b. The addition of Torricellian principle and Galileo's principle of ascent=descent now allows the prior theory to be extended to remove idealizations, in the process removing discrepancies between theory and observation associated with them

c. More exacting agreement with observation is being achieved not by adding independent elements to the theory -- as Kepler would or as a theory of resistance would -- but out of largely the same theory, extended only to include Torricellian principle

6. This is the first place we have seen where discrepancies associated with idealizations are being addressed by removing the idealizations without really changing the theory

a. When successful, an extremely powerful form of evidence for a theory, for not only dispensing with discrepancies, but showing that theory not *ad hoc*, since it can account for things beyond what it was originally intended to

b. Also showing how effective the theory is in allowing information to be extracted from the world, for original theory the basis for interpreting the discrepancies to be from the idealizations, and not from something else

V. "Rational Mechanics": in the Tradition of Huygens

A. Leibniz versus Descartes on Conservation Laws

1. Leibniz, perhaps the foremost protegé of Huygens, had come to Paris from 1672 to 1676, asking Huygens for tutelage toward becoming a philosopher

a. Huygens recommended that he concentrate on mathematics, working with him for the four years he remained in Paris, and corresponding with him from then on

b. Leibniz announced the calculus in the new journal, *Acta Eruditorum*, in 1682, following it in 1684 with the watershed paper: "A new method for maxima and minima as well as tangents, which is impeded by neither fractional nor irrational quantities, and a remarkable type of calculus for this" -- the differential calculus

2. The *Acta Eruditorum* paper of 1686 is famous in part because it challenged a fundamental principle of the leading philosopher of the time, Descartes' third law and more generally the fundamental conservation principle underlying it, conservation of $B*\text{abs}(v)$

a. The paper triggered a strong controversy over the next decades

b. Nowadays, usually (and wrongly) described as a controversy resulting from confusion about the difference between conservation of energy and momentum

3. Leibniz's paper presents the conclusion from Huygens's (not yet fully published) work on impact in a form accessible to the general learned public, at the end citing his solution for center of oscillation

a. In process driving home force of Huygens's result, which he chose not to emphasize in the paper

b. But also generalizing the result -- $B*v^2$ (in relation to $B*h$) conserved not just in impact, but in all situations, just as in Huygens's center of oscillation solution!

4. Leibniz's argument: Descartes' claim is incompatible with Galileo's results, and latter are to be preferred because they are empirically supported -- ultimately an appeal to empirical considerations
    a. Principle: "force" acquired in descent just sufficient for ascent to the same height
    b. Principle (known to Descartes): "force" needed to lift 4 lbs 1 yard = "force" to lift 1 lb 4 yards -- defensible via statics -- that is, the mechanics of machines
5. Leibniz's "*vis viva*" -- "living force" -- which is proportional to $v^2$ and not to v, is what is universally conserved; term "vis viva" introduced in Leibniz's "Specimen Dynamicum" of 1695
    a. The height to which a weight is carried is taken to be a basic measure -- again a more general statement of an idea that runs through all of Galileo's and Huygens's work on natural motion
    b. Analysis shows that what is conserved from bodies of one bulk to another is the $B*v^2$ needed to carry them to that height
6. Leibniz's generalization of Huygens's result not a mere philosophical exercise here, for taking a major step toward the general concept of mechanical energy (potential + kinetic) and toward conservation of mechanical energy as a basic principle in mechanics
    a. With ultimate theoretical foundations in the form of generalizations of certain empirically discovered "rational" principles like the principle of conservation of *vis viva*, instead of such narrower principles as Torricelli's
    b. (Notice how informative this more general principle would have been if they had noticed the discrepancy between rolling and falling along an inclined plane, for they would have immediately looked for the missing $v^2$, and would presumably have found it in rolling -- which seems to be more or less what Huygens did analytically in the early 1690s)

