* for a nonstationary EXP1 variate:

$$M = \frac{x_p(\tau + \Delta\tau)}{x_p(\tau)} = \frac{\frac{1}{\lambda}\exp[\beta(\tau + \Delta\tau)\ln(p_t)]}{\frac{1}{\lambda}\exp[\beta \cdot \tau \ln(p_t)]} = \exp[\beta\Delta\tau] \tag{33}$$

which, interestingly, is identical to the magnification factor derived by Vogel et al. [2011] for a nonstationary lognormal variable. The cdf of a nonstationary EXP1 variable is obtained by inserting (32) into (29) and replacing β with *M* given in (33) leading to:

$$F_x(x,\tau) = 1 - \exp(-\lambda x \cdot M^{-\tau/\Delta\tau}) \tag{34}$$

If the design event X_p is based on conditions at $\tau = 0$, then $X_p = X_o$ and $p = p_o$ and the design event is fixed so that combining (30) and (34) leads to an expression for the hazard rate function $h(\tau) = p_\tau$ as:

$$h(\tau) = 1 - F_x(x_o, \tau) = p_o^{M^{-\tau/\Delta\tau}} \tag{35}$$

After fixing the design event, the random variable of interest is now *T*, the time to failure, with *t* as its realization, and henceforward use τ solely to represent the deterministic time. The survival function $S_T(t)$ and the cumulative hazard function $H(\tau)$ are easily obtained by inserting (35) into the relationships in (25) and (26), respectively, and solving numerically:

$$S_T(t) = \exp\left[-\int_0^t p_o{}^{M^{-s/\Delta\tau}}\, ds\right] \tag{36}$$

$$H(\tau) = \int_0^t p_o{}^{M^{-s/\Delta\tau}}\, ds \tag{37}$$

The cumulative hazard function $H(\tau)$ is the integral of the hazard function over time, interpreted as the total hazard, or equivalently, as the number of failures expected to occur in a given time period, assuming failures can repeat [Cleves, 2008].

The pdf is easily computed from the cdf by $f_T(t) = \dfrac{d}{dt} F_T(t)$ which leads to the following expression, which may be solved numerically:

$$f_T(t) = \frac{d}{dt}\left[\exp\left(\int_0^t p_o{}^{M^{-s/\Delta\tau}}\, ds\right)\right] \tag{38}$$

Above we have shown how HFA can be used to relate the probabilistic properties of an EXP1 flood series *X* combined with a fixed design event, to the properties of the time to failure distributions associated with the resulting design event. Figure 2-1 illustrates the hazard function computed from (35) for a set of trends parameterized by decadal ($\Delta\tau$=10 years in eqn. 33) magnification factors (*M* = 1, 1.05, 1.2, 1.5, 2) assuming [a] p_o = 0.1 and [b] p_o = 0.01. We note from Figure 2-1 that the hazard rate function for the nonstationary EXP1 model is no longer constant through time as it was under stationary conditions; and, that greater trends are associated with more accelerated hazard rates. While h(τ) is an important mathematical function in hazard analysis, and serves as the linkage between the probabilistic properties of the time to failure, *T*, and the properties of the flood series *X*, for the purposes of flood planning and risk communication, the survival function, cdf and pdf of *T* are more useful tools in practice.

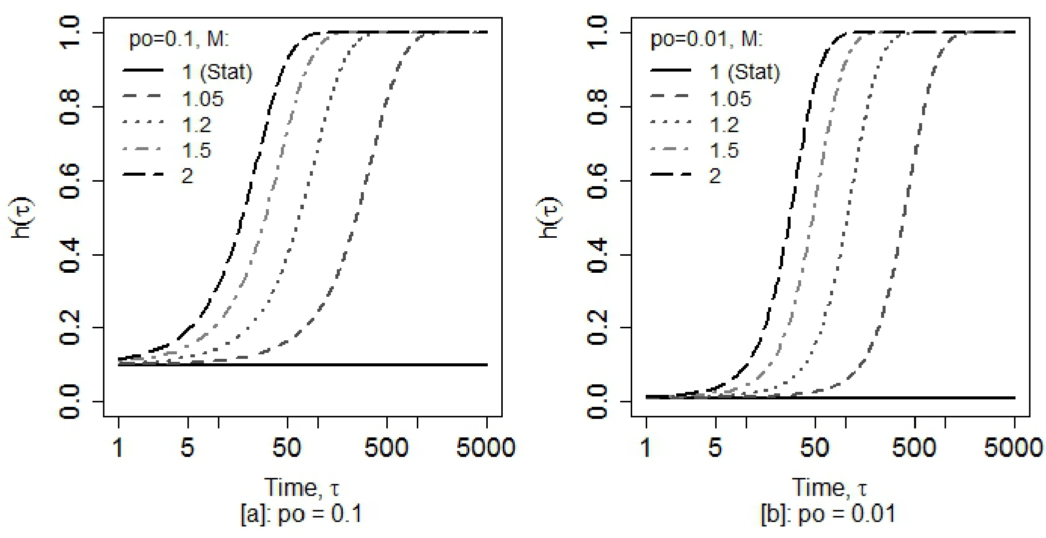


FIGURE 2-1. Hazard rate function h(τ) = p_τ for the nonstationary EXP1 case; lines represent trends parameterized by a range of decadal magnification factors (*M* = 1, 1.05, 1.2, 1.5, 2) for two possible event sizes [a] p_o = 0.1, [b] p_o = 0.01. Note the log scale for the x-axis (time).

The survival function $S_T(t)$ given in eqn (36) and shown in Figure 2 assumes p_o = 0.01 for a range of trends (*M*). The time to failure distribution is clearly impacted by the presence of an increasing trend, evidenced by the departure from the classic exponential curve corresponding to a stationary EXP1 model in Figure 2-2. Realizations of $S_T(t)$ yield important information about the distribution of the time to an exceedance event, or the reliability of a system, and how trends impact this experience.

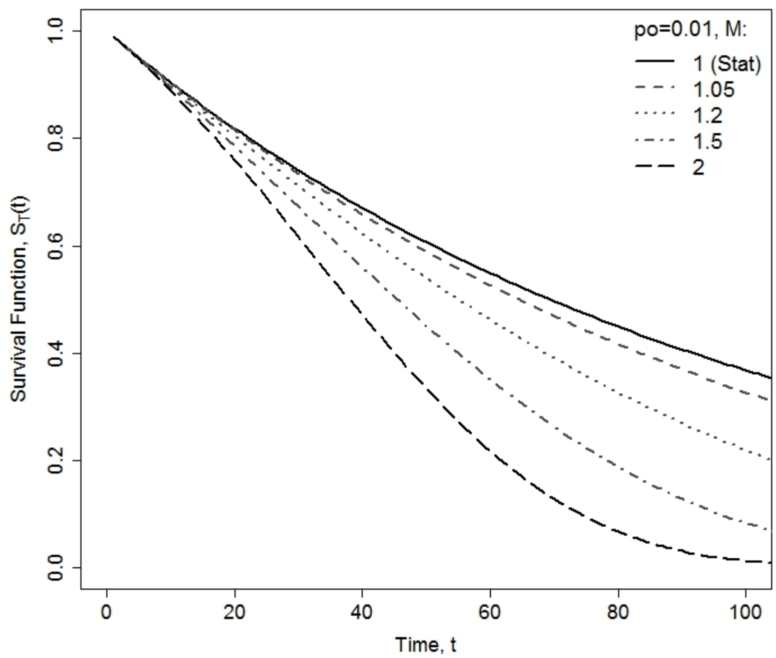


FIGURE 2-2. Survival function, $S_T(t)$ = Reliability(t) for the nonstationary (EXP1) model with p_o = 0.01; lines represent trends parameterized by a range of decadal magnification factors (*M* = 1, 1.05, 1.2, 1.5, 2).

The cumulative hazard function $H(\tau)$ given in eqn (37) and plotted in Figure 3 provides a way to interpret the total hazard through time. From eq. (37) $H(\tau)$ is the integral of the hazard rate function, which represents the number of events up to a particular point in time. Note that for the stationary case, $H(\tau) = 1$ at $\tau = 100$ indicating that only one event is expected within this time period. As the magnitude of the trend increases, the amount of time it takes to experience an event (or magnitude above some threshold) decreases; for example Figure 2-3 illustrates that with M = 1.5, the p_o = 0.01 event may now occur twice in 100 time periods (or once in 60 periods).

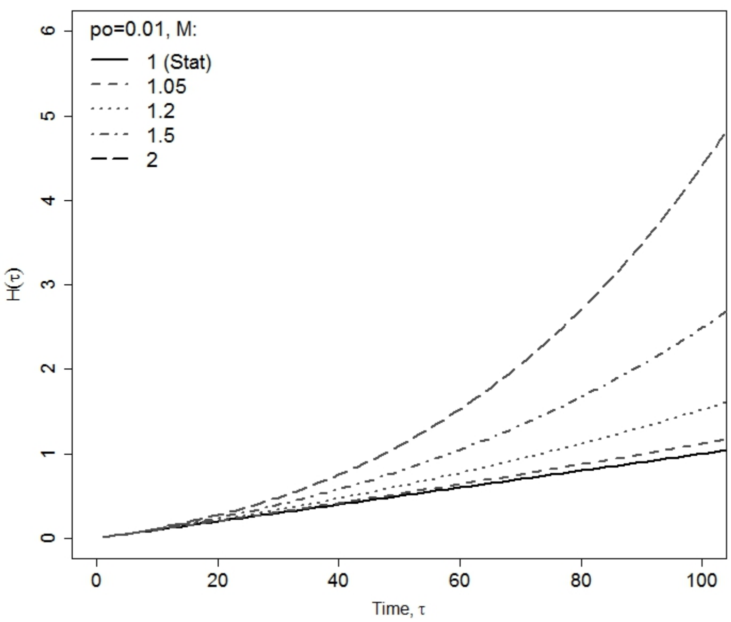


FIGURE 2-3. Cumulative hazard function, H(τ) for the nonstationary EXP1 distribution with p_o = 0.01; lines represent trends parameterized for a range of magnification factors (M = 1, 1.05, 1.2, 1.5, 2).

In the context of flood planning, engineers and managers are concerned with probabilities associated with certain design events, and summary metrics that measure central tendency of these events. The pdf of the time to failure distribution given in equation (38) illustrates important properties of how its shape as trend magnitudes increase. Figure 2-4 (panels [a-c]) illustrates how trends in several event sizes (p_o = 0.1, 0.01, 0.001) impact the behavior of the distribution of the return period in terms of both timing and shape. Figure 2-4 documents that different arrival times are occurring in complex patterns, e.g. the arrival time distribution for a smaller event (p_o = 0.1) is less impacted than a larger event (p_o = 0.001) for the same magnitude trend.

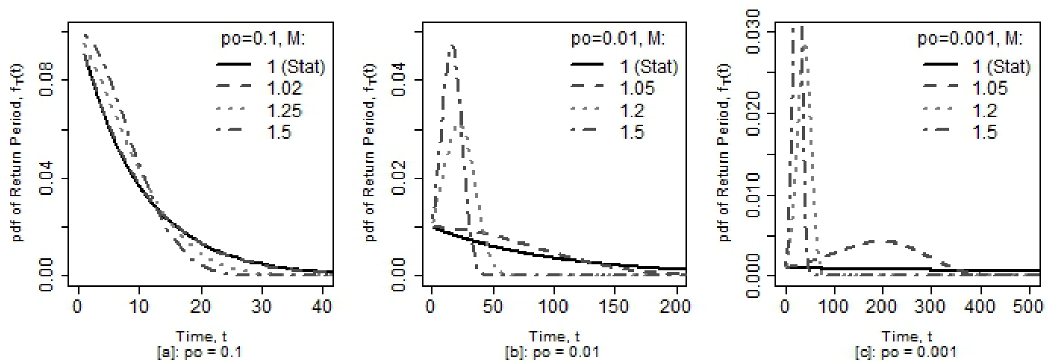


FIGURE 2-4. The pdf of the time to failure distribution for the nonstationary EXP1 model; each figure shows a range of possible trend values by the magnification factor (*M* = 1, 1.02, 1.25, 1.5) and panels are per event size [a] p_o = 0.1, [b] p_o = 0.01, [c] p_o = 0.001.

The HFA corresponding to a nonstationary EXP1 model of flood magnitudes is summarized using the functions h(τ), $S_T(t)$, and H(τ) and provides a window into how the time to failure distribution shape and timing change due to trends, and the type of information we can use for planning under nonstationary conditions. Our results for nonstationary EXP1 flood series based on HFA are analogous to the results of Read and Vogel (2015) for a different nonstationary flood frequency model and using completely different methods of analysis. Their findings and our findings emphasize the importance of extending HFA to other realistic nonstationary flood frequency models.

3 Hazard Function Analysis for Nonstationary 2-Parameter Lognormal (LN2) Floods

In the previous section we derived general results for a nonstationary exponential model of flood series which provides a good representation of the behavior of partial duration series. Here we consider AMS of floods, and thus a more complex pdf of flood series is needed. We employ a nonstationary LN2 model introduced by Vogel et al. [2011] and Prosdocimi et al. [2014] based on evidence that the LN2 distribution is a suitable approximation for representing the pdf of AMS flows [Vogel and Wilson, 1996; IACWD, 1982; Villarini et al., 2009] and that a log-linear (exponential) trend model is simple and effective for approximating a change in the mean of the logarithms of AMS flows through time for thousands of rivers in the USA [Vogel et al., 2011] and the UK [Prosdocimi et al., 2014], particularly in urbanizing areas. The goal of the following experiments is to use Monte-Carlo simulation of AMS flows arising from a nonstationary LN2 model to examine the probabilistic properties of the return period and to apply goodness of fit measures to select a suitable probability distribution for approximation of the survival function associated with the return period. Our application of HFA differs from many previous applications of HFA concerned with modeling and finding a suitable distribution for $h(\tau)$. Instead, we derive $h(\tau)$ from properties of the nonstationary LN2 model and the associated design event, and instead, we seek a parametric distribution to represent the failure time distribution and survival function $S_T(t)$.

Read and Vogel [2015] provide details on steps to derive an expression for p_τ for a LN2 random variable assuming the log-linear trend model

$y_\tau = \ln(x_\tau) = \alpha + \beta \cdot \tau + \varepsilon_\tau$, where an ordinary least squares (OLS) regression yields estimates of the model parameters α and β for the model of the conditional mean of y given by $\mu_{y|\tau} = \alpha + \beta \cdot \tau$. Note that this nonstationary trend model implies a proportional reduction in $\sigma_y{}^2$ compared with stationary conditions that depends on the magnitude of the trend. Read and Vogel [2015] derive a formula for computing the coefficient of variation $C_{x|\tau}$ of the nonstationary series X given by,

$$C_{x|\tau} = \sqrt{(C_x{}^2 + 1)^{(1-\rho^2)} - 1} \qquad (39)$$

Note the two extreme cases of no trend in which case (17) reduces to $C_{x|\tau} = C_x$ and a perfect trend model with $\rho = 1$, which leads to $C_{x|\tau} = 0$.

Again, we employ the decadal magnification factor introduced by Vogel et al. [2011] to parameterize the slope term β into a value with physical meaning, interpreted as the relative change in magnitude of the T-year flood in some future time period $\tau + \Delta\tau$ compared with time τ. Vogel et al. [2011] document that the magnification factor for the nonstationary LN2 model presented here is equivalent to the EXP1 model presented earlier in (35) as $M=\exp(\beta\Delta\tau)$.

Substitution of the log linear trend model $\mu_{y|\tau} = \alpha + \beta \cdot \tau$ into the cdf for a LN2 variable yields an expression for the exceedance probability p_τ in year τ, associated with design discharge X_{p_o}, given by:

$$p_\tau = 1 - \Phi\left[\frac{\ln(x_{p_o}) - \mu_{y|\tau}}{\sigma_{y|\tau}}\right] \qquad (40)$$

where $\sigma_{y|\tau} = \sqrt{\ln(1 + C^2_{x|\tau})}$ and again, X_{p_o} is the fixed design chosen to protect infrastructure over the future planning horizon.

