and in Vellum manuscript had rejected Galileo's value of the fall in 1 second (roughly 5 ft), and then proposed a 45 degree conical pendulum as a means for measuring the correct value

   c. Here he does mention the conical pendulum at the end, but he does not follow up the mention with a full mechanical theory in the way Huygens does

6. Nor does Newton derive any easily testable results, like Huygens's result for the intercepted pendulum, for he is pursuing different objectives
   a. That is, he is working outside the tradition of Galileo's *Two New Sciences*
   b. And he is being provoked by astronomical concerns

B. Four Immediate Applications of the Theory

1. Newton proceeds to derive one primary result about the centrifugal tendency at the surface of the Earth from the basic theoretical result, and then draws conclusions on three other topics
   a. With the exception of the acceleration of gravity, Newton is largely using values taken from Galileo's *Dialogue*
   b. (Interesting that he would reject Galileo's value for the acceleration of gravity and accept other values from the book)
2. Compare the centrifugal tendency at the surface of the earth to gravity, in the process confirming Galileo's explanation for why we do not feel the centrifugal tendency, much less fall off the earth
   a. Assume radius of earth = 3500 (Italian) miles, 5000 ft per mile
   b. Then since acceleration of gravity is 32 ft/sec/sec (corresponding roughly to Huygens's announced value), the force (*vis*) of gravity is 350 times stronger than the centrifugal *conatus* at the equator
   c. The correct value -- using a correct radius of the earth instead of the value Newton has taken from Galileo's *Dialogue* -- is 288
3. Compare the centrifugal tendency of the moon with that on the surface of the earth, in effect exploiting the fact that r/P^2 gives this proportionality
   a. Moon 60 earth radii away, so that centrifugal tendency on the surface of the earth is 12.5 times stronger than that of the moon
   b. Therefore, gravitational force at the surface of the earth is around 4000 (4375) times the centrifugal tendency of the moon -- versus 3600 times if $1/r^2$
4. Infer (incorrectly) a lower bound of the horizontal solar parallax (19 sec) from the assumption that the centrifugal tendency of the moon is responsible for our always seeing the same face
   a. For then the centrifugal tendency of the moon with respect to the earth would have to be greater than that of the moon with respect to the sun, for otherwise the moon would always show the same face to the sun, and not to the earth
   b. Therefore centrifugal *conatus* on surface of earth 132,408 times greater than centrifugal *conatus* with respect to the sun (assuming -- incorrectly -- that horizontal solar parallax is 24 sec)

5. Compare the centrifugal tendencies of the planets from the sun, using fact that this proportionality must be $1/r^2$ since the planets conform with Kepler's 3/2 power rule
    a. Centrifugal tendencies as 614 to 173 to 91 to 39 to 3.33 to 1, from Saturn to Mercury
    b. No reason given or suggested for taking the trouble to obtain this result
6. Final points concerning relation between conical pendulum and circular pendulum probably reflect his earlier idea of using a 45 deg conical pendulum to obtain a more accurate value for strength of gravity than Galileo had proposed
    a. Newton himself carried out a conical pendulum measurement of gravity somewhere around this time, stopping short of full precision once he got within 1 percent of Huygens's value
    b. Perhaps failure to obtain an exact $1/r^2$ for the Moon occasioned him to check Huygens's value

C. Newton's Preoccupation: Copernican Concerns

1. The common element in the three main applications of this manuscript, which appears to have been written for publication judging from how clean it is (see Appendix), is a concern with issues attendant to Copernicanism, more specifically with issues arising within Galileo's *Dialogue*
    a. Even the horizontal solar parallax is pertinent to issues raised in the *Dialogue*
    b. For, the Aristotelian can argue that, if the earth is orbiting the sun, then observed acceleration toward the center of the earth ought to be greater at high noon on the equator than at midnight, since centrifugal conatus additive at noon
2. By contrast, Huygens showed no interest at all in such Copernican issues in his treatment of centrifugal force
    a. Newton prompted by Descartes' discussion of the centrifugal tendency in planetary motion, Huygens prompted by his discussion of the sling
    b. Huygens did not even show interest in the $1/r^2$ implications of Kepler's third "law," something noticed by several others after the *Horologium Oscillatorium* was published in 1673
3. Newton's efforts more akin to those of Kepler's calculations in the *Epitome*, where Kepler was fishing for information about mechanisms underlying the planetary orbits
    a. Newton too appears to be somewhat on a fishing expedition, trying to draw inferences from the theoretical characterization of the *conatus a centro* and various other available information
    b. Almost as if his sole point in deriving the theoretical result on the *conatus a centro* was in the hope of learning more about celestial motion
4. Notice, however, Newton's lack of concern here for exact orbits, which he knew perfectly well from Streete were not circles
    a. In this respect, his effort is less akin to Kepler than to Descartes, for he is showing no interest at all in what might produce trajectories other than circles
    b. Furthermore, the values generated are pertinent to Descartes' vortex model, in which gravity at the surface of the earth is related to vortex pressures

5. The contrast between the moon and the planets is, of course, a Copernican issue discussed in the *Dialogue*, but it is even more so an issue in Descartes' *Principia*
    a. My guess then is that Newton was looking for some way to take Descartes' basic insight concerning the *conatus a centro* associated with curvilinear motion and begin turning it into an argument for Copernicanism
    b. Or at least to bolster, via a few specific numbers, some of the arguments in support of Copernicanism in the *Dialogue*