B. The Approach to Science Exemplified by Huygens

1. Huygens's work gives rise to a tradition called "rational mechanics," where the word 'rational' is intended to stand in some sort of contrast to practical mechanics, but also to generalizing inductively from empirical observation: in the spirit of Huygens's phrase, "not alien to reason"
    a. The idea was to develop mathematical theories covering various phenomena in mechanics by employing reason, informed by a qualitative understanding of the empirical world, to settle on appropriate fundamental hypotheses or axioms
    b. Much like constructing an axiomatized mathematical theory, except axioms here are being honed by requiring compatibility with what we generally know about the empirical world
    c. Idea: because precise experiments not possible, must rely on mathematics and especially telling experiments to filter out errors and block us from garden paths when devising empirical theories
    d. Mathematical tractibility a dominant factor, for needed to elaborate theory
2. The three assigned Huygens's pieces, standing alone, certainly provide a paradigm of such an approach, as does Galileo's work on free fall and projectile motion

   a. Huygens and Galileo are not conducting a series of experiments or observations and generalizing from them or fitting theory to them, in a manner analogous to that of Kepler
   b. Nevertheless, there is much more of an empirical dimension to Huygens's three papers than this description suggests
3. First of all, Huygens sees the elaboration of rich theory not just as an end to itself, but also as a means to higher quality empirical evidence
   a. Problems in direct measurement of g solved by elaborating a theory that provides means for precise measure
   b. Theories in the form of question-answering devices in general can lead to technological devices and experimental set-ups that yield much higher quality evidence
   c. Huygens held a strict hypothetico-deductive conception of evidence in science, as indicated by perhaps the best succinct statement of this approach (see Appendix from Class 7), from the Preface to his *Treatise on Light*, published in 1691 (under the same covers as his *Discourse on the Cause of Gravity*)
4. Second, Huygens's insistence on integrating the narrow problems he is addressing into a broader theoretical context allows diverse evidence to be brought to bear on the whole edifice
   a. So far as possible, proceed from principles used in theories of other phenomena -- preferably weak forms of these principles
   b. Look for ways to tie problem being addressed into other problems for which already have theories -- e.g. centrifugal forces and even motion under impact -- tied to gravity
   c. Can then bring evidence to bear on new theory -- e.g. evidence for centrifugal force claims via free fall theory
   d. And in turn diverse evidence accrues to original theory as it successfully lends itself to extension into new areas
   e. Huygens exploits this as a means for learning -- e.g. conservation of *vis viva* as a potential refinement to the theory
5. Third, Huygens's pursuit of stable, precise complementary measurements of fundamental quantities like the coefficient of proportionality g yields a much stronger form of evidence than does corroboration of salient predictions
   a. Fundamental quantities: those that are ubiquitous throughout the theory
   b. But for just this reason, ought to be open to diverse precise measurement if theory used to design requisite experiments
   c. Stable measure from one experimental paradigm is by itself strong evidence for the fragment of the theory in question, serving to define bounds of precision for it!
   d. Still stronger evidence, and evidence for overall theory, when obtain same value from experiments based on other parts of theory -- e.g. evidence for centrifugal force theory and for free fall

        from obtaining same value of g from conical pendulum as from cycloidal pendulum
    e. Occasional deviations then a basis for further empirical elaboration and refinement -- e.g. as in response to Richer's results from Cayenne
    f. As remarked more than once above, Huygens seems unaware of the power of this further form of evidence, but it is central to Newton's *Principia*
  6. Fourth, Huygens's success in extending the theory (without modification) to remove the point-bob idealization adds significant empirical evidence: initial theory an extendable first approximation! -- a move, like the emphasis on measuring fundamental quantities, that goes beyond Galileo
    a. Keep in mind how easy the solution for center of oscillation is to test, with two pendulums in synchrony if solution is correct
    b. Indeed, duration of their synchrony even determines a well-defined bound on accuracy
  7. Thus, while others in the tradition of rational mechanics may have managed to limit the detailed empirical dimension of their efforts in subsequent decades, Huygens surely was not trying to do so