We define h(τ) = p_τ given in (40), which unlike the nonstationary EXP1 model, does not lead to a closed form solution for $S_T(t)$ and H(τ). In order to estimate a survival model, we use Monte-Carlo simulation to generate a large number of failure times, along with their associated average return periods, and then employ goodness of fit measures to select a reasonable probability distribution to represent both $S_T(t)$ as well as the mean survival times (return periods). Our methodology is as follows: given p_o, X_{p_o}, *M*, and C_x, we generated traces of floods (for τ = 1…1000) using the nonstationary LN2 quantile function:

$$x_{p|\tau} = \exp\left[\bar{y} + \beta(\tau - \bar{\tau}) + z_{p_\tau}\sigma_y\sqrt{1-\rho^2}\right] \qquad (41)$$

where $\bar{y} = \frac{1}{n}\sum_{\tau=1}^{n}\ln(x_\tau)$, z_p is the standard normal variate randomly generated by sampling the exceedance probability p_o from a uniform distribution U(0,1); β is obtained from the fact that M=exp($\beta\Delta\tau$), $\Delta\tau$=10, and the standard deviation σ_y is computed from $\sigma_y = \sqrt{\ln(1 + C^2_x)}$ if ρ is assumed to be zero, otherwise from $\sigma_{y|\tau}$. Each of these 1000-year traces results in a failure time, *t*, i.e. the year in which $_{p|\tau}$ > $x_{\tau,stat,}$ and produces a single realization of the return period, t_i. This process is then repeated for *i* = 100,000 experiments to obtain the pdf and L-moments of t_i. We vary the experiments over a realistic range of *M* (1 – 2) and C_x (0.25 – 1.5) values and for large and small events (p_o = 0.001 to 0.1).

Figure 2-5 shows that similar to the EXP1 model, the hazard rate function for the LN2 model is constant under stationary conditions (equal to p_o when M = 1), and increases rapidly towards unity as the trend in the mean increases. The panel in Figure 5 illustrates h(τ) for three events p_o = 0.1, 0.01 and 0.001, with C_x = 1 and a range of trends (note the log scale on x-axis). The shape of h(τ) informs our search for a probability distribution to fit the survival times corresponding to a nonstationary LN2 model, pointing toward one that can accommodate increasing hazards.

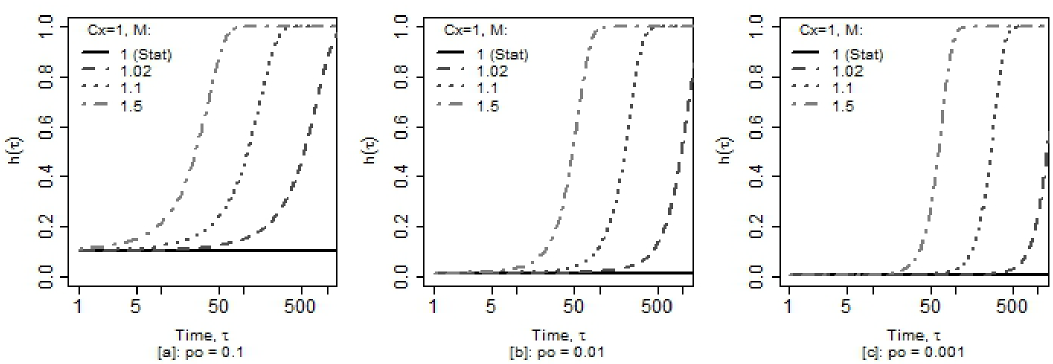


FIGURE 2-5. Hazard function h(τ) = p_τ for a nonstationary LN2 variable with C_x = 1; dotted and dashed lines represent increasing trends (M = 1.01, 1.05, 1.1, 1.5) from stationary (M = 1 solid black) illustrating evolution in each panel for different sized events [a] p_o = 0.1, [b] p_o = 0.01, [a] p_o = 0.001.

In selecting a probability distribution to fit the failure times of the simulated nonstationary LN2 data, probability plot correlation coefficients (PPCC) were employed to assess goodness of fit for potential 2 or 3-parameter probability distributions. The PPCC hypothesis test was introduced by Filliben [1975] and shown to compare favorably with some of the most powerful alternative hypothesis tests for a number of alternative distributional assumptions [Vogel, 1986]. The PPCC tests are now widely used, as evidenced by the recent availability of equations for evaluating the significance level of the resulting test for a wide class of probability distributions [see Heo et al., 2008]. Figure 6 shows box plots illustrating the PPCC values over a range of distributions corresponding to the full range of M, C_x, and p_o parameters described in the experimental design. Figure 6 indicates that the distribution of the failure times is no longer well approximated by an exponential pdf, as it is under stationary conditions. Figure 2-6 documents that among all the distributions considered, the Weibull-2 distribution provides a best overall approximation of the distribution of return periods with PPCC values which ranged from 0.9970 to 0.9999, and a median value of 0.9979. .

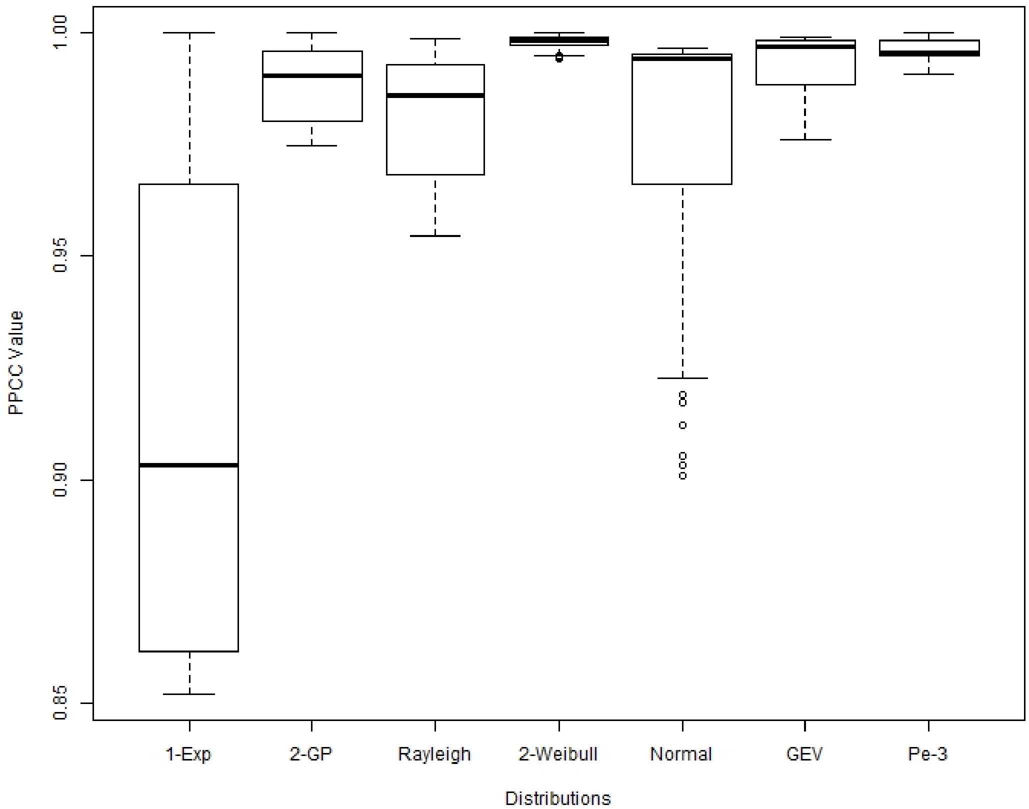


FIGURE 2-6. Boxplots of PPCC values that illustrate fit of nonstationary LN2 simulated failure time data for a range of parameters (*M* : 1 – 2; $C_{x|}$= 0.25 – 1.5; p_o = 0.001, 0.01, 0.1) to a range of models including the Exponential (EXP1), Generalized Pareto (GP2), Rayleigh, Weibull, Normal, Generalized Extreme Value (GEV) and Pearson type III (PE3) distributions.

Given our results in Figure 2-6, we further explore the Weibull-2 model as a distribution for describing the time to failure distribution corresponding to the nonstationary LN2 model over a feasible range of trends (*M*), event sizes of interest (p_o), and hydrologic variability (C_x).

3.1 *Characterization of Weibull-2 model for LN2 time to failure distribution*

Interestingly, in the HFA literature, the two-parameter Weibull model is one of the most common hazard models applied to characterize many different types of data. This is due to its ability to model both increasing and decreasing hazards due to its flexibility [Mudholkar et al., 1996; Klein and Moeschberger, 1997; Wienke, 2010]. Examples of hazard types that give rise to Weibull-2 survival models include software reliability (i.e. time between failures of software) [Pham and Pham, 2000], bank failure rates [Evrensel, 2008], and occurrences of earthquakes from crustal strain [Hagiwara, 1974]. In addition, several researchers have proposed a modified Weibull survival models, for example the Beta-Weibull to characterize breast cancer occurrence rates [Wahed et al., 2009].

In the following section we show how the cdf of the time to failure associated with a design based on a series of AMS floods from the nonstationary LN2 model can be linked with a Weibull-2 survival model, using an empirical analysis. Consider a two parameter Weibull cdf of the random variable time, *T*:

$$F_T(t) = 1 - \exp\left(\left[-\frac{t}{\sigma}\right]^{\kappa}\right) \tag{42}$$

where the scale and shape parameter are given by σ and κ, respectively. The survival function is easily written as $S_T(t) = 1-F_T(t)$ and the corresponding hazard function is given by Mudholkar et al. [1996] as:

$$h(\tau = t) = \frac{\kappa}{\sigma}\left[\frac{t}{\sigma}\right]^{(\kappa-1)} \tag{43}$$

O'Quigley and Roberts [1980] use the Weibull quantile function to derive the 2-parameter Weibull survival model as

$$\log[-\log(S_T(t))] = \log(\sigma) + \kappa \log(t) \qquad (44)$$

The Weibull survival model provides a good fit to data if the a plot of the left-hand side of (44) vs. log(*t*) produces a straight line; then the slope = *κ* and intercept = *σ* [Vogel and Kroll, 1989].

Here we need to estimate the shape (*κ*) and scale (*σ*) parameters of the Weibull-2 distribution from the true LN2 failure time data in order to develop a plausible model for $S_T(t)$. Using the *survival* package in R [Therneau, 2015], we compute the maximum likelihood estimates for *κ* and *σ*; we then develop regression models to predict the *κ* and *σ* Weibull parameters from the physical/design parameters of the hydrologic system, i.e. *κ* (M, C_x, p_o) and *σ* (M, C_x, p_o). Regression analyses resulted in the following approximations:

$$\sigma = \frac{1}{-0.0249 + 0.0489 \cdot M - 0.0194 \cdot C_x + 1.0882 \cdot p_o} \qquad (45)$$

$$\kappa = e^{-0.3252} M^{0.6583} C_x^{-0.1196} p_o^{-0.1858} \qquad (46)$$

To test how well the regression models in (45) and (46) predict the true values of *σ* and *κ*, we computed the Nash-Sutcliffe Efficiencies (NSE) between the true values of the parameters and the regression estimates of the parameters as 0.796 and 0.968, respectively. The adjusted R^2 values for the *κ* and *σ* parameter models are 0.961 and 0.987, respectively. The reported values of NSE are obtained in cross validation using the leave-one-out method. Based on the high NSE and adjusted R^2 of (45) and (46) we are confident in using these equations to approximate values of κ and σ based on a given set of hydrologic parameters, M, C_x, p_o. Below we describe the utility of these estimates for hydrologic planning and design.

With the relationships in (45) and (46) we can compute $S_T(t)$ and $H(\tau)$ for any system, given M, C_x, p_o; several sample traces of the resulting $S_T(t)$ and $H(\tau)$ functions are shown in Figures 2-7 and 2-8, respectively. Recall that the cumulative hazard function $H(\tau)$ can be computed using the theoretical relationship between $S_T(t)$ and $h(\tau)$ shown in eqn. (26), resulting in $H(\tau) = \left[\frac{\tau}{\sigma}\right]^{\kappa}$ for the Weibull-2 pdf. As expected from the nature of $h(\tau)$ as an increasing function, $H_T(t)$ increases through time, more so for higher trends. The $H(\tau)$ function is yet another way to interpret the expected number of events within a certain period of time, and an easy way to visually compare expected exceedances under stationary conditions to those under nonstationary conditions.

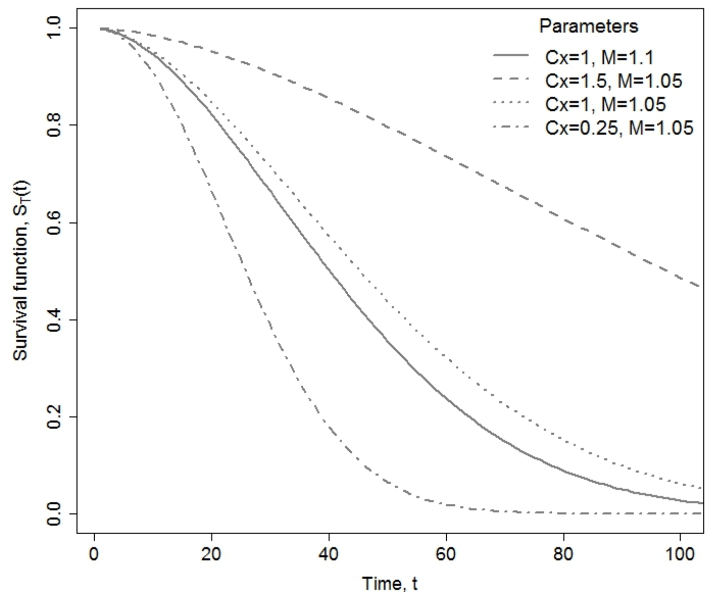


FIGURE 2-7. Survival function of simulated time to failure data corresponding to a nonstationary LN2 model; traces illustrate parameter subsets for the p_o = 0.01 event

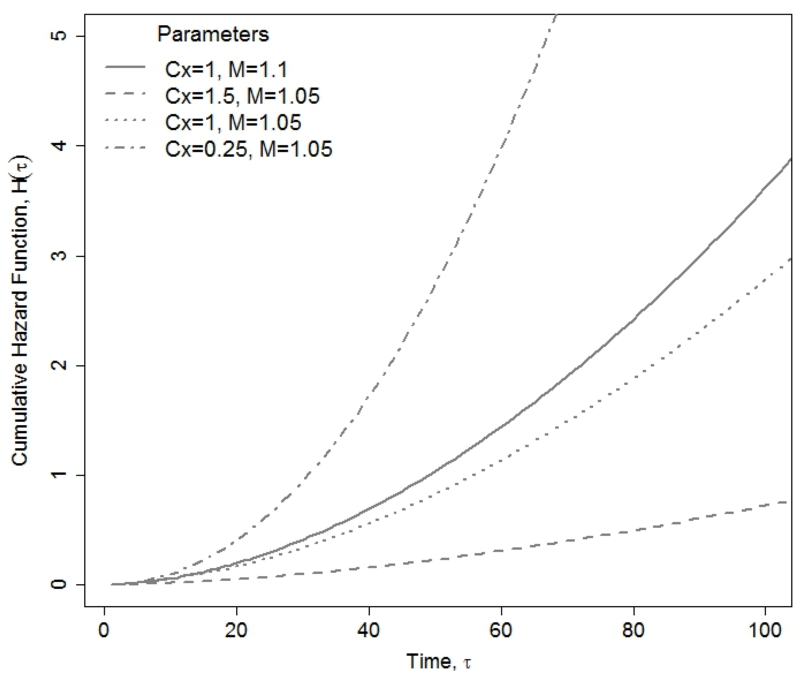


FIGURE 2-8. Cumulative hazard function H(τ) for simulated time to failure data from a nonstationary LN2 model; traces illustrate parameter subsets for the p_o = 0.01 event

With estimates of the σ and κ obtained from (45) and (46), we can compute the average return period, E[T], and reliability based on probability theory. The expected value of a Weibull-2 distribution is:

$$E[T] = \sigma \cdot \Gamma(\frac{1}{\kappa} + 1) \tag{47}$$

Where Γ() is the Gamma function. A regression estimate of E[T] is easily obtained from (47) by using the regression equations for σ and κ in (45) and (46). As a comparison of central tendency, we can compute the average return period of *T* when *X* follows a nonstationary LN2 model, in three independent ways: (1) calculation of the mean of the simulated failure times for a nonstationary LN2 model for a given set of parameters, (2) an exact result from T_1 in (27) where p_τ in (35) is derived from the nonstationary LN2 model, and (3) computation of (47) with estimates of the κ and σ obtained from (45) and (46) respectively. Figure 2-9 illustrates the exact average return period versus the regression estimates from the Weibull-2 model computed from (47) with regression estimates of the σ and κ obtained from (45) and (46), for the entire range of M, C_x and p_o values.

