c. Then carried through a mathematical analysis to explain why the longitudes would be so much more sensitive to the distance between the sun and equant than to the location of the center of the circle, thus explaining Tycho's success with the sun (Chapter 31)

7. The combination of the actual sun and the bisection of the eccentricity of the earth-sun orbit reduces the discrepancies in Mars's orbit from greater than 4 degrees to a few minutes of arc -- a reform that can be introduced just as well to the Ptolemaic system (see Voelkel and Gingerich)
   a. In effect, an order of magnitude-plus reduction in discrepancies between observation and Ptolemaic theory when latter revised for actual sun and epicycles have bisected eccentricity
   b. Had these two reforms been carried out earlier, a very high standard of observational accuracy would have been required to give reason to pursue the further refinements of the area rule and the ellipse

F. "Phase 3" Continued: the Area Rule
   1. Now adopts new hypothesis in place of the equant -- arc length velocity everywhere varies inversely with distance from the sun
      a. Kepler had long believed that the sun provides the motive power driving the planets, and thought it must diminish with distance in keeping with the longer periods of outer planets
      b. In Ptolemaic type orbit as in vicarious theory, with equant but with bisected eccentricity, velocity exactly inversely proportional to distance from sun at perigee and apogee
      c. So, now replace the equant with this feature of it, but generalizing across entire orbit: a minimal move beyond the vicarious theory
   2. But now finds this new rule for locating Mars versus time calculationally taxing (needed calculus)
      a. Substitutes an approximate method, dividing circular orbit into equal 1 deg segments, computing distance of each arc from sun and adding up these distances, so that the time in each arc determined by the ratio of its distance to the sum of the distances
      b. Verifies this simplification against Tycho's solar theory (within 9 arcseconds)
   3. Then happens upon a still simpler approximation to the inverse distance from the Sun rule: equal areas in equal times (Chapter 40: "An Imperfect Method ...")
      a. I.e. area of sector as a measure of all the distances within the sector (which were being summed on above method): assume areas proportional to times for the equal arcs
      b. Again verifies against Tycho's solar theory (within 34 arcseconds)
      c. Note the reasoning here: so long as within observational error bands, both methods okay
   4. Proceeds through remainder with two separate motion rules to replace the equant hypothesis, the inverse distance rule and the area rule, though he never states the latter in a fully perspicuous manner until after *Astronomia Nova*

G. "Phase 4": The Orbit is Definitely Oval
   1. Given rule (in fact two rules) for motion, now ready to address question of trajectory

a. But not in a position to infer distance from sun via motion rule alone, for directional orientation of each small arc not determined
b. I.e. still had to make some assumption about trajectory

2. Asked what physical factor might conceivably cause an eccentric circle, as in revised "solar" theory, if sun controlling the velocity (via magnetic effect)
   a. Concluded that planets must have independent source of motion superposed on sun effect
   b. This representable by means of a small epicycle superposed on circle with sun at center, to produce a circle with sun off center (a classic Apollonian move, as discussed in *Apologia*)
   c. For orbit to be circle and area rule to be satisfied, motion in epicycle must not be uniform; but proceeded anyway
3. I.e. proceeded to use assumed circular trajectory and the area rule to determine heliocentric longitudes of Mars, comparing them with values obtained from the vicarious theory
   a. Result: good agreement in apsides and quadrants, but error in octants
   b. +8 min 21 sec for first octant, -8 min 1 sec for third (see figure from Wilson in Appendix) -- i.e. planet moving too rapidly in apsides, too slowly in quadrants
   c. Verify that these discrepancies not just from using area rule by confirming that comparable discrepancies emerge as well with the 1/r rule
   d. To satisfy area rule, then, orbit must be oval rather than circular, for need less area around the quadrants; same true with 1/r rule as well
4. Triangulation calculations of the sort described last week (using the modified earth-sun theory) confirmed that the orbit is an oval of some sort (see Appendix), but sensitivity to observational errors prevented him from concluding what specific oval
   a. These triangulations used heliocentric longitudes from the vicarious theory, earth-sun distances from the modified solar theory, and geocentric longitudes from Tycho's observations -- all elements that had not been available e.g. to Copernicus
   b. Calculations difficult, given need to control for observational errors; Kepler tried several ways
   c. Conclusion: orbit comes in around 800/152,500 parts, with estimated errors of 100 to 200 parts
5. Two upshots: (1) orbit some kind of oval; and (2) area rule survived a test, for it predicts an oval and the distance calculations, though inexact, confirm an oval
   a. (Moreover, the conclusion that it is some kind of an oval is theory-dependent to a sufficiently large extent that some would have seen it as piling hypothetical conclusions on top of hypothetical conclusions in just the way that elicits objections and complaints in courtrooms
   b. Maybe why Kepler hesitated for two years before finally abandoning the circle)
6. Did manage to get some valuable conclusions out of the triangulations (see Appendix):
   a. To within the bounds of uncertainty, the oval is bi-laterally symmetric with respect to the line of apsides (Chapter 51)