C. The Emergence of Theoretical Physics as a Subfield
  1. The obvious way of characterizing the progress from Galileo's *Two New Sciences* to Huygens's *Horologium Oscillatorium* is to say that Huygens has extended the original Galileo theory to a much wider range of phenomena, yielding something that can legitimately be called a general theory of uniformly accelerated motion (along parallel lines)
    a. Now have a unified network of theoretical solutions to problems of motion involving gravity in one way or another, all built off a small number of basic principles (or hypotheses)
    b. Evidence for overall theory from salient predictions and from converging diverse measures of the fundamental quantity g
    c. In the process Huygens answered a range of outstanding questions including the twelve listed at the end of the Appendix, all within a unified theoretical framework
    d. And that in turn created a much larger range of opportunities to test, in one way of another, the fundamental principles of that theoretical framework
  2. But, at a higher level of abstraction, can also characterize the step from Galileo to Huygens as the emergence of theoretical physics as a distinct subfield, for that is just what theoretical physics typically does, develop rich mathematical theories, leaving extended testing of them to others
    a. I.e. theoretical physics, in contrast to speculative natural philosophy of sort found in Descartes
    b. It has remained a distinct subfield ever since Huygens (which may be a more instructive way of looking at rational mechanics)
  3. A subfield involving theoretical conjectures, but within a strict framework, the main elements of which have been empirically motivated and, to some extent, molded
    a. Start from problems and questions that seem to call for a question-answering type of theory, a set of lawlike relations among quantities licensing inferences from one quantity to another

    b. Propose theoretical solutions, proceeding as much as possible from principles -- as weak in form as possible -- devised for prior problems, and adding further principles only under demands of the problem at hand
    c. In process expose new principles, the generalizations of which can sometimes be regarded as empirically based foundations of the sort Descartes wanted
4. Although theoretical in character, such effort is empirically oriented in two crucial respects
    a. The theory is aimed at yielding new means for bringing empirical evidence to bear not just on new parts of the theory, but also on theoretical framework as a whole
    b. Integrating new problems into existing framework allows evidence to be brought to bear on the new, and diverse evidence from different applications to accrue to the whole
    c. Indeed, success-in-being-extended a major form in which evidence accrues to an initial fragment, as illustrated by the enormous increase in evidence for Galileo's fragment resulting from Huygens's efforts!
    d. Notice how what unified the theory is not an underlying mechanism -- e.g. gravity -- for impact and centrifugal force not governed by the underlying mechanism of gravity; what unifies the theory, besides uniform acceleration, is the means for acquiring evidence for the various interrelated parts of it, through tieing them together!
5. One thing that we can see happening as theoretical mechanics emerges in the work of Huygens is a concern for preferred foundations
    a. Huygens being the opposite of bold in his initial hypotheses; the theory itself then yields special cases of potentially bolder principles, and also a potentially simpler set of basic principles
    b. For example, conservation of mechanical energy principle is in process of emerging from Huygens's work, and would have emerged even more strongly if others had noticed the difference between rolling and falling
    c. Approach being taken is allowing the empirical world to mold the foundations -- not just by restricting them, but by justifying generalizations that can serve as alternatives to them
6. Another thing we can see happening is the proliferation of different concepts of force that will have to be reconciled at some point in the future
    a. Not just *vis* and *conatus*, but (1) static balance and elastic deformation; (2) gravity and tension in a string; (3) resistance of media; (4) *vis inertia* (Newton’s phrase) and centrifugal *conatus*; (5) Descartes' change in quantity of motion; (6) forces in impact -- percussion, impetus, and change of motion; (7) *vis viva*, proportional to $v^2$ and bulk, and to product of height and bulk
    b. As theory becomes elaborated, empirical bases for sorting out concepts become more available, suggesting that the way to go about reconciling them is through further theory development
7. In short, all the empirical advantages of having a subfield of theoretical physics are beginning to become apparent in Huygens's early work in mechanics

    a. Rational mechanics therefore need not be construed as a reflection of rationalistic philosophy and Platonism on the continent
    b. It can be construed as a sound response to the goal of developing high quality empirical evidence, letting the world be the ultimate arbiter of theory, in the face of difficulties in designing straightforward experiments that control for "externalities" like friction
    c. Would have been even more evident if Huygens had published his intended book on mechanics
8. Why with Huygens? Because he was historically situated to build on the work done before him, especially by Galileo and Descartes, but also Mersenne, Gassendi, Torricelli, Riccioli, and Stevin
    a. Huygens assimilated the prior work, where "assimilated" means not merely understanding it
    b. It means extending it beyond what those before him had done, which in this case resulted in a much richer theory, in part because he was combining Galileo and Descartes
    c. Can think of Huygens in relation to his predecessors in mechanics as the counterpart of Horrocks in relation to Keplerian theory -- only Huygens lived to work 40 years, versus Horrocks's 5 years

