Discourse on the Method of rightly conducting one's reason and seeking the truth in the sciences, and in addition the Optics, the Meteorology, and the Geometry, which are essays in this method

Originally proposed title: ***The Plan of a universal Science which is capable of raising our nature to its highest degree of perfection. In addition, the Optics, the Meteorology and the Geometry, in which the Author, in order to give proof of his universal Science, explains the most abstruse Topics he could choose, and does so in such a way that even persons who have never studied can understand them***

Descartes on Reflection, Refraction, and "Snel's" Law

Fig. 8.

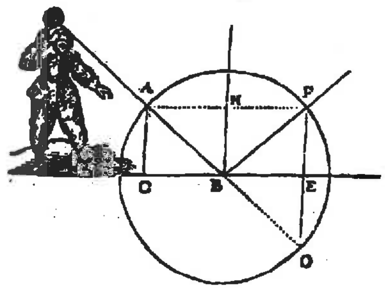


Fig. 6.

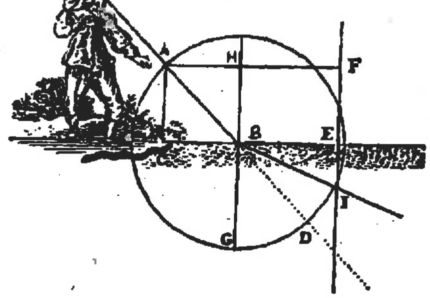
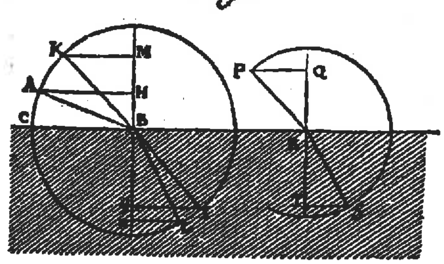


Fig. II.



Descartes on Lenses for Telescopes

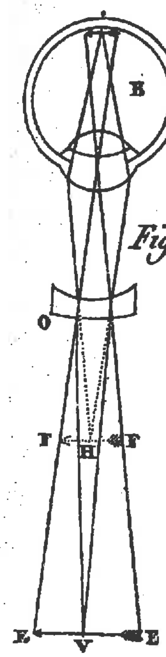


Fig.27.

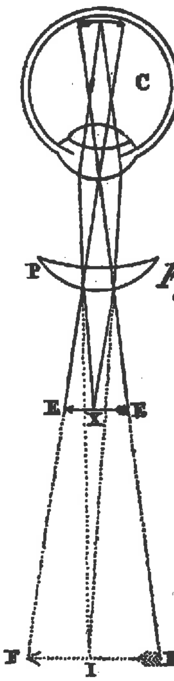


Fig.28.

Having accordingly thus chosen the glass which is the purest and the least colored, and which causes the least possible reflection, if we wish by means of it to correct the defect of those who cannot see objects at some distance as well as close ones, or the close as well as the distant, the most appropriate shapes for this effect are those which are traced by hyperbolas. For example, if the eye *B* or *C* is disposed to cause all the rays coming from point *H* or *I* to converge exactly in the center of its base, and not those from point *V* or *X*, in order to make it see the object near *V* or *X* distinctly, it is necessary to place the lens *O* or *P* between the two, whose surfaces, the one convex and the other concave, have the shapes traced by two hyperbolas which are such that *H* or *I* is the burning point of the concave surface, which must be turned toward the eye, and *V* or *X* that of the convex.

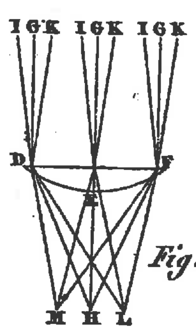


Fig.53.

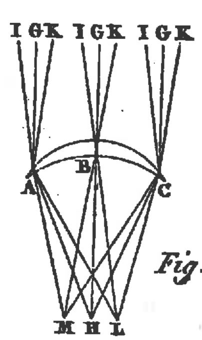
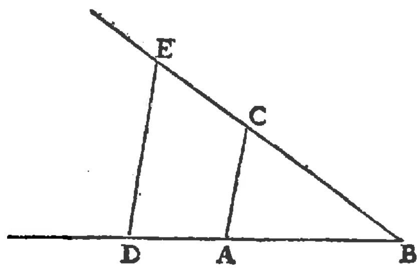
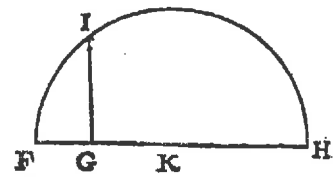


Fig.54.



