b. Ptolemy's and Copernicus's treatments of latitude so complicated, and so unsuccessful, in large part because of this

c. Kepler ends up offering an explanation of the inclination in terms of an interaction between the (hypothesized) magnetic actions between the sun and Mars

4. Kepler's approach was to use certain privileged observations -- e.g. when the Earth is on the line of nodes and the line from Earth to Mars is perpendicular to the line of nodes (Wilson, Fig 4)

   a. Used a number of privileged observations of this sort, obtaining stable values for the inclination

   b. Then at end, after orbit defined and inclination corrected to 1 deg 50 min, 30 sec, compared calculated and observed geocentric latitudes for a large number of observations at opposition, concluding, contrary to Copernicus, that no other element needed to account for latitudes

   c. Finally, takes the trouble to check Ptolemy's observed latitudes (for fear of changes over long periods of time): consistent with little change (Ch. 68-70)

   d. (First pass needed because Kepler had to locate the nodes of Mars preparatory to using Tycho's data to determine oppositions to the actual sun in deriving his first theory of the orbit)

5. A tremendous step forward in the history of planetary astronomy

   a. E.g. comparison of calculated and observed latitudes for the 12 oppositions used by Kepler shows several in the 2-5 min range and one of 13 min -- owing to a mistake in calculation: 3 deg 20 min should be 3 deg 26 min (p. 388)

   b. Attributes remaining discrepancies to uncertainties in parallax and atmospheric refraction

D. "Phase 2": The "Vicarious Theory"

1. To use triangulation to infer Mars-sun distances, need earth-sun distances and, more important, heliocentric longitudes for Mars and earth

   a. Tycho's (improved) theory for the sun was known to give accurate heliocentric longitudes for earth -- this via comparison of theory and measurement for Tycho's large number of observations of midday altitudes of the sun

   b. Worst problem, then, was to obtain trajectory-independent heliocentric longitudes of Mars

2. Approach: use observations at opposition, when heliocentric longitude of Mars is the same as that of earth -- i.e. when heliocentric and geocentric longitudes of Mars coincide

   a. Calculated 12 oppositions from groups of observations made roughly at the times of opposition -- extracting an "observation" from Tycho's observations

   b. Took opposition relative to true sun, and not to mean sun (as had been the customary way to define it, in keeping with the mean sun being the reference point for the basic time unit)

   c. Not just redefining "opposition", but then also inferring the time when opposition occurs from observations near, but not at opposition

3. Assumed a quasi-Ptolemaic model for Mars -- an eccentric circle, with an equant at an independent eccentricity along the line of apsides

a. Knew the radius of the circle, but had to determine line of apsides, locations of eccenter and equant, and the time of aphelion
b. I.e. four unknowns requiring four observations and solution of what amounted to a 16th-order equation
c. No direct calculational methods then available, so had to proceed by trial and error (first requiring the four to lie on a single circle of known radius and then requiring the center of the circle to lie on line between sun and equant)
d. Selected four observations from the 12 and proceeded through 70 trials until requirements met
e. Then checked result against the other 8 observations, finding an average discrepancy around 50 sec, with a maximum of 2 min 12 sec

4. Upshot was a model -- he called it his "Vicarious Hypothesis" -- that predicted Mars's heliocentric longitudes to within the accuracy of Tycho's observations everywhere around the zodiac, provided that Mars orbits the sun
    a. Could henceforth use this theory in place of observations to determine heliocentric longitudes for Mars at any time under the proviso
    b. Thus freeing him from having to depend on specific observations that were primarily acronychal -- i.e. near opposition
    c. If instead of the proviso a Ptolemaic account of Mars is adopted, the vicarious hypothesis gives the longitudinal position of the center of the Mars epicycle on its deferent at all times
5. The trouble, however, was that the vicarious theory was demonstrably false
    a. Angles in earth-sun-Mars triangle known from Tycho's solar theory, Kepler's vicarious theory, and observation
    b. Earth-sun distances known to reasonable accuracy from Tycho's solar theory; Kepler confirmed this, by using parallax observations, to show earth-sun distances do not change very much
    c. Can thus calculate Mars-sun distances from observations; Kepler did so, and found that the center of Mars's orbit falls near the midpoint between the Sun and the equant, and not at a little more than 0.6 of the distance between the two as the vicarious theory had it
    d. Changing the center in the vicarious theory to the midpoint yielded heliocentric errors in longitude as large as 8 min, well outside the error-bands of Tycho's observations
    e. (This was the second cross-check; the first revealed discrepancies in latitude near opposition, discrepancies that could be removed by bisecting the eccentricity)
6. (From parallel reasoning, the violation of bisected eccentricity when the vicarious hypothesis is taken to give accurate longitudes along Mars's deferent shows its inadequacy in a Ptolemaic theory as well)
7. Upshot: the model underlying the vicarious hypothesis must be wrong -- either equant or circular trajectory or both wrong (see quote from end of Chapter 20 in appendix); this freed Kepler from having to stay within the constraints of a 1500 year tradition in astronomy

a. Corollary: if trajectory a circle, no point of equiangular motion at all (see Appendix); at best a point oscillating in location along the line of apsides
b. Obvious question: how can the theory give heliocentric longitudes within observational accuracy
c. Kepler took the trouble to answer (Chapter 21), showing that the location of the equant is more critical than the location of the center

8. Notice finally that without Tycho's accurate measurements, no Keplerian reform of trajectories, for modified vicarious theory would have been found accurate to within 10 min of arc -- the old observational standard
   a. Names and dates in the history of science would be different
   b. Kepler's view of the relative accuracy of Tycho's observations was also crucial here

E. "Phase 3": Bisect the Eccentricity of the Earth-Sun Orbit

1. Kepler chose first to abandon the equant, not just because it had long been criticized, but also because he saw no hope of giving a physical explanation of equal angular motion about a mere point in space
   a. Note here that two separate elements involved in any planetary theory, the trajectory and the variable rate of motion along it
   b. Really then two separate assumptions being adopted in prior theories
2. From before he had begun with Mars, he had conjectured that the (arc length) velocity of the planets was inversely proportional to their distance from the sun -- in keeping with his conjecture that a magnetic flux from a rotating sun drove all the planets, a flux that diminished inversely with distance
   a. He showed that this variation in velocity is approximately correct, via comparison with planets
   b. Moreover, with Ptolemy's bisected eccentricity, exactly correct at two ends of lines of apsides (on the deferent) for each planet, though not elsewhere
   c. Thus reconcilable with classic approaches to fixing apsides and eccentricity
3. Problem: in prior theories of the sun, including Tycho's superior theory, no bisected eccentricity of earth-sun orbit; instead equiangular motion about the center
   a. Therefore a direct empirical objection to hypothesis that planet velocities vary inversely with distance from the sun (taking earth to be a planet)
   b. Had to dispense with this objection before proceeding
   c. Provided multiple arguments from different observations, considering Ptolemaic, Copernican, and Tychonic theories, as well as his own, and ended with an argument laying out his physical conjecture
   d. (A numerical error made the first of these arguments appear better than it was)
4. In his main effort to show that bisected eccentricity should hold for the earth-sun orbit too, Kepler used combinations of three observations 687 days apart so that Mars (presumably) in the same place each time (or, on Ptolemaic view, center of Mars's epicycle in the same place on its deferent)