D. Huygens and the Continental Tradition in Mechanics

1. In a position now to give a picture of the history of the development of mechanics, showing where Huygens (and Newton) fit in
2. Four distinct traditions from the first half of the 17th century involved in the evolution of mechanics (see figure at end of Appendix) -- i.e. of "Newtonian" mechanics, which emerged in the form we know it at the hands of Euler in the mid-18th century
    a. One tradition comes out of orbital astronomy, with Kepler as the chief figure, and such other figures as Horrocks, Cassini, and Flamsteed, leading into Newton's *Principia*
    b. English "magnetical" philosophy forms a second tradition, coming out of Gilbert and continued by Wilkins, Wren, Hooke, etc., again leading (via Streete) into Newton's *Principia*
    c. A third tradition in "rational" mechanics has Galileo as the key figure in the first half of the 17th century, though Beeckman and Stevin deserve mention because of their influence on Huygens; Torricelli, Mersenne, and Riccioli are part of the bridge leading from Galileo to Huygens, the person who, on my view, becomes the key, pivotal figure within this tradition, for it was his work that made clear how solutions for individual problems can be unified into a theory
    d. Finally, a fourth tradition, emphasizing comprehensive theories with universal foundations, comes out of Descartes, influencing Newton and Huygens in somewhat different ways, and continuing to exert some influence well into the 18th century
3. A "rational" mechanics tradition grows out of Huygens initially through his direct protegés -- Leibniz, Johann and Jakob Bernoulli, and those working with them in developing mechanics and the calculus, starting in mid-1680's
    a. They in turn were followed by Johann's sons, Daniel and Johann, and their boyhood friend Euler, along with a group coming out of the Paris Academy, with d'Alembert the leading figure

   b. This group dominated both mechanics and mathematics during the first half of the 18th century, passing on the mantle to such people as Laplace and Lagrange
   c. "Rational" because proceeded overwhelmingly from theoretical principles that seemed to be appropriate, and not at all inductively from experimental results
   d. Experiments serve to verify salient test results, in the manner of Huygens, but lie mostly in the background: "theory agrees with experience"
   e. Opens up a potentially vast array of experimental tests and responses, and hence not inappropriate that experimental ramifications left to others, leading to a split of sorts between theoretical and experimental physics
   f. The way mechanics of motion is still taught today, with experiments presented almost as an afterthought

4. The influence of that tradition of "rational" mechanics on Newton's *Principia* comes not directly from Galileo, but from Huygens and his impact on various figures in England
   a. Newton read the *Dialogue*, but unless I am sorely mistaken, he never read *Two New Sciences* (at least before 1700) and most of his knowledge of the theory of "natural" motion comes from *Horologium Oscillatorium* (along with Digby, Charleton, Collins, and, after 1686, Halley)
   b. I.e. he came to know the theory in rich form it had taken after Huygens's work in the late 1650's
5. "Newtonian" mechanics does not come so directly out of Newton's *Principia* as it does out of the influence that book had on the 18th century figures in the tradition of "rational" mechanics deriving directly from Huygens
   a. One of the things they did was to recast the results of the *Principia* into calculus, using them as a check against their own results and, in some cases, building on them
   b. Euler was the one member of this group comfortable with working with forces; he ended up formulating the mechanics they were all generating within a Newtonian framework, using what we now call Newton's three laws of motion (though not the first, Euler explicitly put forward F=ma in 1745 and called it the fundamental principle of all mechanics in 1750)
   c. Others in the tradition tended to avoid forces and instead used global principles like conservation of *vis viva*, but their mechanics was nevertheless fully compatible with Euler's -- as of course was Lagrange's *Analytical Mechanics* (1788), the ultimate culmination of the rational mechanics tradition of Huygens and the Continent
   d. Ultimate culmination of tradition out of Newton: Euler’s monographs on pure mechanics (1750-1780s), yielding the Eulerian equations of motion, and Laplace’s synthesis with gravitational mechanics in *Mécanique Céleste* (1799-1805)
6. Upshot: Newton’s *Principia*, as important as it was, ultimately must be viewed as part of a larger unfolding context that had gained its momentum well before it, and continued to have influence somewhat independently of it indefinitely into the future