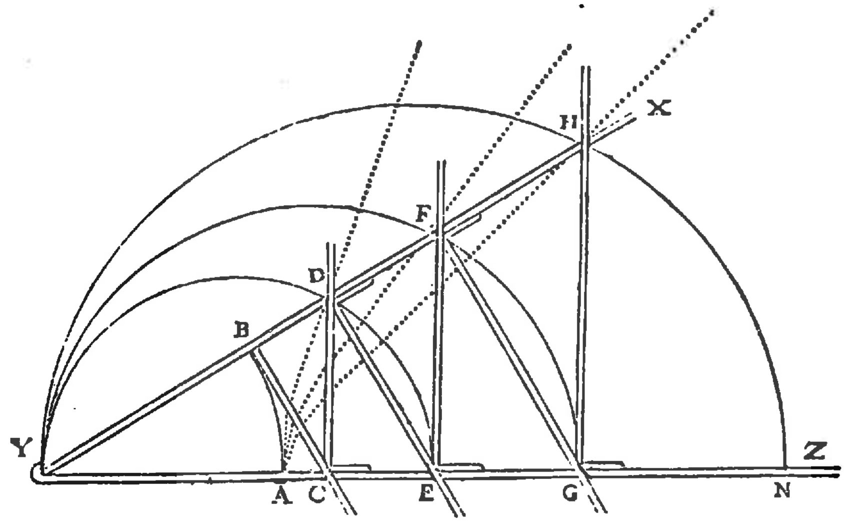


For example, let AB be taken as unity, and let it be required to multiply BD by BC. I have only to join the points A and C, and draw DE parallel to CA; then BE is the product of BD and BC.

If it be required to divide BE by BD, I join E and D, and draw AC parallel to DE; then BC is the result of the division.

If the square root of GH is desired, I add, along the same straight line, FG equal to unity; then, bisecting FH at K, I describe the circle FIH about K as a center, and draw from G a perpendicular and extend it to I, and GI is the required root. I do not speak here of cube root, or other roots, since I shall speak more conveniently of them later.

Often it is not necessary thus to draw the lines on paper, but it is sufficient to designate each by a single letter. Thus, to add the lines BD and GH, I call one a and the other b, and write $a+b$. Then $a-b$ will indicate that b is subtracted from a; ab that a is multiplied by b; $\frac{a}{b}$ that a is divided by b; aa or a^2 that a is multiplied by itself; a^3 that this result is multiplied by a, and so on, indefinitely.[4] Again, if I wish to extract the square root of a^2+b^2, I write $\sqrt{a^2+b^2}$; if I wish to extract the cube root of $a^3-b^3+ab^2$, I write $\sqrt[3]{a^3-b^3+ab^2}$, and similarly for other roots.[5] Here it must be observed that by a^2, b^3, and similar expressions, I ordinarily mean only simple lines, which, however, I name squares, cubes, etc., so that I may make use of the terms employed in algebra.[6]



Now as the angle XYZ is increased the point B describes the curve AB, which is a circle; while the intersections of the other rulers, namely, the points D, F, H describe other curves, AD, AF, AH, of which the latter are more complex than the first and this more complex than the circle. Nevertheless I see no reason why the description of the first[73] cannot be conceived as clearly and distinctly as that of the circle, or at least as that of the conic sections; or why that of the second, third,[74] or any other that can be thus described, cannot be as clearly conceived of as the first: and therefore I see no reason why they should not be used in the same way in the solution of geometric problems.[75]

[73] That is, AD.

[74] That is, AF and AH.

[75] The equations of these curves may be obtained as follows: (1) Let $YA=YB=a$, $YC=x$, $CD=y$, $YD=z$; then $z:x=x:a$, whence $z=\frac{x^2}{a}$. Also $z^2=x^2+y^2$; therefore the equation of AD is $x^4=a^2(x^2+y^2)$. (2) Let $YA=YB=a$, $YE=x$, $EF=y$, $YF=z$. Then $z:x=x:YD$, whence $YD=\frac{x^2}{z}$. Also

$$x : YD = YD : YC, \text{ whence } YC=\frac{x^4}{z^2}\div x=\frac{x^3}{z^2}.$$

But $YD : YC = YC : a$, and therefore

$$\frac{ax^2}{z}=\left(\frac{x^3}{z^2}\right)^2, \text{ or } z=\sqrt[3]{\frac{x^4}{a}}.$$

Also, $z^2=x^2+y^2$. Thus we get, as the equation of AF,

$$\sqrt[3]{\frac{x^8}{a^2}}=x^2+y^2, \text{ or } x^8=a^2(x^2+y^2)^3.$$

(3) In the same way, it can be shown that the equation of AH is

$$x^{12}=a^2(x^2+y^2)^5.$$