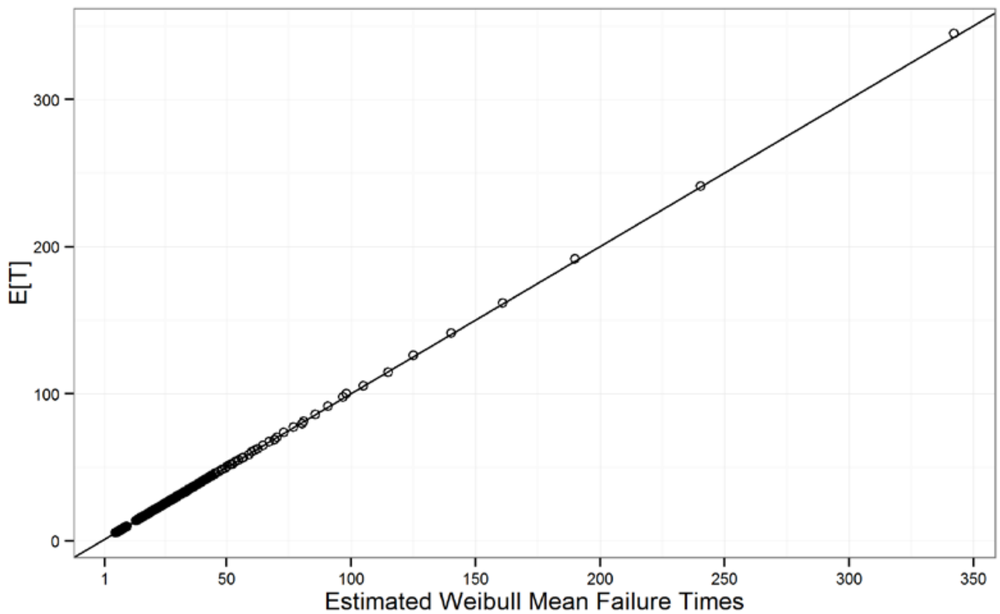


FIGURE 2-9. Comparison of exact values of average return period from (27) and (35) with regression estimates from a fitted Weibull-2 model in (45), (46) and (47). The data range (points) represent a reasonable range of parameters from LN2 distributed flood flows (M = 1 – 2; C_x= 0.25 – 1.5; p_o = 0.001, 0.01, 0.1).

We note that average return periods, T_1, are extremely well approximated by the Weibull-2 model which is fit using regression estimates of its model parameters, and that in practice, it may be easier to estimate the Weibull parameters and use $S_T(t)$ to describe the likelihood of experiencing an event than using (27), which may require difficult derivations of p_τ. Thus our result could be quite practical and useful in flood planning and we recommend that future studies develop similar approximations for wider range of models of nonstationary flood series.

Read and Vogel [2015] and others [see Bayazit, 2015; Serinaldi, 2015; Sivapalan and Samuel, 2009] document concerns with use of the expected return period in practice and instead, recommend use of the reliability of a system over a planning horizon. For a given system with physical and design parameters M, C_x and p_o, "exact" values of reliability are computed by using the expression given in Read and Vogel, 2015 and others: $\mathrm{Re}\,l_n = \prod_{i=1}^{n}(1-p_i)$. Regression estimates Weibull-2 of $S_T(t)$ in each year, are computed from the Weibull-2 model by taking the inverse of (42) with estimates of σ and κ obtained from (45) and (46). Figure 10 compares reliability values over a planning horizon for the exact (solid black lines) and Weibull-2 regression estimates (solid grey lines) for a range of experiments: [a] trends: M, [b] C_x, and [c] p_o. The Weibull-2 estimates based on regression estimates of the model parameters reproduces the exact reliabilities very well for a range of increasing trends and event sizes (panels [a] and [b] in Figure 10). The greatest difference between exact and estimated reliability values is ± 0.04 years, occurring when C_x is higher.

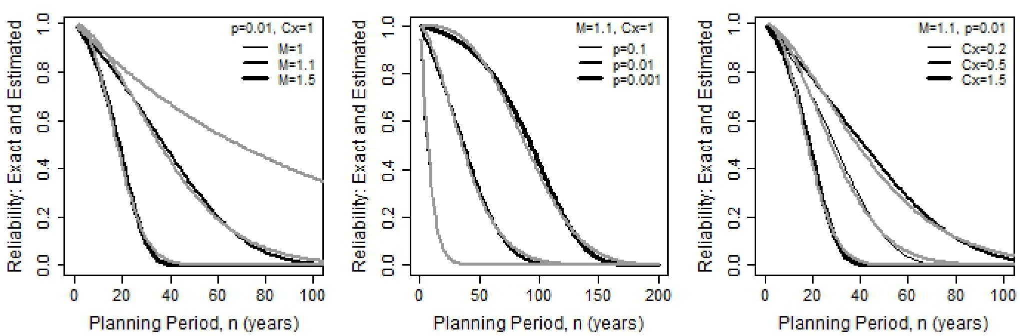


FIGURE 2-10. Comparison of nonstationary reliability values between exact (black lines) and Weibull-2 regression model estimates (grey lines) for a range of experimental values: [a] trends, M = 1 (stationary), 1.1, 1.5; [b] p_o = 0.1, 0.01, 0.001; [c] C_x = 0.2, 0.5, 1.5

4 Conclusions

The purpose of this paper is to introduce the field of hazard function analysis (HFA) to flood planning under nonstationarity. When trends are present in historic flood series or predicted in the future due to climate, urbanization or other causes, the assumption of stationary probabilities of exceedance and the associated distribution of the time to failure fall into serious question. This has important implications on hydrologic design and the metrics by which we rely on to build infrastructure to protect against floods. HFA provides a set of tools for analyzing flood series whose probabilities of exceeding a design flood (threshold) are changing in time. HFA has served numerous other fields well over the past few decades, thus we expect this initial study to result in numerous extensions in hydrology and water resource planning.

We have presented two approaches to the application of HFA to flood frequency analysis. Our first example provides a relatively simple yet relevant and application of HFA to flood series which arise from a nonstationary exponential model applicable for peaks-over-threshold flood frequency analysis. That simple example, resulted in elegant, and general analytical results. A second example provides an application of HFA to a more realistic and more complicated nonstationary LN2 model. Here, even for this relatively simple nonstationary model, analytical results could not be obtained, and instead, we resorted to a Monte-Carlo analysis to simulated failure time data, applicable for flood frequency analysis with AMS flows.

With the simple nonstationary EXP1 model of PDS, we demonstrated how the properties of a hydrologic variable X can be explicitly linked to the hazard rate, survival function, and cumulative hazard function associated with associated return periods, T. Since an EXP1 model is limited in its relevance to hydrology beyond a POT analysis, we also performed an analogous HFA to a nonstationary

LN2 model of AMS. Due to the complexity of the nonstationary LN2 model, numerical approximations were needed for this analysis. Using the probability plot correlation coefficient (PPCC) as a distributional goodness of fit measure, the Weibull-2-2model was identified as a suitable model for representing $S_T(t)$ corresponding to a nonstationary LN2 model of AMS flows; this finding is in line with the widespread use of the Weibull survival model in other fields [O'Quigley and Roberts, 1980; Mudholkar, et al., 1996; Cleves, 2008]. Weibull-2We then developed a regression model to estimate the Weibull shape and scale parameters based on known hydrologic system parameters and design requirements (M, C_x, p_o). We show how these estimated shape and scale parameters can be employed to calculate important metrics for hydrologic planning, such as the average return period E[T], and reliability, achieving very reasonable estimates without complex numeric computations required using alternative approaches (see Read and Vogel, 2015).

The primary goals of this paper were to introduce HFA to the water resources community and to encourage other extensions to our analysis to enable more efficient water resource systems planning under both uncertainty and nonstationarity. We envision a great deal of future work on this topic, including the application of HFA to other reasonable nonstationary models of AMS flood series besides those presented here, such as Gumbel, GEV, and LP3, which have been recommended for nonstationary flood frequency analysis [Villarini and Smith, 2010; Salas and Obeysekera, 2014; Serinaldi and Kilsby, 2015]. We also expect future extensions to our example for a nonstationary EXP1 model of PDS floods which derived for analytical estimates of the hazard rate function $h(\tau)$, the expected return period and the survival function $S_T(t)$. For example, the natural extension to that example would be to apply HFA to PDS series which follow a nonstationary Generalized Pareto model, and to consider alternate forms of nonstationarity to the exponential model considered here. For cases that PH models are appropriate, such as testing whether certain climatic or human activities act as co-variates to influence a particular hydrologic process, we see further work that builds on Smith and Karr [1986], Maia and Meinke [2010] and Villarini et al. [2012] to identify semi-parametric Cox models for characterizing these nonstationarities through time.

We have shown how hazard function analysis can provide theory for examining nonstationary processes where the variables of concern are a design event and its associated time to a failure event. . This field has benefited from researchers in many disciplines who have been asking similar types of questions, working with a range of data types, and advancing analysis methods that we can apply to hydrology. We can use hazard analysis to independently compare measures of central tendency, for example computing the average return period E[T] of an event arising from a nonstationary probabilistic process.

While outside the scope of this paper, the work presented here further draws attention to the need for a risk-based decision analysis to choose an “optimal” design event given tremendous uncertainty about future nonstationarities [see Rosner et al., 2014 for a related example]. Future work should consider deriving more general guidance for design decisions under nonstationarity using HFA coupled with the risk-based framework introduced by Rosner et al. [2014] for nonstationary AMS series.

5 Acknowledgment

The authors would like to acknowledge support in part by an appointment to the U.S. Army Corps of Engineers (USACE) Research Participation Program administered by the Oak Ridge Institute for Science and Education (ORISE) through an interagency agreement between the U.S. Department of Energy (DOE) and the U.S. Army Corps of Engineers (USACE). ORISE is managed by ORAU under DOE contract number DE-AC05-06OR23100. All opinions expressed in this paper are the author's and do not necessarily reflect the policies and views of USACE, DOE or ORAU/ORISE. All data are freely available and analyzed with the R statistical software package version 3.1.1.

6 References

Asquith, W.H., 2011, Distributional analysis with L-moment statistics using the R environment for statistical computing: Createspace Independent Publishing Platform, ISBN 978–146350841–8

Bayazit, M. (2015), Nonstationarity of Hydrological Records and Recent Trends in Trend Analysis: A State-of-the-art Review. *Environmental Processes*, 1-16.

Bean, M. A. (2001), Probability: the science of uncertainty with applications to investments, insurance, and engineering (Vol. 6). American Mathematical Soc.

Chow, V. T. (1965), Handbook of Applied Hydrology, McGraw Hill Book Co., N.Y.

Cleves, M. (2008), An introduction to survival analysis using Stata. Stata Press.

Cohn, T. A., and H. F. Lins, (2005), Nature’s style: Naturally trendy. Geophysical Research Letters, 32(23): 1-5, doi: 10.1029/2005GL024476

Condon, L. E., S. Gangopadhyay, and T. Pruitt (2015), Climate change and non stationary flood risk for the upper Truckee River basin, Hydrology and Earth System Sciences, 19(1), 159-175, doi:10.5194/hess-19-159-2015

Cooley, D. (2009), Extreme value analysis and the study of climate change, a commentary on Wigley 1988, Climatic Change 97, 77-83, doi: 10.1007/s10584-009-9627-x

Cooley, D. (2013), Return periods and return levels under climate change, in Extremes in a Changing Climate: Detection, Analysis and Uncertainty, edited by A. AghaKouchak etal., Springer Science + Business media, Dordrecht

Cox, D. R. (1972). Regression models and life-tables (with discussion). J. R. Statist. Soc. B 34, 187-220.

Cunderlik, J. M., and Burn, D. H. (2004). Linkages between regional trends in monthly maximum flows and selected climatic variables. Journal of Hydrologic Engineering, *9*(4), 246-256. doi: 10.1061/(ASCE)1084-0699(2004)9:4(246)

Evrensel, A.Y. (2008) Banking crisis and financial structure: A survival-time analysis, International Review of Economics & Finance, *17*(4), 589-602, ISSN 1059-0560, doi:10.1016/j.iref.2007.07.002.

Finkelstein, M. (2008). Failure rate modelling for reliability and risk. Springer Science & Business Media.

Futter, M. R., Mawdsley, J. A., & Metcalfe, A. V. (1991). Short-term flood risk prediction: A comparison of the Cox Regression Model and a Conditional Distribution Model. *Water Resources Research*, *27*(7), 1649-1656. doi: 10.1029/91WR00692

Gumbel, E. J. (1941). The return period of flood flows. Annals of Mathematical Statistics, *12*(2), 163–190.

Gumbel, E.J. (1958). *Statistics of Extremes.* Columbia University Press, New York

Hagiwara, Y. (1974) Probability of earthquake occurrence as obtained from a Weibull distribution analysis of crustal strain, Tectonophysics, 23:(3), 1974, Pages 313-318, ISSN 0040-1951, doi:10.1016/0040-1951(74)90030-4

Hensher, D. A., & Mannering, F. L. (1994). Hazard-based duration models and their application to transport analysis. *Transport Reviews*, *14*(1), 63-82.

Heo, J. H., Y. W. Kho, H. Shin, S. Kim, and T. Kim (2008). Regression equations of probability plot correlation coefficient test statistics from several probability distributions. *Journal of Hydrology*, *355*(1), 1-15. doi: 10.1016/j.jhydrol.2008.01.027

Hosking, J.R.M., (1990), L-moments—Analysis and estimation of distributions using linear combinations of order statistics: Journal of the Royal Statistical Society, Series B, v. 52, pp. 105–124, http://www.jstor.org/stable/2345653

IACWD (Interagency Committee on Water Data) (1982), Guidelines for determining flood flow frequency, Bulletin 17B, Office of Water Data Coordination, U.S. Geological Survey, Reston,VA

Katz, R. W., and B. G. Brown (1992), Extreme events in a changing climate: variability is more important than averages. *Climatic Change*, *21*(3), 289–302. doi: 10.1007/BF00139728

Katz, R. W., M. B. Parlange, and P. Naveau (2002), Statistics of extremes in hydrology. *Advances in Water Resources*, *25*(8), 1287-1304. doi: 10.1016/S0309-1708(02)00056-8

Klein, J. P., & Moeschberger, M. L. (1997). Survival analysis: techniques for censored and truncated data. Springer Science & Business Media.

Kottegoda, N. T., & Rosso, R. (2008), *Applied statistics for civil and environmental engineers*. Malden, MA: Blackwell.

