a. A famous problem to which Galileo had offered a solution that he had already learned to be mistaken (from Mersenne and Fermat) -- see Koyré's monograph

b. Concession drops claim that parabola even an idealization, for no longer said to hold exactly in absence of resistance

c. Raises a question for others: what trajectory does a projectile truly follow in the absence of resistance, with the earth's curvature taken into consideration

4. Galileo has a real problem here that he does not resolve, namely how to reconcile local motion near the surface of the earth with circular motion

a. Response here simply withdraws claim that results would hold exactly without resistance -- hold only approximately, but still of practical value

b. In process opens up a number of problems for others to address in future -- exact motions, relation between local motion and (celestial) circular motion, etc.

c. These questions will receive increasing attention in subsequent years

5. Galileo's response to question about resistance is maybe his clearest statement that no science of resistance possible at all [275ff]

"No firm science can be given of such events [*accidenti*] of heaviness, speed, and shape, which are variable in infinitely many ways. Hence to deal with such matters scientifically, it is necessary to abstract from them. We must find and demonstrate conclusions abstracted from the impediments, in order to make use of them in practice under those limitations that experience will teach us. And it will be of no little utility that materials and their shapes shall be selected which are least subject to impediments from the medium, as are things that are very heavy, and rounded."

a. Practical defense: use scientific result as basic approximation and then fudge it as needed -- engineering defense

b. Further argument: fudge not that large in cases of interest -- in particular, small enough that need not worry about how resistance separately undermines the individual patterns of the two compounded motions; so, combining separate components okay

c. {Note reference to "supernatural" motion in cannon -- speeds greater than generable by nature, [278]}

C. The Problem of Compound Impetus: Which Parabola

1. Galileo's next concern is with the impetus and hence speed that occurs in compound motion

a. Remember again that impetus relates to percussive effects on impact, which Galileo took to be proportional to weight and (mistakenly) speed

b. Speed is the scalar magnitude of the velocity vector, always taken in the direction of the motion

2. Impetus and hence speed of two compounded uniform motions is simply the vector sum of the respective uniform impetuses and speeds

a. For, spaces traversed in equal times can be used as a measure of speed and hence of impetus in the case of uniform motion

b. And combined space is the vector sum of the individual spaces

c. (Fortunately, Galileo did not resort to impact measurements here)

3. But cannot thus use spaces traversed as measures of speed or impetus when one component of the compound motion is not uniform

   a. For speed and hence impetus not proportional to space covered -- i.e. speed = $\sqrt{(2*s*a)}$ with uniform acceleration a

   b. Galileo is reluctant to appeal to some notion of instantaneous velocity, which was considered suspect (from Zeno's paradoxes)

   c. Galileo not mathematically permitted to express speed or impetus in units of length versus time

   d. He needs some geometric way to characterize compound impetus and speed, and cannot use our way, simply vectorially combining algebraic expressions for speed

4. A related problem Galileo faces is to be able to specify the specific parabola the projectile describes, given its initial horizontal motion and given the uniform vertical acceleration from nature

   a. Cannot just specify a value of initial velocity and then use algebra for the rest, as we would, for cannot specify such a value in units of distance/time

   b. Free fall rate, g, is set by nature, but uniform horizontal motion is arbitrary, so that the specific dimensions of the parabola depend on it – i.e. the scaling factor p in $x^2 = 4py$ (see Appendix)

   c. Galileo needs some measure of uniform horizontal motion that makes it (geometrically) commensurate with free fall

5. Salviati poses the problem in just this way when explaining Proposition IV [286, 288, and 289]

   a. Need unit of length, unit of time, unit of impetus, and unit of speed so that ratios possible

   b. Last unit should enable geometric comparisons and compounding of uniform horizontal and uniformly accelerated vertical motions

D. Galileo's Solution for Compound Impetus

1. Galileo correctly insists on some naturally governed phenomena as the basis for measurements generally, such as the pendulum for time [286]

   a. Need phenomena that are the same everywhere (and at all times) so that measurements made at different places (and times) are commensurate with one another

   b. I. e., need an "invariant" measure of speed and impetus

2. Galileo's proposal: use as measure of speed (and hence impetus) that gained in free fall from a specified height, so that *height* of fall from rest becomes a ***proxy*** for *speed*^{2} and hence for *speed*

   a. He asserts that this is a natural and invariant relationship everywhere around entire surface of the earth, and hence meets the needed condition to be a universal measure

   b. This assertion has really been an implicit part of the physical theory throughout, but it has been made fully explicit for the first time here

c. Notice that Galileo's theory is what licenses a height to be a measure of speed -- a theory-mediated measure, using a proxy for speed (squared)!

(1) Same height of fall, same speed -- a prerequisite

(2) Regardless of path, regardless of weight, regardless of shape of the movable: all those Galilean principles, and not just the principle of free fall, needed to license the use of height as a proxy for speed squared; so more thoroughly theory-mediated than one might notice

(3) Further claim: invariant over the surface of earth

3. Given some specified height as a unit of measure of speed and impetus, then measure of speed and impetus generally will be specified in terms of the square root (i.e. the mean proportional) of distance an object would have to fall to gain impetus in question

a. I.e. Given AB as basic measure of impetus, and want to know impetus acquired in fall through AC; measure is the length that is the mean proportional between AB and AC

b. Because, of course, speed = $\sqrt{(2*g*h)}$, so that ratio of speeds is as sqrt of ratios of lengths: AD = $\sqrt{(AB*AC)}$, so that AD/AC = $\sqrt{(AB/AC)}$

c. (Mean proportional determinable by ruler and compass; see Euclid, VI, 13)

4. Using vertical distance in this way provides a conveniently commensurate measure for comparing horizontal and vertical impetus and speed even when the vertical acceleration g -- the fall in one second -- is unknown

a. Horizontal distance traversed in time required for free fall from specified height = twice that height

b. Thus earlier theorem provides way of expressing uniform horizontal speed and impetus in terms of speed and impetus gained in free fall without as such requiring a specific value of g!

5. Note that Galileo might have tried to use impact measurements for impetus, as suggested by the discussion on [291-293]

a. Fortunately he did not, but even if he had, the rules of his mathematics would have forced him into some form of extensive representation of the measure, some form of proxy

b. This way not only gives him the latter, but also avoids problems of measuring percussive force, a parameter of great interest in military ballistics (see "Added Day")

E. Galileo's Determination of the Parabola

1. Suppose now we are given a projectile with uniform horizontal motion of a specified speed and impetus

a. Let the vertical distance p be that required for projectile to gain this speed and impetus in free fall -- i.e. the sublimity

b. But then the parabola is the one obtained by allowing the projectile to fall a further distance p while proceeding horizontally a distance 2p