Lawless, J. F. (2011). *Statistical models and methods for lifetime data* (Vol. 362). John Wiley & Sons

Lee, K. S., J. Sadeghipour, and J. A. Dracup (1986), An approach for frequency analysis of multiyear drought durations. *Water Resources Research*, *22*(5), 655-662. doi: 10.1029/WR022i005p00655

Maia, A. H. and H. Meinke (2010), Probabilistic methods for seasonal forecasting in a changing climate: Cox-type regression models. *International journal of Climatology*, *30*(15), 2277-2288. doi:10.1002/joc.2042

Matalas, N. C. (2012), Comment on the announced death of stationarity, *J. Water Resour. Plann.Manage.*, *138*, 311–312. doi: 10.1061/(ASCE)WR.1943 5452.0000215

McCabe, G. J., and D.M. Wolock (2014). Spatial and temporal patterns in conterminous United States streamflow characteristics. *Geophysical Research Letters*, *41*(19), 6889-6897. doi: 10.1002/2014GL061980

Merz, B., Vorogushyn, S., Uhlemann, S., Delgado, J., & Hundecha, Y. (2012). HESS Opinions "More efforts and scientific rigour are needed to attribute trends in flood time series" *Hydrology and Earth System Sciences*, *16*(5), 1379-1387. doi:10.5194/hess16-1379-2012

Milly, P. C. D., J. Betancourt, M. Falkenmark, R.M. Hirsch, Z.W. Kundzewicz, D.P. Lettenmair and R.J. Stouffer (2008), Stationarity is dead: wither water management. *Science*, *319*, 73-574.

Mondal, A. & Mujumdar, P. P., 2015. Return levels of hydrologic droughts under climate change. *Advances in Water Resources*, Volume 75, pp. 67-79. doi:10.1016/j.advwatres.2014.11.005

Mudholkar, G. S., D. K. Srivastava, and G. D. Kollia (1996), A generalization of the Weibull distribution with application to the analysis of survival data. *Journal of the American Statistical Association*, *91*(436), 1575-1583.

Obeysekera, J., and J. Park (2013), Scenario-Based Projection of Extreme Sea Levels. *Journal of Coastal Research*, *286*, 1–7. doi:10.2112/JCOASTRES-D-12-00127.1

O'Quigley, J. and A. Roberts (1980), Weibull: a regression model for survival time studies. *Computer programs in biomedicine*, *12*(1), 14-18.

Olsen, J. R., J. H. Lambert and Y. Y. Haimes (1998), Risk of extreme events under nonstationary conditions. *Risk Analysis*, *18*(4), 497–510.

Parey, S., Hoang, T. T. H., & Dacunha-Castelle, D. (2010). Different ways to compute temperature return levels in the climate change context. *Environmetrics*, *21*(7-8), 698–718. doi:10.1002/env.1060

Parey, S., Malek, F., Laurent, C., & Dacunha-Castelle, D. (2007). Trends and climate evolution: Statistical approach for very high temperatures in France. *Climatic Change*, *81*(3-4), 331–352. doi:10.1007/s10584-006-9116-4

Pham, L., & Pham, H. (2000) Software reliability models with time-dependent hazard function based on Bayesian approach. Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions, 30(1), 25-35

Prosdocimi, I., Kjeldsen, T. R., & Svensson, C. (2014). Non-stationarity in annual and seasonal series of peak flow and precipitation in the UK. *Natural*

Hazards and Earth System Sciences, *14*, 1125-1144. doi:10.5194/nhess-14-1125-2014

Read, L., and R. M. Vogel (2015), Reliability, return periods, and risk under nonstationarity, *Water Resour. Res.*, *51*, doi:10.1002/2015WR017089

Rootzén, H., & Katz, R. W. (2013). Design Life Level: Quantifying risk in a changing climate: Design Life Level. *Water Resources Research*, *49*(9), 5964–5972. doi:10.1002/wrcr.20425

Salas, J.D. and J. Obeysekera (2014), Revisiting the Concepts of Return Period and Risk for Nonstationary Hydrologic Extreme Events, J. Hydrol. Eng., 19(3), 554–568, doi: 10.1061/(ASCE)HE.1943-5584.0000820

Serinaldi, F. (2015), Dismissing return periods! Stochastic Environmental Research and Risk Assessment, 1-11, doi: 10.1007/s00477-014-0916-1

Serinaldi, F. and C.G. Kilsby (2015), Stationarity is undead: uncertainty dominates the distribution of extremes, *Advances in Water Resources*, 77, 17-36, doi:10.1016/j.advwatres.2014.12.013

Sivapalan, M. and Samuel, J. M. (2009), Transcending limitations of stationarity and the return period: process-based approach to flood estimation and risk assessment. Hydrol. Process., 23: 1671–1675. doi: 10.1002/hyp.7292

Smith, J. A., & Karr, A. F. (1986). Flood frequency analysis using the Cox regression model. *Water Resources Research*, *22*(6), 890-896.

Stedinger, J.R., R.M. Vogel and E. Foufoula-Georgiou (1993), Frequency Analysis of Extreme Events in The Handbook of Hydrology, edited by David R. Maidment, McGraw-Hill Book Company

Therneau T (2015). *A Package for Survival Analysis in S*. version 2.38, http://CRAN.R-project.org/package=survival.

Todorovic, P., and E. Zelenhasic (1970), A stochastic model for flood analysis, *Water Resources Research*, 6(6), 1641-1648.

Villarini, G., J. A. Smith, F. Serinaldi, J. Bales, P. D. Bates, and W. F. Krajewski (2009). Flood frequency analysis for nonstationary annual peak records in an urban drainage basin. *Advances in Water Resources*, *32*(8), 1255-1266.

Villarini, G. and J. A. Smith (2010). Flood peak distributions for the eastern United States. *Water Resources Research*, *46*(6). doi: 10.1029/2009WR008395

Vogel, R. M. (1986). The probability plot correlation coefficient test for the normal, lognormal, and Gumbel distributional hypotheses. *Water Resources Research*, *22*(4), 587-590. doi: 10.1029/WR022i004p00587

Vogel, R. M., and C. N. Kroll (1989). Low-Flow Frequency Analysis Using Probability Plot Correlation Coefficients. *J. Water Resour. Plann. Manage.*, 115(3), 338-357. doi: 10.1061/(ASCE)0733-9496(1989)115:3(338)

Vogel, R.M. and I. Wilson, (1996). The Probability Distribution of Annual Maximum, Minimum and Average Streamflow in the United States, *Journal of Hydrologic Engineering*, ASCE, 1(2), pp.69-76

Vogel, R.M., C. Yaindl, and M. Walter (2011), Nonstationarity: Flood Magnification and Recurrence Reduction Factors in the United States, *Journal of American Water Resources Association*, 47(3) 464-474, doi: 10.1111/j.1752-1688.2011.00541.x.

Wahed, A. S., Luong, T. M. and Jeong, J.-H. (2009) A new generalization of Weibull distribution with application to a breast cancer data set. *Statist. Med.*, 28: 2077–2094. doi: 10.1002/sim.3598

Wang, D., A. D. Hutson, and J. C. Miecznikowski (2010). L-moment estimation for parametric survival models given censored data. *Statistical Methodology*, *7*(6), 655–667. doi:10.1016/j.stamet.2010.07.002

Westra, S., L. V. Alexander, and F. W. Zwiers (2013). Global increasing trends in annual maximum daily precipitation. *Journal of Climate*, *26*(11), 3904-3918. doi: 10.1175/JCLI-D-12-00502.1

Wienke, A. (2010). Frailty models in survival analysis. CRC Press.

Wigley, T. M. L. (2009). The effect of changing climate on the frequency of absolute extreme events. *Climatic Change*, *97*(1-2), 67–76. doi:10.1007/s10584-009-9654-7

Zhong, M. and J. D. Hunt (2010). Exploring best-fit hazard functions and lifetime regression models for urban weekend activities: case study. *Journal of Transportation Engineering, Am. Soc. Civ. Eng., 61*, 62.

Chapter 3: Hazard function theory for nonstationary natural hazards

Journal reference: Read, L.K. and R. M. Vogel (2015, submitted), Hazard function theory for nonstationary natural hazards, Nat. Hazards Earth Syst. Sci

ABSTRACT

Impact from natural hazards is a shared global problem that causes tremendous loss of life and property, economic cost, and damage to the environment. Increasingly, many natural processes show evidence of nonstationary behavior including wind speeds, landslides, wildfires, precipitation, streamflow, sea levels, and earthquakes. Traditional probabilistic analysis of natural hazards based on peaks over threshold (POT) generally assumes stationarity in the magnitudes and arrivals of events, i.e. that the probability of exceedance of some critical event is constant through time. Given increasing evidence of trends in natural hazards, new methods are needed to characterize their probabilistic behavior. The well-developed field of hazard function analysis (HFA) is ideally suited to this problem because its primary goal is to describe changes in the exceedance probability of an event over time. HFA is widely used in medicine, manufacturing, actuarial statistics, reliability engineering, economics, and elsewhere. HFA provides a rich theory to relate the natural hazard event series (X) with its failure time series (T), enabling computation of corresponding average return periods, risk and reliabilities associated with nonstationary event series. This work investigates the suitability of HFA to characterize nonstationary natural hazards whose POT magnitudes are assumed to follow the widely applied Generalized Pareto (GP) model. We derive the hazard function for this case and demonstrate how metrics such as reliability and average return period are impacted by nonstationarity and discuss the implications for planning and design. Our theoretical analysis linking hazard event series X, with corresponding failure time series T, should have

application to a wide class of natural hazards with rich opportunities for future extensions.

1. Introduction

Studies from the natural hazards literature indicate that certain hazards show evidence of nonstationary behavior through trends in magnitudes over time. Such trends in the magnitudes of natural hazards have been attributed to changes in climate patterns, e.g. for wind speeds (de Winter et al., 2013), wildfires (Liu et al., 2010), typhoons (Kim et al., 2015), and extreme precipitation (Roth et al., 2014), and also linked directly to human activities, e.g. increase in earthquakes from wastewater injection associated with hydraulic fracturing (Ellsworth, 2013). Other natural hazards such as floods resulting from streamflow (Di Baldassarre et al., 2010; Vogel et al., 2011) and from sea level rise (Obeysekera and Park, 2012) may be a result of a myriad of anthropogenic influences including climate change, land use change and even natural processes such as land subsidence. In addition to evidence of magnitude changes of many natural hazards, recent reports document a corresponding surge in human exposure to natural hazards (Blaikie et al., 2014), along with a 14-fold increase in economic damages due to natural disasters since 1950 (Guha-Sapir et al., 2004). Given evidence of trends and the consequent expected growth in devastating impacts from natural hazards across the world, new methods are needed to characterize their probabilistic behavior and communicate event likelihood and the risk of failure associated with infrastructure designed to protect society against such events. The existing rich and evolving field of hazard function analysis (HFA) is ideally suited to problems in which the probability of an event is changing over time, yet to our knowledge has not been applied to natural hazards. Our primary goal is to apply HFA to characterize the likelihood of nonstationary natural hazards and to better understand the expected time until the next natural hazard event occurrence for design and planning purposes.

Probabilistic analysis of natural hazards normally takes one of two approaches in fitting a probability distribution to hazard event data series. As we summarize in Table 3-1, a very common approach to probabilistic analyses of natural hazards employs the peaks over threshold (POT) dataset, also commonly referred to as the partial duration series (PDS), to characterize exceedances above some defined magnitude (threshold) that occur over an interval of time. The second approach is to fit a probability distribution to the annual maximum series (AMS), common practice in hydrology (Gumbel, 2012; Stedinger et al., 1993) and also appropriate for earthquakes (Thompson et al., 2007) and many other processes (Beirlant et al., 2006; Coles et al., 2001). Hydrologists have extensively studied the theoretical relations between POT and AMS methods (Stedinger et al., 1993; Todorovic, 1978) and compared the two for characterizing the probabilities of flood events (Madsen et al., 1997). In general the POT approach appears to provide a larger dataset to draw from, however as the threshold of exceedance used to define the POT series is lowered, the series of maxima begin to exhibit temporal dependence which complicates the probabilistic analysis considerably. Further complexities arise in POT analyses due to subjectivity of the threshold and difficulty in confirmation of independence between events (Stedinger et al., 1993).

Still the POT method is often the most widely used approach for many natural phenomena either because there is an intuitive choice for the threshold of exceedance, as in the case of earthquakes where the magnitude of completeness is often selected, or because the analyst wishes to maximize the use of data, and does not always understand the tradeoff between the length of the POT series and the inherent increase in its temporal dependence structure and the associated consequences. Due to the wide-spread application of the POT method, a substantial number of textbooks and articles have studied methods for minimizing the difficulty of implementation, with emphasis on the subjectivity of threshold selection and evaluation of independence of events (Davison and Smith, 1990; Smith, 2003). This study assumes a POT approach is taken as is so often the case for the probabilistic analysis of magnitudes of natural hazards including extreme winds (Palutikof et al., 1999), earthquakes (Pisarenko and Sornette, 2003), wildfires (Schoenberg et al., 2003), and wave heights (Lopatoukhin et al., 2000), among others. Future work will extend our analyses to AMS series as has been attempted recently by Read and Vogel (2015ab) for floods.

1.1 Application of general POT model in natural hazards

Here we focus on the POT approach to characterize exceedance events and their frequencies for natural hazards. The GP distribution, a generalization of the exponential distribution, was first introduced as a limiting distribution for modelling high level exceedances by Pickands (1975), and later developed by

Hosking and Wallis (1987), who discuss its theory and an application for extreme floods. Davison and Smith (1990) provide techniques for dealing with serially dependent and seasonal data for modeling exceedances above a threshold with the GP distribution. Hosking and Wallis (1987) discuss the fundamental properties of the GP distribution. See Pickands (1975) and Davison and Smith (1990) for further theoretical background on the application of the GP distribution for modeling POT series. In general, the GP distribution arises for variables whose distributions are heavy-tailed, in cases where the lighter-tailed exponential distribution does not provide sufficient robustness (Hosking and Wallis, 1987).

The GP distribution has been widely applied to natural hazards (see Table 3-1) and in many other fields including financial risk, insurance, and other environmental problems (Smith, 2003) to characterize the magnitude of exceedances above a threshold. Hosking and Wallis (1987), Stedinger et al. (1993) and others show that if the time between the peaks corresponding to a POT follow a Poisson distribution and the POT magnitudes follow an exponential distribution, then the AMS follow a Gumbel distribution. Similarly, Hosking and Wallis (1987) and others show that if time between the peaks of the POT series are Poisson and the POT magnitudes follow a 2-parameter GP (GP2) distribution then the AMS follows a Generalized Extreme Value (GEV) distribution.

TABLE 3-1. Summary of natural hazards employing Poisson-GP model

Natural Hazard	**References**	**Evidence of nonstationarity in process**	**Nonstationary** POT **model formulated?**
Floods	Todorovic, 1978; Madsen et al., 1997	Bayazit, 2015 for a review	Yes - Strupczewski et al., 2001; Villarini et al., 2012
Earthquakes	Gutenberg and Richter, 1954; Utsu, 1999	Ellsworth, 2013 for a review	No
Extreme rainfall	Sugahara et al., 2009; Bonnin et al., 2011	Begueria et al., 2011; Tramblay et al., 2013; Roth et al., 2014; Sugahara et al., 2009	Yes, in all references
Wildfires	Holmes et al., 2008	Liu et al., 2010	No
Extreme wind speeds	Palutikof et al., 1999 for review; Jagger and Elsner, 2006	Young et al., 2011; Pryor and Barthelmie, 2010 for review	Young et al., 2011
Wave height (proxy for storm surge)	Davison and Smith, 1990	Mendez et al., 2006; Young et al., 2011; Ruggerio et al., 2010	Yes - in Mendez et al., 2006; and Ruggerio et al., 2010
Daily max and min temperature	Waylen, 1988	Keellings and Waylen, 2014	Yes, but not derived in text
Ecological extremes	Katz et al., 2005	Katz et al., 2005 (sediment yield)	Yes

Table 3-1 lists many natural hazards problems that apply the Poisson-GP model for their probabilistic analysis. Here we consider a POT that follows the GP model (the exponential distribution is a special case of the GP model when the limit of the shape parameter approaches zero; see Davison and Smith (1990) for details). For example the Gutenberg-Richter model developed for earthquake magnitudes is a 2-parameter exponential model (Gutenberg and Richter, 1954).

For the natural hazards listed in Table 3-1, the common approach is to assume that the probability of exceedance (*p*) for a given magnitude event is constant from year to year, i.e. stationary through time. Under the assumption of stationarity in the time series and resulting exceedance probabilities associated with a particular design event, the theoretical relationships between POT and AMS enable straightforward computation of summary and design metrics such as the quantile or percentile of the distribution associated with a particular average return period and/or reliability (Stedinger et al., 1993). When evidence of nonstationarity, or a trend in either or both the frequency or the magnitude of the exceedance events occurring through time is present (Table 3-1), then *p* can no longer be assumed as a constant and the traditional Poisson-GP (or other) model must be modified to account for dependence on time and/or some other explanatory co-variate. Not adjusting the probabilistic analysis for a positive trend when it is present can lead to gross over-estimation of the expected return period and reliability of a system, as shown for floods by several recent studies (Salas and Obeysekera, 2014; Read and Vogel, 2015a). Importantly, Vogel et al., (2013) document that without a rigorous probabilistic analysis of trends in natural hazards, we may overlook and fail to prepare for a wide range of societal outcomes which they document may have occurred repeatedly in the past..

We conclude from a brief review of the literature that the GP POT model is widely used to model natural hazards, and that it can provide a foundation for a nonstationary analysis. What distinguishes this work from the expanding literature on nonstationary natural hazards, is that we attempt to draw a formal linkage between the magnitude of the natural hazard event and the waiting time or failure time until we experience another hazard in excess of some design event.

1.1 *Brief introduction to hazard function theory and implications for natural hazards*

Despite the similarity in name, the application of the theory of hazard function analysis (HFA), also commonly referred to as survival analysis, is practically absent from general literature in the field of natural hazards. HFA is a well-established set of tools useful for conducting a “time-to-event” analysis, or for understanding the distribution of survival (failure) times for a given process (e.g. survival rate of a chronic disease, time until electrical burnout of a device, age-specific mortality rate). This is precisely the concern of those modeling natural hazards that are changing over time. Generally, HFA is comprised of three primary functions: (1) the hazard function, $h(\tau)$, which is defined as the failure rate, or as the likelihood of experiencing a failure at a particular point in time; (2) the survival function, $S_T(t)$, defined as the exceedance probability for the random variable time (T), or in reliability engineering as the cumulative distribution function (cdf) of t, $F_T(t)$, where $S_T(t) = 1 – F_T(t)$; and (3) the cumulative hazard function, $H(\tau)$, interpreted as the total number of failure events over a period of time τ. Note that in natural hazards work, we normally begin with the POT random variable X, which denotes the magnitude of the natural hazard of interest above some threshold. Thus a connection is needed between the variable of interest *X*, and the time to failure, *T*, associated with some design event chosen from the probability distribution of *X*. This is the focus of our work and distinguishes it from most previous work in HFA as well as most previous studies in natural hazards, because normally HFA only focuses on the random variable *T*, without any formal connection to the variable of interest *X*.

Most applications of HFA are interested in computing design metrics based on knowledge of $h(\tau)$, $S_T(t)$ and $H(\tau)$, e.g. the mean time to failure (MTTF), or the reliability of *surviving* a certain amount of time without at least one failure event. In nearly all of the literature on HFA, the process for defining these three functions begins in one of two ways: either by first identifying an appropriate hazard function $h(\tau)$, a possible path if sufficient knowledge (or empirical evidence) of the failure process is known, (e.g. does the probability of failure increase, decrease, or is it constant over time); or, by estimating the survival function $S_T(t)$ by fitting a set of survival time data to a distribution (Klein and Moeschberger, 1997). Neither of these two approaches are suited to natural hazards, because until this paper, there has not been any guidance on how to choose a suitable hazard function $h(\tau)$ for a natural hazard event, and in natural hazards work, we do not have adequate empirical data on the time to failure to enable fitting a survival function to data.

The theory of hazard function analysis is derived elsewhere and summarized in numerous textbooks (Finkelstein, 2008; Kleinbaum and Klein, 1996; Klein and Moeschberger, 1997), hence we only summarize the fundamental and useful results here including the relationships among the hazard rate function

$h(\tau)$, the probability distribution function of the time to failure $f_T(t)$, the cumulative distribution function of the time of failure $F_T(t)$, and its corresponding survival function $S_T(t)$, as well as the cumulative hazard function $H(\tau)$:

$$h(\tau) = \frac{f_T(t)}{1 - F_T(t)} \tag{48}$$

$$S_T(t) = 1 - F_T(t) = \exp\left(-\int_0^t h(s)ds \right) \tag{49}$$

$$H_T(t) = -\ln(S_T(t)) = \int_0^t h(s)ds \tag{50}$$

HFA has been applied in the fields of bio-statistics and medicine (Cox, 1972; Pike, 1966) as well as many other disciplines including economics (Kiefer, 1988) and engineering (Finkelstein, 2008; Hillier and Lieberman, 1990). Similar to the field of natural hazards, these fields need summary metrics associated with the time to failure, such as the concepts of reliability and the average return period (known in HFA as $S_T(t)$ and MTTF, respectively). Very little attention has been given to the use of HFA to natural hazards (Katz and Brown, 1992; Lee et al., 1986), with the exception of the recent paper by Read and Vogel (2015b), who apply HFA to nonstationary floods. Concepts from hazard function theory were applied to develop dynamic reliability models for characterizing evolving risk of hydrologic and hydraulic failures in conveyance systems in the 1980s (Landsey, 1989; Tung, 1985; Tung and Mays, 1981).

The primary goal of this work is to use the theory of HFA to link the probabilistic properties of *t* with properties of the probability distribution for a nonstationary natural hazard event *X*. We begin by explicitly relating the properties of h(τ) to the event magnitudes for a general natural hazard, *X,* assuming the natural hazard POT follows a GP distribution. We then derive $S_T(t)$, H(τ) and MTTF=E[T] for the case of 2-parameter Generalized Pareto (GP2) model. Since an exponential model is a special case of the GP model, our results also apply to POT series which follow an exponential model. Recall that the exponential distribution is of interest when the AMS is Gumbel, which is applicable for many natural hazards. See Read and Vogel (2015b) for an application of HFA to POT hazards which follow an exponential distribution, in which case very elegant analytical expressions for $S_T(t)$ and E[T] result. Since both the exponential and GP2 models are widely used for representing natural event magnitudes in a POT, the analysis presented here is relevant for a wide range of nonstationary natural hazards. We hope to demonstrate that HFA can be a useful methodology for characterizing nonstationary natural hazards, for communicating natural hazard event likelihood under nonstationarity, and for computing corresponding design metrics that reflect the changing behavior of the both the magnitude and frequency of a natural hazard through time.

2. Hazard function analysis for nonstationary natural hazard magnitudes

To relate the properties of h(τ) with the magnitudes of a particular natural hazard (*X*), we first consider the stationary situation in which the exceedance probability

p_o for a particular natural hazard event to exceed some threshold magnitude value is constant through time. In the stationary case, the hazard failure rate, h(τ), is constant so that h(τ) = p_o. For the stationary case, the time to failure, *T*, always follows an 1-parameter exponential distribution (or the geometric distribution for a discrete random variable); and, computation of the average return period (or MTTF) is easily obtained from probability theory as the expected value of the exponential series E[T] = 1/*p*.

If the magnitudes of a natural hazard exhibit an increasing trend through time, this indicates that the exceedance probability associated with a particular design event is changing with time, which we denote as p_τ. In such a situation, the expectation E[T] or MTTF, is no longer a sufficient statistic for the distribution of *T*, and a more complex analysis is needed.

For a nonstationary natural hazard, h(τ) is no longer constant and can be computed directly from a probabilistic analysis of the natural hazard of interest. For example, suppose our interest is in the probability distribution of the time to failure for a natural hazard which has been designed to protect against an event with exceedance probability p_o at time $\tau = 0$. Such a design event can be expressed using the quantile function x_p for the natural hazard of interest. We term this design event $x_o(p_o)$. Suppose we also have defined a nonstationary cumulative probability distribution for the natural hazard *X*, which we term $F_x(x,t)$. Then it is possible to compute the changing exceedance probability associated with this design event, p_τ, using the fact that $p_\tau = 1 - F_x(x_o,t)$. This forms the fundamental linkage between the probabilistic properties of *X* and *T*, because the changing exceedance probabilities associated with the design event are identical to the hazard rate function so that h(τ)= p_τ. Our use of probabilistic theory to link the properties of *X* and *T*, rather than empirical evidence (fitting) to explicitly relate h(τ) to the event magnitudes (*X*) of a natural hazard is the primary difference in our analysis compared with other HFA applications in the literature. We believe this is a fundamental difference which should enable future researchers to formulate even more general conclusions concerning the probabilistic behavior of nonstationary natural hazards.

3. Two-parameter Generalized Pareto (GP2) model for magnitudes of natural hazards

As discussed earlier, the GP distribution is widely used in modeling the magnitudes above a pre-defined threshold for a variety of natural hazards. In this section we present the GP2 stationary and nonstationary models, reviewing literature to support the selection of our nonstationary natural hazard model formulation. We then use HFA theory to derive h(τ), $S_T(t)$ and H(τ) for the GP2 nonstationary model and discuss the findings and interpretations for each function. Our goal is to show that HFA is ideally suited for modeling the probabilistic behavior of a wide range of natural hazards whose behavior is changing through time.

3.1 Stationary Generalized Pareto two-parameter model

We assume that the POT natural hazard series follows the GP2 distribution. The definitions of the stationary probability density function (pdf) and cumulative distribution function (cdf) for a GP2 distribution for random variable *X*, introduced by Hosking and Wallis (1987) are:

$$f_x(x) = \frac{1}{\alpha}\left[1-\kappa\left(\frac{x}{\alpha}\right)\right]^{\frac{1}{\kappa}-1} \qquad \text{for } \kappa \neq 0 \qquad (51)$$

$$F_x(x) = 1-\left[1-\kappa\left(\frac{x}{\alpha}\right)\right]^{\frac{1}{\kappa}} \qquad \text{for } \kappa \neq 0 \qquad (52)$$

where α is the scale parameter and κ is the shape parameter. Note that when $\kappa=0$ $C_x = 1$, (4) and (5) reduce to the exponential distribution with a mean of α; this form corresponds to a Gumbel distribution for the AMS. The reliability function is simply Rel(x) = 1-F_x(x), or the probability of nonexceedance associated with *X*. The first and second moments of *X* are:

$$\mu_x = \frac{\alpha}{1+\kappa} \qquad (53)$$

$$\sigma_x^2 = \frac{\alpha^2}{(1+\kappa)^2(1+2\kappa)} \qquad (54)$$

We use the coefficient of variation $C_x = \sigma_x / \mu_x$ to represent the variability of the system. Combining (53) and (54) for the GP2 model yields

$$C_x = \frac{1}{\sqrt{2\kappa+1}} \qquad (55)$$

The quantile function for the GP2 distribution for a design event, X_p, associated with exceedance probability, *p*, is written as

$$x_p = \frac{\alpha}{\kappa}\left[1 - p^{\kappa}\right] \tag{56}$$

These equations serve as the foundation for developing a nonstationary GP2 model, discussed in the next section.

3.2 Nonstationary GP2 model

Although we could not locate any previous research combining HFA and nonstationary natural hazards, there are numerous papers that employ a nonstationary GP model for the POT magnitudes of specific natural hazards (shown in Table 1). We briefly review those models to provide context for the trend model adopted here. Literature on nonstationary GP models for specific natural hazards has employed a variety of parameterizations. For example, Roth et al. (2012, 2014) considered models of changes in the POT threshold over time and Strupczewski et al., (2001) modeled the arrival time distribution of the POT with time-varying Poisson parameters. Strupczewski et al., (2001) also modeled the changes in the magnitudes of the POT events over time by modeling changes in the GP model parameters over time as we do here.

Nearly all previous studies that employed nonstationary POT models in the context of natural hazards adopt some form of the Poisson-GP model, and many whose concerns regard increasing magnitudes have been specific to extreme rainfall. With respect to extreme daily rainfall, most have built nonstationary GP2 models assuming a trend in the scale parameter (α), either modeled linearly (Beguería et al., 2011; Sugahara et al., 2009), or log-linearly (Tramblay et al., 2013). Roth et al., (2014) notes that modeling a trend in the threshold level itself indicates a comparable trend in the scale parameter. Tramblay et al., (2013) used time-varying co-variates in the Poisson arrivals (occurrence of seasonal oscillation patterns) and in the magnitudes (monthly air temperature) to model heavy rainfall in Southern France and found improvement from the stationary model. As pointed out by Khaliq et al., (2006) and Tramblay et al., (2013) and others, it is less common to vary the shape parameter (κ) through time due to difficulty with precision and a lack of evidence on model improvement with a time-varying shape parameter.

Studies from other natural hazards are consistent with those in extreme rainfall for nonstationary Poisson-GP model formulations, though with more examples of time-variation in the shape parameter. For example, Strupczewski et al., (2001) used linear and parabolic trends in both α and κ to model flood magnitudes; others have explored linear models in κ for extreme winds (Young et al., 2011), and in sediment yield (Katz et al., 2005). For wave height, several assumed a trend in the location parameter either as linear (Ruggiero et al., 2010) or log-linear (Méndez et al., 2006) formulations. Renard et al., (2006) used a Bayesian approach to explore step-change and linear trend models in α for general purpose with an application to floods. The Bayesian framework was also used by Fawcett and Walshaw, (2015) to present a new hybridized method for estimating more precise return levels for nonstationary storm surge and wind speeds.

3.3 Derivation of nonstationary GP2 hazard model

Our approach is to derive the primary HFA functions $f_T(t)$, $F_T(t)$, $S_T(t)$, $h(\tau)$ and $H(\tau)$ from the probability distributions of the random variable *X*, $f_x(x)$ and $F_x(x)$ of the GP2 distribution. To create a nonstationary GP model, we employ an exponential trend model in the scale parameter $\alpha_x(\tau)$, so that:

$$\alpha_x(\tau) = \alpha_o \exp(\beta\tau) \tag{57}$$

The model in (57) is equivalent to a model of the conditional mean of the natural hazard *X* and has been found to provide an excellent representation of changes in the mean annual flood for flood series at thousands of rivers in the United States (Vogel et al. 2011) and in the United Kingdom (Prosdocimi et al., 2014). This model is described by Khaliq et al., (2006) and was also used in Tramblay et al., (2013) for extreme rainfall.

We assume that the shape parameter κ is constant through time as consistent with previous studies discussed earlier. This assumption implies that C_x is fixed (Equation 10), or that the variability of the system is assumed constant over the time period, defined at $\tau = 0$, and thus the standard deviation changes in step with the mean (parameterized by α). Again, there is reasonable evidence that this is the case for floods (see Vogel et al., 2011; and Prosdocimi et al. 2014).

Following Vogel et al. (2011), Prosdocimi et al. (2014), and Read and Vogel (2015), we replace the trend coefficient β in (57) with the more physically meaningful magnification factor *M* to represent the ratio of the magnitude of the natural hazard quantile at time period ($\tau + \Delta\tau$) to the natural hazard quantile at time τ. For the model developed here, the magnification factor, *M*, can be derived by combining the GP2 quantile function in (56) and the trend model in (57), inserting into the expression below:

$$M = \frac{x_p(\tau + \Delta\tau)}{x_p(\tau)} = \frac{\frac{1}{\lambda}\exp\left[\beta(\tau + \Delta t)\ln(p_\tau)\right]}{\frac{1}{\lambda}\exp\left[\beta\tau\ln(p_\tau)\right]} = \exp\left[\beta\Delta\tau\right] \qquad (58)$$

Thus *M* reflects the change in the magnitude of the natural hazards over time. So for example, a magnification factor of two corresponding to particular time interval $\Delta\tau$, indicates that the natural hazard has increased twofold over that time period, for all values of *p*.

We consider a magnification factor *M* corresponding to ten time periods, ($\Delta\tau$ =10), since that is what others have done and because it provides a physically meaningful interpretation of the degree of change in the design events over time. For example, if the time periods were equal to a year as in an AMS series, this would correspond to a decadal magnification factor. In this section we derive h(τ), $S_T(t)$, H(τ), and $f_T(t)$ for the nonstationary GP2 model. First recall that the hazard function is equal to the exceedance probability through time for a natural hazard event series, h(τ) = p_τ. Using the relationships above we can now derive an expression for h(τ) dependent only on those fundamental parameters *M*, p_o, and C_x. describing the behavior of the natural hazard *X* and our design exceedance probability to protect against future hazards.

Consider that the design event in (56) is fixed and set at time *t* = 0, and denoted as x_o, and associated with p_o and α_o; we can use the fact that p_τ =1-F_X(x) in (52), combined with $x_p = x_o$ from (56) and the definition of *M* in (57) which yields,

$$h(\tau) = p_\tau = \left[\frac{1-(1-p^{\frac{1-C_x^2}{2C_x^2}})}{\exp(M\cdot\tau)} \right]^{\frac{2C_x^2}{1-C_x^2}} \quad (59)$$

where κ is replaced with C_x after rearranging (55). Combining the theoretical relationships in Equations (47-50) with (59), leads to expressions for $S_T(t)$, $H(\tau)$, and $f_T(t)$ which are solved easily by numeric integration.

$$S_T(t) = \mathrm{Re}\,liability(t) = \exp\left(-\int_0^s \left[\frac{1-(1-p^{\frac{1-C_x^2}{2C_x^2}})}{\exp(M\cdot\tau)} \right]^{\frac{2C_x^2}{1-C_x^2}} ds \right) \quad (60)$$

$$H(\tau) = \int_0^s \left[\frac{1-(1-p^{\frac{1-C_x^2}{2C_x^2}})}{\exp(M\cdot\tau)} \right]^{\frac{2C_x^2}{1-C_x^2}} ds \quad (61)$$

Finally, the pdf of the time to failure distribution for the GP2 nonstationary model is

$$f_T(t) = b\cdot\exp\left(-\int_0^t b\,ds \right) \quad (62)$$

$$\text{where } b = \left[\frac{1-(1-p^{\frac{1-C_x^2}{2C_x^2}})}{\exp(M\cdot s)} \right]^{\frac{2C_x^2}{1-C_x^2}}$$

For the case of the 1-parameter exponential, these functions simplify to

$h(t) = p_o M^{-t/\Delta\tau}$, $S_T(t) = \exp\left[-\int_0^t p_o M^{-s/\Delta t} ds\right]$, $H_T(t) = \int_0^t p_o M^{-s/\Delta\tau} ds$, and

$f_T(t) = \frac{d}{dt}\left[\exp(\int_0^t p_o M^{-s/\Delta\tau} ds)\right]$ (see Read and Vogel, 2015b for a complete derivation).

3.4 Investigation of impacts of nonstationarity on probabilistic analysis of natural hazards using HFA

In this section we explore how HFA can characterize the behavior of nonstationary natural hazards whose PDS magnitudes follow a GP2 model. Our results are exact (within the limitations of numerical integration) because they result from the derived analytical equations in (59-62) for the HFA functions $f_T(t)$, $F_T(t)$, and $S_T(t)$, $h(\tau)$ and $H(\tau)$ corresponding to a natural hazard *X* which follows a GP2 model. With no loss in generality, we assume the mean of the GP2 natural hazard of unity. We investigate the impact of small and large trends (corresponding to magnification factors, *M*, ranging from 1 to 1.25 with $\Delta\tau$=10) for a range of physical systems characterized by a range in variability corresponding to a range in the coefficient of variation of *X*, C_x, from 0.5 to 1.5 corresponding to a range in the GP2 shape κ between -0.28 to 1.5) for three event sizes (p_o = 0.01, 0.002, 0.001).

Figure 1 presents the hazard function $h(\tau)$, examining how it is influenced by the variability of the natural hazard and the magnitude of the trend for a particular design event p_o = 0.002 (500-year event): [a] increasing variability, C_x = 0.75, 1.25, 1.5 for a set M = 1.1, and [b] increasing trend values, M = 1.1, 1.25, 1.5 for a set C_x = 0.75. Note that in the stationary case, $p_t = p_o = 0.002$ results in a constant horizontal line, and as the magnitude of the trends increases (as M increases), the hazard rate h(τ) tends toward unity earlier in time. Even from this initial relatively simplistic investigation we find that the hazard functions exhibit complex shapes, with some exhibiting inflection points, and other without an inflection. This is extremely important, because most applications of HFA assumes a particular hazard function without deriving their generalized shapes in advance as we do here in Figure 3-1. Another important point is that the variability of the hazard magnitudes (characterized by the shape of the pdf of X) impacts the rate at which h(τ) increases, so that less variable hazards tend to have higher hazard rates than more variable hazards. This point is perhaps initially counter-intuitive, our interpretation is that if a hazard is more consistent (with less variability), a larger trend ensures exceedance more so than a less consistent system that has a wider range of small and large events. This finding is relevant for planning purposes as it indicates which systems may be greater impacted by nonstationarity.

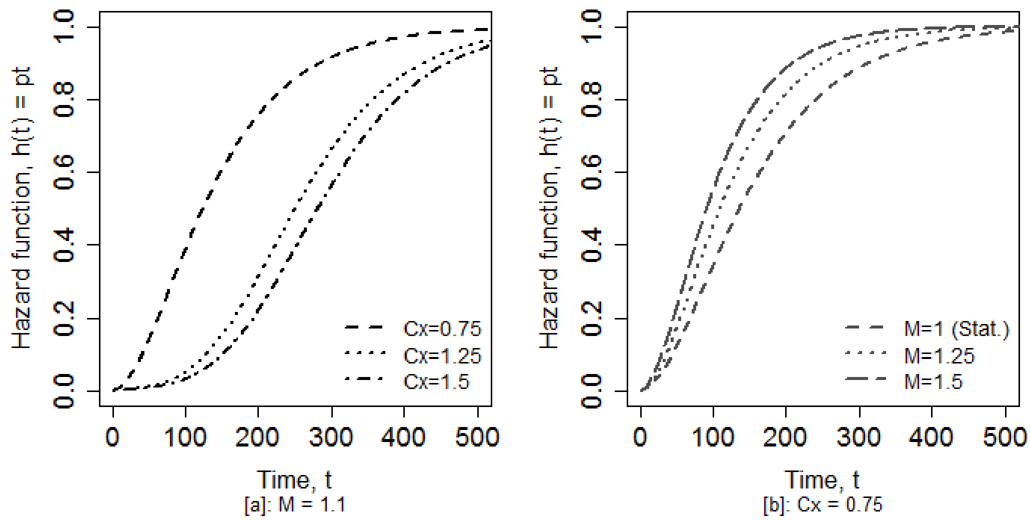


FIGURE 3-1. Hazard function h(τ) for the nonstationary GP2 model, $p_o = 0.002$ for [a] a range of variability (C_x = 0.75, 1.25, 1.5), given M = 1.1; [b] a range of trend values (M = 1.1, 1.25, 1.5), given C_x = 0.75.

Typically in HFA work, the survival function $S_T(t)$ is presented as a primary figure in understanding risk of failure and likelihood of experiencing an exceedance event within a given period of time. Since $S_T(t)$ also represents the relationship between system reliability and time and because many fields employ the concept of reliability to protect against natural hazards, the $S_T(t)$ function is also very relevant for planning purposes in this context (see Read and Vogel 2015ab for further discussions relating to flood management and design). Figure 3-2 illustrates $S_T(t)$ for a $p_o = 0.002$ event with a fixed $C_x = 0.75$ representing a slightly lower variable system, and a range of increasing trends (M = 1.02, 1.1, 1.25) compared with stationary conditions (M = 1). Clearly even a small trend significantly reduces the system reliability compared with our expectations under stationary conditions. For example, the reliability of a structure designed to protect against a 500-year event under stationary conditions after 50 time periods is quite high (Rel = 0.90), however, as M increases, the reliability decreases significantly, approaching zero for M = 1.1 and 1.25 at τ = 50. This suggests that if one was designing infrastructure to withstand a particularly large magnitude event over a planning period, under nonstationary conditions, the design would need to be significantly larger, and it may not even be possible to design a structure to achieve the same reliability as expected under stationary conditions.

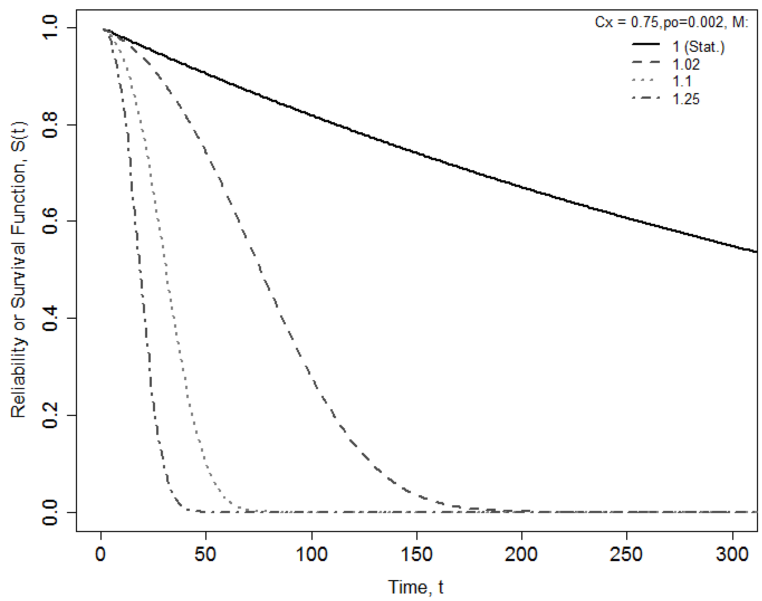


FIGURE 3-2. Reliability or Survival function S(t) for the nonstationary GP2 model, p_o = 0.002 and C_x = 0.75, for a range of trend values (M = 1, 1.02, 1.1, 1.25)

A unique tool offered by HFA that can provide advancements in planning for nonstationary natural hazards is the cumulative hazard function H(τ) which represents the total hazard over a given amount of time (Wienke, 2010). For example if p_o = 0.002, as expected under stationary conditions H(τ) = 1 for τ = 500, or we will experience, on average, one exceedance event every 500 years. However, if a trend with a magnification factor M = 1.1 ($\Delta\tau$ =10) is introduced in the same system, the time it takes for H(τ) = 1 is about 36 time periods, or another view, H(τ) = 333 events for τ = 500. Figure 3-3 illustrates these interpretations for two exceedance event sizes, fixed C_x = 0.75: [a] p_o = 0.002, showing the number of time periods until H(τ) = 1; and [b] p_o = 0.001, showing the total number of events over time. In Figure 3-3a, we note that the stationary M = 1 line corresponds with the H(τ) = 1 for τ = 500 as expected, and that as M increases, the time until an exceedance event occurs dramatically decreases (note the log x-axis scale). Figure 3-3b depicts a similar story, but illustrates an alternate interpretation: the total number of exceedance events over a time period for the rarer p_o = 0.001 event, where H(τ) ranges from 1 for τ = 1000 as expected under stationary conditions, to H(τ) = 10+ events in under 50 time periods with a large M.

When one wishes to communicate the risk of failure and event likelihood, the cumulative hazard function is a useful metric for describing total risk (or reliability) over a certain planning horizon. While our analysis assumes that the trend would increase over the entire time period, perhaps a 'worst case' scenario, our results show that in the presence of an increasing trend in the POT of a natural hazard series, we may experience far more exceedance events than expected under stationary conditions. Ignoring such trends may result in significant increased damages and losses from under-design of infrastructure or insufficient planning in populated areas.

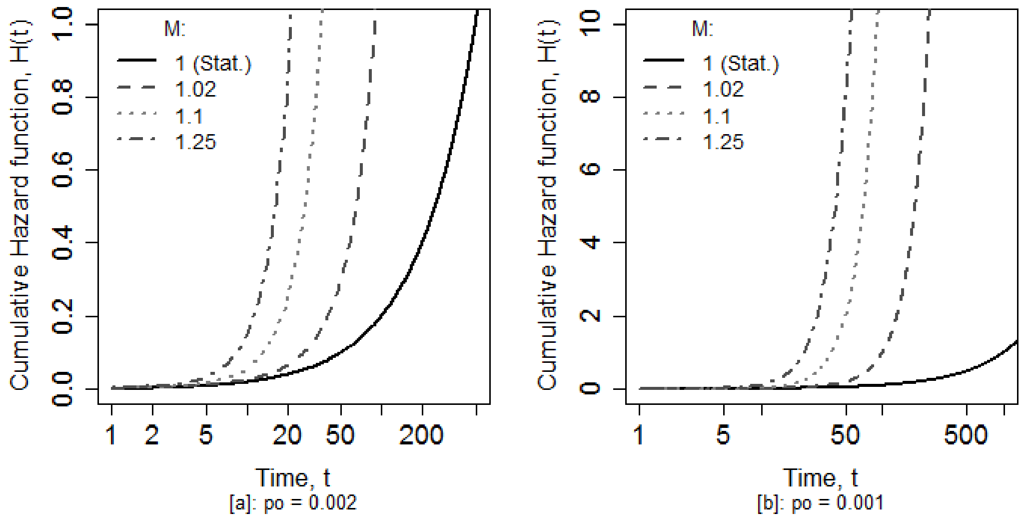


FIGURE 3-3. Cumulative hazard function H(τ) for the nonstationary GP2 model, with a fixed C_x = 0.75 for a range of trend values (*M* = 1, 1.02, 1.1, 1.25); panels show two different size exceedance events [a]: p_o = 0.002 and [b]: p_o = 0.001

After computing $S_T(t)$ and h(τ) we can easily use (1) to determine the pdf of the time to failure distribution for the nonstationary GP2 model by (62). Since we are interested in the behavior of $f_T(t)$ due to trends on a range of physical systems and for extreme events, we plot $f_T(t)$ for a fixed C_x = 0.75 in Figure 3-4, for a range of increasing trends (*M* = 1, 1.02, 1.1, 1.25) and three event sizes (p_o = [a] 0.01, [b] 0.002, [c] 0.001). We note that the shape of $f_T(t)$ evolves from the expected exponential curve under stationary conditions, to a more symmetric or normally distributed shape as *M* increases. These results complement those by Read and Vogel, (2015ab) who show similar behavior for a nonstationary 2-parameter lognormal model of an AMS series of floods. Interestingly, Figure 3-4 [b] and [c] show little difference in timing of the peak, especially for larger *M* values (*M* > 1.1), suggesting that for systems experiencing large increasing trends, rare events (p_o = 0.002) and extremely rare events (p_o = 0.001) may exhibit similar probabilistic behavior.

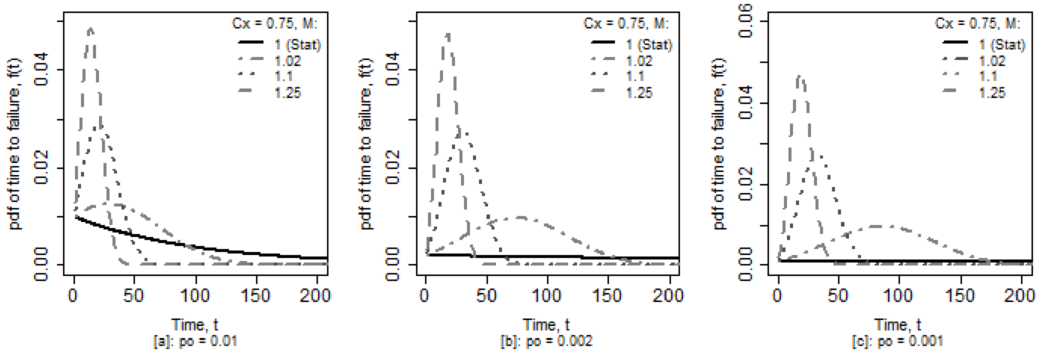


FIGURE 3-4. Probability density function (pdf) of the time to failure distribution for the nonstationary GP2 model, with a fixed C_x = 0.75 for a range of trend values (M = 1, 1.02, 1.1, 1.25); panels show three exceedance event sizes increasing in extremity [a]: p_o = 0.01, [b]: p_o = 0.002 and [c]: p_o = 0.001

Similarly, in Figure 3-5 we fix the trend at M = 1.05 and explore the behavior of $f_t(t)$ over a realistic range of C_x values (0.5, 0.75, 1.5), for the same three event sizes (p_o = [a] 0.01, [b] 0.002, [c] 0.001). As consistent with Figure 3-1 showing $h(\tau)$ for various C_x values, the shape of $f_T(t)$ in less variable systems (lower C_x) is more impacted by a trend than a more variable system, as indicated by the sharp peaks and shift in timing of the peaks (Figure 3-5 [a-c]). We again highlight the similar shape and timing of peaks in the p_o = 0.002 and p_o = 0.001 events, an unanticipated yet consistent result with Figure 3-4.

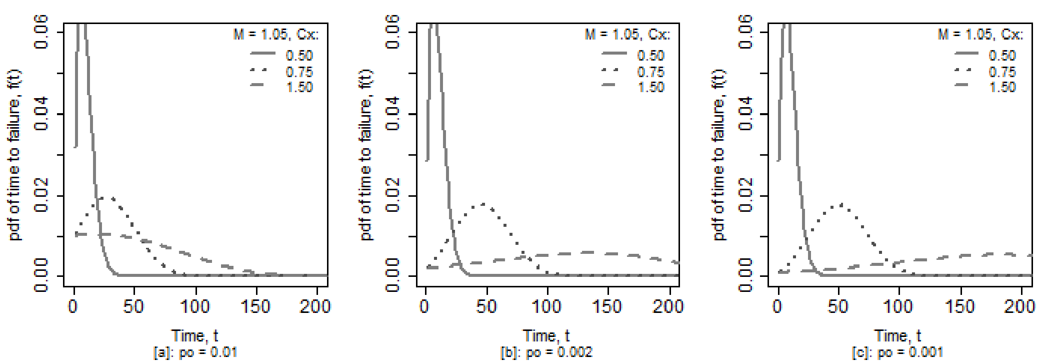


FIGURE 3-5. Probability density function (pdf) of the time to failure distribution for the nonstationary GP2 model, with a fixed M = 1.05 for a range of trend values (C_x = 0.5, 0.75, 1.50); panels show three exceedance event sizes increasing in extremity [a]: p_o = 0.01, [b]: p_o = 0.002 and [c]: p_o = 0.001

Our investigation of the behavior of $f_T(t)$ for the nonstationary GP2 model indicates that the shape and timing of the distribution changes with both the magnitude of the trend and variability of natural hazard. We also note that $f_T(t)$ exhibits complex patterns under nonstationary conditions, e.g. $f_T(t)$ is less impacted in shape/timing by M for smaller events (p_o), and that the presence of a trend leads to a range of shapes (approaching normal for large positive M) of the time to failure distribution. This more complicated behavior implies that under the premise of nonstationarity, we can no longer assume the failure time distribution is exponential in shape and that the MTTF is equal to $1/p$. In fact, the mean of the distribution of T is no longer a sufficient statistic as is the case under stationary conditions. We are the first to document such changes in the context of natural hazards for the GP2 distribution and anticipate that others will continue to do so for specific events that exhibit nonstationarity. Using the derivations of $h(\tau)$, $S_T(t)$, $H(\tau)$, and $f_T(t)$ that we have provided here, one can use knowledge of the system (M, C_x) and existing design metrics (p_o and reliability standards) in combination with HFA to better understand and characterize natural hazards as they change through time.

4. Summary and Conclusions

We have presented a general introduction to the probabilistic analysis of nonstationary natural hazards using the well-developed field of hazard function analysis (HFA). We cited numerous sources of evidence which suggests that the magnitudes of a variety of natural hazards are increasing, thus our study should be of considerable interest in the coming years. To the authors' knowledge, the analysis and discussion presented here provides the first formal probabilistic analysis of the link between a natural hazard with design event *X*, and the corresponding properties of the probability distribution of the time to failure *T* associated with a structure designed to protect against that design event. Through the lens of HFA, we have investigated the impacts of a positive trend in natural hazard event magnitudes for a random variable *X* that follows a GP2 distribution on the likelihood of future hazards under nonstationary conditions. We introduce a complete probabilistic analysis useful for re-evaluating event likelihood, reliability (survival) and cumulative hazard under nonstationary conditions which should prove useful for a wide range of natural hazards. Our results are applicable for understanding the probabilistic behavior of the time to occurrence of natural hazards subject to increasing trends.

By explicitly linking properties of the time to failure *T* with the exceedance probability *p* of a natural hazard (*X*), we have derived the primary hazard analysis equations: the hazard function h(τ), the survival (reliability) function $S_T(t)$, the cumulative hazard function H(τ), and the pdf of the time to failure distribution $f_T(t)$ corresponding to a POT series of natural hazards which follows the GP2 distribution. We parameterize this GP2 model such that it only depends on the design exceedance probability at time $\tau = 0$, p_o, the known system variability C_x, and the magnification factor *M*, and use this to explore the impact of positive trends on the reliability, or survival $S_T(t)$ until an exceedance event. Findings of this investigation suggest that under nonstationary conditions, medium and large events could occur with much greater frequency than under stationary conditions (no trend). We find that the total number of hazards as characterized by H(τ) within a given planning period may substantially increase in the presence of a positive trend in the POT magnitudes of a natural hazard. Perhaps most importantly, under nonstationary conditions, the distribution of the time to the natural hazard is no longer exponentially distributed, and instead takes on a distribution with extremely complex shapes, depending on the variability of the hazard and the magnitude of the trend. As trend magnitudes increase for a range of event sizes (p_o), the shape of the distribution of the survival time approaches normality in shape and exhibits a sharp peak with a heavy upper-tail. We also find that variability impacts the shape and timing of this peak in $f_T(t)$, such that less variable systems (lower C_x) are more affected by larger M values, i.e. produce a more pronounced peak and a greater shift in timing.

The implications of these findings for planning and design for nonstationary natural hazards are significant. Given a historic (or future) increasing trend in the magnitudes of some hazard, we should prepare to experience exceedance events much more frequently. For fields that use reliability as a primary design standard, our analysis suggests that the presence of a positive trend corresponds to a lower system reliability for a given design event (x_o) than under stationary conditions. Through exploration of $S_T(t)$ for various trend factors, we also note that determining the reliability of a system over time is more complicated given uncertainty in the magnitude of the trend and how it will manifest through time. In either case, continuing to assume stationary conditions when computing system reliability for design purposes, when a positive trend in the POT magnitudes has been observed historically, may pose a significant risk to the populations and infrastructure in that region. Thus we recommend that design practices should be reviewed and adapted for cases where nonstationary behavior of natural hazards is evident in order to avoid under-design (also see Vogel et al., 2013).

Overall, we have shown that HFA provides a set of tools for understanding the probabilistic behavior of nonstationary natural hazards for application to a wide range of natural phenomena. Using the well-studied theory of HFA, engineers and planners can use language from HFA – hazard rate, survival, cumulative hazard – to relate to risk and reliability under nonstationarity for natural hazards, advancing risk communication in this field. We intend for this analysis to inform future work on modeling nonstationary natural hazards with HFA, for example by developing other models that may include co-variates, extensions to AMS series, and also exploring the impact of decreasing trends. We expect that additional research on this topic will contribute to the emerging conversation on planning for nonstationary natural hazards and shed light on innovative methods to determine best practices for infrastructure design. Results of this work further support the need for a risk-based decision analysis framework for selecting a design event under nonstationarity (Rosner et al., 2014). Such a framework can provide guidance in choosing infrastructure that minimizes the risk of under-design (protection) and over-design (excess spending) through probabilistic decision trees.

5. Acknowledgments

The authors are grateful to the United States Army Corps of Engineers (USACE) Institute of Water Resources for funding this work through support in part by an appointment to the USACE Research Participation Program administered by the Oak Ridge Institute for Science and Education (ORISE) through an interagency agreement between the U.S. Department of Energy (DOE) and the USACE. ORISE is managed by ORAU under DOE contract number DE-AC05-06OR23100. All opinions expressed in this paper are the author's and do not necessarily reflect the policies and views of USACE, DOE or ORAU/ORISE. All data are freely available and analyzed with the R statistical software package version 3.1.1.

6. References

Di Baldassarre, G., Montanari, A., Lins, H., Koutsoyiannis, D., Brandimarte, L. and Blöschl, G.: Flood fatalities in Africa: From diagnosis to mitigation, Geophys. Res. Lett., 37(22), L22402, doi:10.1029/2010GL045467, 2010.

Bayazit, M.: Nonstationarity of hydrological records and recent trends in trend analysis: a state-of-the-art review. Environmental Processes, 2(3), 527-542, 2015.

Beguería, S., Angulo-Martínez, M., Vicente-Serrano, S. M., López-Moreno, J. I. and El-Kenawy, A.: Assessing trends in extreme precipitation events intensity and magnitude using non-stationary peaks-over-threshold analysis: a case study in northeast Spain from 1930 to 2006, Int. J. Climatol., 31(14), 2102–2114, doi:10.1002/joc.2218, 2011.

Beirlant, J., Goegebeur, Y., Segers, J. and Teugels, J.: Statistics of Extremes: Theory and Applications, John Wiley & Sons., 2006.

Blaikie, P., Cannon, T., Davis, I. and Wisner, B.: At Risk: Natural Hazards, People’s Vulnerability and Disasters, Routledge, 2014.

Bonnin, G. M., Maitaria, K., and Yekta, M.: Trends in Rainfall Exceedances in the Observed Record in Selected Areas of the United States. Journal of the American Water Resources Association, 47(6), 1173-1182, 2011.

Coles, S., Bawa, J., Trenner, L. and Dorazio, P.: An introduction to statistical modeling of extreme values, Springer, 2001.

Cox, D. R.: Regression Models and Life-Tables, J. R. Stat. Soc. Ser. B Methodol., 34(2), 187–220, 1972.

Davison, A. C. and Smith, R. L.: Models for Exceedances over High Thresholds, J. R. Stat. Soc. Ser. B Methodol., 52(3), 393–442, 1990.

Ellsworth, W. L.: Injection-Induced Earthquakes, Science, 341(6142), 1225942, doi:10.1126/science.1225942, 2013.

Fawcett, L. and Walshaw, D.: Sea-surge and wind speed extremes: optimal estimation strategies for planners and engineers, Stoch. Environ. Res. Risk Assess., 1–18, doi:10.1007/s00477-015-1132-3, 2015.

Finkelstein, M.: Failure rate modelling for reliability and risk, Springer Science & Business Media, 2008.

Guha-Sapir, D., Hargitt, D. and Hoyois, P.: Thirty Years of Natural Disasters 1974-2003: The Numbers, Presses univ. de Louvain, 2004.

Gumbel, E. J.: Statistics of Extremes, Courier Corporation., 2012.

Gutenberg, B. and Richter, C.: Seismicity of the earth and associated phenomena, Princeton University Press., 1954.

Hillier, F. S. and Lieberman, G. J.: Introduction to stochastic models in operations research, McGraw-Hill Companies., 1990.

Holmes, T. P., Jr, R. J. H. and Westerling, A. L.: Statistical Analysis of Large Wildfires, in The Economics of Forest Disturbances, edited by T. P. Holmes, J. P. Prestemon, and K. L. Abt, pp. 59–77, Springer Netherlands, 2008.

Hosking, J. R. M. and Wallis, J. R.: Parameter and Quantile Estimation for the Generalized Pareto Distribution, Technometrics, 29(3), 339–349, doi:10.1080/00401706.1987.10488243, 1987.

Katz, R. W. and Brown, B. G.: Extreme events in a changing climate: variability is more important than averages, Clim. Change, 21(3), 289–302, doi: 10.1007/BF00139728, 1992.

Katz, R. W., Brush, G. S. and Parlange, M. B.: Statistics of Extremes: Modeling Ecological Disturbances, Ecology, 86(5), 1124–1134, doi: 10.1890/04-0606, 2005.

Keellings, D., & Waylen, P.: Increased risk of heat waves in Florida: Characterizing changes in bivariate heat wave risk using extreme value analysis. Applied Geography, 46, 90-97, doi: 10.1016/j.apgeog.2013.11.008, 2014.

Khaliq, M. N., Ouarda, T. B. M. J., Ondo, J.-C., Gachon, P. and Bobée, B.: Frequency analysis of a sequence of dependent and/or non-stationary hydro-meteorological observations: A review, J. Hydrol., 329(3–4), 534–552, doi:10.1016/j.jhydrol.2006.03.004, 2006.

Kiefer, N. M.: Economic Duration Data and Hazard Functions, J. Econ. Lit., 26(2), 646–679, 1988.

Kim, Y., Jang, S.-C. and Lim, T.-J.: Hazard analysis of typhoon-related external events using extreme value theory, Nucl. Eng. Technol., 47(1), 59–65, doi:10.1016/j.net.2014.08.001, 2015.

Kleinbaum, D. G. and Klein, M.: Survival analysis, Springer, New York, 1996.

Klein, J. P. and Moeschberger, M. L.: Statistics for biology and health, Stat. Biol. Health, 1997.

Lansey, K. E., Duan, N., Mays, L. W., and Tung, Y. K.: Water distribution system design under uncertainties. *Journal of Water Resources Planning and Management*, *115*(5), 630-645,doi: 10.1061/(ASCE)0733-9496(1989)115:5(630), 1989.
Liu, Y., Stanturf, J. and Goodrick, S.: Trends in global wildfire potential in a changing climate, For. Ecol. Manag., 259(4), 685–697, doi:10.1016/j.foreco.2009.09.002, 2010.
Lopatoukhin, L. J., Commission, I. O., Joint, W. M. O. and others: Estimation of extreme wind wave heights, Secretariat of the World Meteorological Organization, 2000.
Madsen, H., Rasmussen, P. F. and Rosbjerg, D.: Comparison of annual maximum series and partial duration series methods for modeling extreme hydrologic events: 1. At-site modeling, Water Resour. Res., 33(4), 747–757, doi:10.1029/96WR03848, 1997.
Méndez, F. J., Menéndez, M., Luceño, A. and Losada, I. J.: Estimation of the long-term variability of extreme significant wave height using a time-dependent Peak Over Threshold (POT) model, J. Geophys. Res. Oceans, 111(C7), C07024, doi:10.1029/2005JC003344, 2006.
Obeysekera, J. and Park, J.: Scenario-Based Projection of Extreme Sea Levels, J. Coast. Res., 1–7, doi:10.2112/JCOASTRES-D-12-00127.1, 2012.
Palutikof, J. P., Brabson, B. B., Lister, D. H. and Adcock, S. T.: A review of methods to calculate extreme wind speeds, Meteorol. Appl., 6(2), 119–132, doi:10.1017/S1350482799001103, 1999.
Pike, M. C.: A Method of Analysis of a Certain Class of Experiments in Carcinogenesis, Biometrics, 22(1), 142–161, doi:10.2307/2528221, 1966.
Pisarenko, V. F. and Sornette, D.: Characterization of the Frequency of Extreme Earthquake Events by the Generalized Pareto Distribution, Pure Appl. Geophys., 160(12), 2343–2364, doi:10.1007/s00024-003-2397-x, 2003.
Prosdocimi, I., Kjeldsen, T. R. and Svensson, C.: Non-stationarity in annual and seasonal series of peak flow and precipitation in the UK, Nat Hazards Earth Syst Sci, 14(5), 1125–1144, doi:10.5194/nhess-14-1125-2014, 2014.
Read, L. K., and Vogel, R. M.: Reliability, return periods, and risk under nonstationarity. Water Resour. Res., 51(8), 6381-6398, doi: 10.1002/2015WR017089, 2015a.
Read, L. K., and Vogel, R. M.: Hazard function analysis for flood planning under nonstationarity. Water Resour. Res., *under review as* 2015b.
Renard, B., Lang, M. and Bois, P.: Statistical analysis of extreme events in a non-stationary context via a Bayesian framework: case study with peak-over-threshold data, Stoch. Environ. Res. Risk Assess., 21(2), 97–112, doi:10.1007/s00477-006-0047-4, 2006.
Rosner, A., Vogel, R. M. and Kirshen, P. H.: A risk-based approach to flood management decisions in a nonstationary world, Water Resour. Res., doi:10.1002/2013WR014561, 2014.
Roth, M., Buishand, T. A., Jongbloed, G., Klein Tank, A. M. G. and van Zanten, J. H.: Projections of precipitation extremes based on a regional, non-stationary peaks-over-threshold approach: A case study for the

Netherlands and north-western Germany, Weather Clim. Extrem., 4, 1–10, doi:10.1016/j.wace.2014.01.001, 2014.
Ruggiero, P., Komar, P. D. and Allan, J. C.: Increasing wave heights and extreme value projections: The wave climate of the U.S. Pacific Northwest, Coast. Eng., 57(5), 539–552, doi:10.1016/j.coastaleng.2009.12.005, 2010.
Salas, J. D. and Obeysekera, J.: Revisiting the Concepts of Return Period and Risk for Nonstationary Hydrologic Extreme Events, J. Hydrol. Eng., 19(3), 554–568, doi:10.1061/(ASCE)HE.1943-5584.0000820, 2014.
Schoenberg, F. P., Peng, R. and Woods, J.: On the distribution of wildfire sizes, Environmetrics, 14(6), 583–592, doi:10.1002/env.605, 2003.
Smith, R. L.: Extreme Values in Finance, Telecommunications, and the Environment, CRC Press., 2003.
Stedinger, J. R., Vogel, R. M. and Foufoula-Georgiou, E.: Frequency analysis of extreme events, Chapter 18 in Handbook of Hydrology, edited by DR Maidment, McGraw-Hill., 1993.
Strupczewski, W. G., Singh, V. P. and Feluch, W.: Non-stationary approach to at-site flood frequency modelling I. Maximum likelihood estimation, J. Hydrol., 248(1–4), 123–142, doi:10.1016/S0022-1694(01)00397-3, 2001.
Sugahara, S., da Rocha, R. P. and Silveira, R.: Non-stationary frequency analysis of extreme daily rainfall in Sao Paulo, Brazil, Int. J. Climatol., 29(9), 1339–1349, doi:10.1002/joc.1760, 2009.
Thompson, E. M., Baise, L. G. and Vogel, R. M.: A global index earthquake approach to probabilistic assessment of extremes, J. Geophys. Res. Solid Earth, 112(B6), B06314, doi:10.1029/2006JB004543, 2007.
Todorovic, P.: Stochastic models of floods, Water Resour. Res., 14(2), 345–356, doi:10.1029/WR014i002p00345, 1978.
Tramblay, Y., Neppel, L., Carreau, J. and Najib, K.: Non-stationary frequency analysis of heavy rainfall events in southern France, Hydrol. Sci. J., 58(2), 280–294, doi:10.1080/02626667.2012.754988, 2013.
Tung, Y.-K.: Models for Evaluating Flow Conveyance Reliability of Hydraulic Structures, Water Resour. Res., 21(10), 1463–1468, doi:10.1029/WR021i010p01463, 1985.
Tung, Y.-K. and Mays, L. W.: Risk models for flood levee design, Water Resour. Res., 17(4), 833–841, doi:10.1029/WR017i004p00833, 1981.
Utsu, T.: Representation and analysis of the earthquake size distribution: a historical review and some new approaches, in: Seismicity Patterns, their Statistical Significance and Physical Meaning. Birkhäuser Basel, 509-535, 1999.

Villarini, G., Smith, J. A., Serinaldi, F., Ntelekos, A. A., and Schwarz, U.: Analyses of extreme flooding in Austria over the period 1951–2006. International Journal of Climatology, 32(8), 1178-1192, doi:10.1002/joc.2331, 2012.

Vogel, R. M., Yaindl, C. and Walter, M.: Nonstationarity: Flood Magnification and Recurrence Reduction Factors in the United States1, JAWRA J. Am.

Water Resour. Assoc., 47(3), 464–474, doi:10.1111/j.1752-1688.2011.00541.x, 2011.
Vogel, R. M., Rosner, A. and Kirshen, P. H.: Brief Communication: Likelihood of societal preparedness for global change: trend detection, Nat Hazards Earth Syst Sci, 13(7), 1773–1778, doi:10.5194/nhess-13-1773-2013, 2013.
Waylen, P. R.: Statistical analysis of freezing temperatures in Central and Southern Florida. Journal of climatology, 8(6), 607-628, 1988.
Wienke, A.: Frailty models in survival analysis, CRC Press, 2010.
de Winter, R. C., Sterl, A. and Ruessink, B. G.: Wind extremes in the North Sea Basin under climate change: An ensemble study of 12 CMIP5 GCMs, J. Geophys. Res. Atmospheres, 118(4), 1601–1612, doi:10.1002/jgrd.50147, 2013.
Young, I. R., Zieger, S. and Babanin, A. V.: Global Trends in Wind Speed and Wave Height, Science, 332(6028), 451–455, doi:10.1126/science.1197219, 2011.

Appendix A: Derivation of Coefficient of Variation of *X* in nonstationary lognormal model.

The nonstationary LN2 flood hazard model employed in this paper assumes a linear trend in the mean of logarithms of the annual maximum flood series *Y*=ln(*X*). Interestingly, this single trend model of *Y* implies a trend in both the mean and the variance of the annual maximum floods *X*. Importantly, although the coefficient of variation of the flood series *X*, denoted C_x, is assumed fixed, the nonstationary LN2 model implies a different coefficient of variation of *X* which we term the conditional coefficient of variation and denote as $C_{x|\tau}$ and for which we derive an expression below. Recall the well-known relationship between the stationary standard deviation of *Y*=ln(*X*) and C_x:

$$\sigma_y = \sqrt{\ln(1 + C_x^2)} \qquad \text{(A1)}$$

The nonstationary LN2 model introduces a trend in the mean of *Y* so that the variance of *Y*, conditioned on the time τ, is obtained by simply taking the variance of *Y*, given in (15), assuming α, β and τ are constants which leads to:

$$\sigma_{y|t}^2 = \sigma_\varepsilon{}^2 = \sigma_y{}^2 - \beta^2\sigma_t^2 \qquad \text{(A2)}$$

This is equivalent to

$$\sigma_{y|t} = \sigma_y\sqrt{(1-\rho^2)} \qquad \text{(A3)}$$

because ρ is the Pearson correlation coefficient defined as $\rho = \beta\dfrac{\sigma_t}{\sigma_y}$ which measures the strength of the linear relationship between the flood series *Y* and the covariate time, τ. Consider two extreme cases. If the regression model has perfect explanatory power, then ρ=1 and the conditional variance of *Y* in (A3) is equal to zero. If the regression model has no explanatory power, then ρ=0 and the conditional variance in (A3) is exactly equal to the unconditional variance of *Y*. Of course the unconditional variance of *Y* is always the same, regardless of the explanatory power of the regression. Since the residuals are assumed to be normally distributed, the values of exp(ε) are lognormal, thus analogous to (A1):

$$\sigma_{y|\tau}{}^2 = \ln(1 + C_{x|\tau}{}^2) \tag{A4}$$

where $C_{x|\tau}$ is the coefficient of variation of the nonstationary variable *X*. Combining equations (A1), (A3) and (A4) leads to a relationship between the coefficient of variation of the stationary series C_x and the coefficient of variation of the nonstationary series, $C_{x|\tau}$ given by

$$C_{x|t} = \sqrt{(C_x{}^2 + 1)^{(1-\rho^2)} - 1} \tag{A5}$$

For a reasonable range of ρ, we explore the relationship between C_x and $C_{x|t}$ in Figure A1. As expected, there is a reduction in the coefficient of variation of the nonstationary series $C_{x|\tau}$ as ρ increases, i.e. as the trend increases. Since the true value of ρ is unknown, is generally quite small in practice, and will depend on the level of sophistication of the trend model employed, we assume that C_x = $C_{x|\tau}$ for all trends analyzed in this paper, but we recommend that in practice (A5) be used in any particular application of the nonstationary lognormal model.

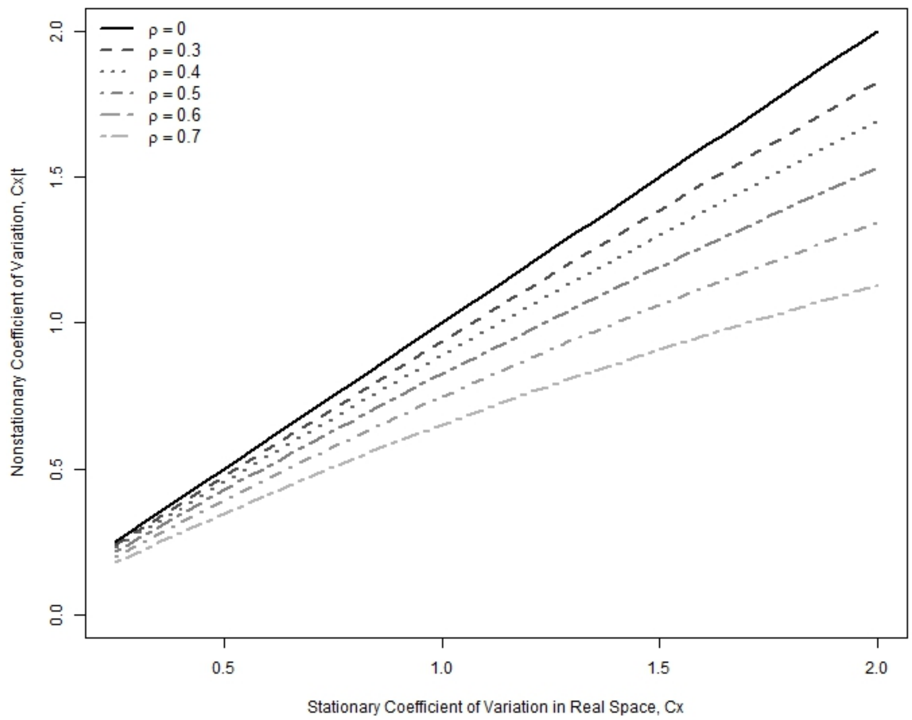


Figure A1. Relationship between coefficient of variation of the stationary series C_x and the coefficient of variation of the nonstationary series $C_{x|\tau}$ for a feasible range of correlation coefficients, ρ (grey lines). Note that the black solid line represents the 1:1 line for stationary conditions where ρ=β=0.

Appendix B: Analytical example: Exponential distribution

The following example uses the simplest case of the exponential distribution to demonstrate how behavioral properties of flood flows (*X*) can be adapted to hazard function analysis where the random variable is now time (*T*). We use the standard equations for the exponential distribution pdf and cdf presented in Equations 36-37.

The corresponding quantile function is reprinted here for convenience:

$$x_p = -\frac{1}{\lambda}\ln(p) \qquad \text{(B1)}$$

Assuming the annual maximum flow series follow an exponential distribution and under stationary conditions we can now write the survival function of time to relate the stochastic properties of time to the random variable of interest, *X*. The expected value and variance of *X* are equal so that E[x]= Var[x], 1/ λ . Since the expected waiting time is exponential we can use parallel logic to express E[t] = 1/*p*, the survival function for a fixed hazard function, as it depends on the probability of *X*, the exponential random variable. From here our goal is to use the theory of hazard function analysis to derive the pdf $f_T(t)$, cdf $F_T(t)$, and reliability (now called survival function $S_T(t)$) of the variable *T*, defined as the waiting time until we exceed the design event.

Now consider that the random variable *X* is not stationary, or that the annual probability *p* of exceeding a certain design event *X* * is increasing every year due to an increasing trend in the mean, E[X] = 1/(λ_0 - bτ). Since we now know how the probability of *X* relates to the survival function for time, we can write the hazard function equations for a nonstationary exponential random variable *X* by deriving an equation for the changing exceedance probability p_t and equating this to h(τ). First, at time zero we rearrange (B1) to compute the exceedance probability p_0 defined at time zero with a known initial value of λ_0 and a design flow x_o* as:

$$p_0 = \exp(-\lambda_0 x_o *) \qquad \text{(B2)}$$

For the design flow x_0* we can write the time-varying p_t as:

$$p_\tau = \exp(-(\lambda_0 - b\tau)x_0 *) \qquad \text{(B3)}$$

We can combine terms in (B2) and (B3) to arrive at a useful form of p_τ which we can equate with h(τ) and continue on to derive H(τ) and S_T(t).

$$p_\tau = h(\tau) = p_0^{\,1-b\tau/\lambda_0} \qquad \text{(B4)}$$

Note that when *b*=0, (B4) reduces to the stationary case. Since the survival function S_T(t) is related to the hazard function we can derive a model of the survival function that depends on the rate of change in the mean (*b*) and the probability of exceedance and scale parameters at time zero, p_o and λ_0.

$$S_T(t) = \exp\left[\frac{-p_0}{bx_0 *}\left(\exp(b\tau x_0 *) - 1\right)\right] \qquad \text{(B5)}$$

Alternatively we can write S_T(t) without the design event *X** in the equation by substituting (B2) into (B4) and solving for S_T(t):

$$S_T(t) = \exp\left[\frac{p_0\lambda_o}{b\ln(p_o)}\left(\frac{1}{p_o^{\,b\tau/\lambda_o}} - 1\right)\right] \qquad \text{(B6)}$$

We can also derive the cumulative hazard function for this case, where the interpretation of H(τ) is the total number of expected failures (flows exceeding the design event) over a certain number of years.

$$H(\tau) = -\frac{p_0\lambda_o}{b\ln(p_o)}\left(\frac{1}{p_o^{\,b\tau/\lambda_o}} - 1\right) \qquad \text{(B7)}$$

We can now derive $F_T(t)$ and $f_T(t)$ from (B5) using known relationships.

$$F_T(t) = 1 - S_T(t) = 1 - \exp\left[\frac{-p_0}{bx_0*}\left(\exp(b\tau x_0*) - 1\right)\right] \qquad \text{(B8)}$$

$$f_T(t) = \frac{d}{dt}F_T(t) = p_0 \exp\left[b\tau x_0* - \frac{p_0}{bx_0*}\left(\exp(b\tau x_0*) - 1\right)\right] \qquad \text{(B9)}$$

or written shorthand as $f_T(t) = \frac{p_0\lambda_o}{p_o^{\,b\tau/\lambda_o}} S_T(t)$. The beta term can be easily replaced by the magnification factor described in Chapter 1. While the exponential hazard function family is important to survival analysis, the inflexibility of a constant hazard function limits its use for modeling survival times in many applications.