D. Universal Gravitation: The Historical Issue

1. The issue is whether Newton considered the hypothesis of inverse-square universal gravitation in the 1660s and rejected it because of the substantial empirical discrepancy between 4000 (4375) and 3600
    a. Or even the weaker hypothesis, gravity varies as $1/r^2$ and extends from the earth to the moon
    b. At least the latter hypothesis compatible with ones being entertained at the time and subsequently by Hooke and Wren, although surely this was not then known to Newton
2. The legend that Newton was already looking to gravity to explain planetary motion in 1666 derives from Whiston and Pemberton, and from Newton too, all after 1700 (see accounts in the Appendix)
    a. The legend of the apple is from Stuckley and Conduit, apparently originally owing to Newton sometime after 1715
    b. Part of Newton's embellished account of how much of the *Principia* was developed in 1666, years before Hooke had suggested anything to him
3. The source of the legend was undoubtedly Newton's defense against Hooke's accusation of plagiarism, an accusation that took on more bite following the dispute with Leibniz
    a. Newton's 1686 defense against Hooke's charge pointed out only the derivation of $1/r^2$ from the 3/2 power rule, and added that Wren and Gregory had derived the same thing before 1679
    b. Thus the 1686 defense is compatible with the manuscript on circular motion (dated by Turnbull as ca. 1669), and the later embellished defense is much less so
4. Note that throughout this manuscript Newton is comparing centrifugal tendencies -- *conati a centro* -- with one another and with the gravitational force at the surface of the earth
    a. No mention is made of gravity in the case of the sun and planets at all
    b. And, as Wilson points out, nothing remotely akin to universal gravity is ever suggested
5. Equally, however, one should notice that the comparison between 4000 and 3600 -- a comparison not in the manuscript -- does not make much sense unless Newton is either assuming or hoping to show that the 3/2 power rule holds for the moon
    a. There is of course no evidence for this at all at the time, though one might nevertheless hypothesize it on the basis that the 3/2 power rule is a general property of all celestial orbiting systems
        (1) Horrocks's improvement in Kepler's orbits from using the 3/2 power rule to determine semi-major axis is grounds for saying this

(2) Newton knew of Horrocks's success as well as the 3/2 power rule from Streete

b. And this hypothesis might well lead one to hypothesize a common mechanism for the planets and the moon, which might then lead to the question, does this mechanism have anything to do with terrestrial gravity

c. But this is a far reach from anything in this or the attendant manuscripts

6. Thus, Wilson's conclusions about the discovery of universal gravity, which are now almost universally accepted, continue to stand up under scrutiny

   a. Suppose even that Newton had used the correct earth radius and obtained a value very close to 3600, in contrast to 4375; what would he have done next?

   b. Equally, supposing Hooke had been clever enough to do the same thing in the late 1670's, what would he, having long been committed to celestial gravity as the mechanism governing planet and comet motion, have done next?

7. (One possible answer in Newton's case is that he would have concluded that the 3/2 power rule is a general feature of celestial orbiting bodies -- or at least a feature of bodies orbiting the earth -- and he might then have proceeded to argue for Copernicanism over Tychonism on the grounds that the horizontal solar parallax is quite incompatible with the 3/2 power law applied to the sun as well as the moon orbiting the earth

   a. That is, Newton may well have been trying to devise an argument against the Tychonic system on the basis of the 3/2 power rule

   b. To this end he needed to show that the 3/2 power rule holds around the earth

   c. Since he cannot get this from comparing bodies orbiting the earth, he instead gets the clever idea of trying to show that it holds by showing that there is an inverse-square relation between the *conatus a centro* of the moon and the acceleration of gravity at the surface of the earth

   d. The seemingly radical accompanying thought -- that terrestrial gravity holds the moon in orbit -- he had already encountered in a brief passage in Streete

   e. This view of what he was up to has the added virtue of fitting in perfectly with the preoccupation with Copernicanism of the rest of the manuscript, as well as the interest in the horizontal solar parallax)

E. The Implicit Conception of Empirical Science

1. As Westfall remarks, the story of universal gravitation coming to Newton in a flash during 1666 "vulgarizes universal gravitation by treating it as a bright idea" -- p. 155

   a. It also leads to a vulgarized conception of empirical science -- geniuses somehow in tune with reality who have profound insights into the nature of the universe, insights that the rest of us ultimately catch up to when nothing disproves the insight

   b. A conception under which the evidential side of science plays a negligible role in discovery and hence in the major events in the history of science

2. Given our central preoccupation with the development of increasingly sophisticated conceptions of empirical evidence in 17th century science, appropriate to ask what conception of empirical science is implicit in Newton's short tract on circular motion
3. One thing for certain is that it is far removed from the idea of achieving exceptionally high quality evidence through convergent, precise measures of a fundamental quality like *g*
   a. At the beginning of this class we listed three ways in which Huygens extended the conception of empirical evidence prevailing at the time
   b. All three are clearly present in Huygens's efforts on uniform circular motion
   c. The only one of the three remotely present in Newton's efforts is the idea that theory is a vehicle for evidence, for Newton is adopting various theoretical claims and using them to draw inferences from observations that could not otherwise be drawn
4. Equally, however, the conception of empirical science implicit in Newton's short tract is far removed from the one Descartes exhibits in his *Principia*
   a. Not putting forward bold hypotheses about underlying mechanisms and then drawing out an explanatory account of phenomena from them
   b. Indeed, no real mention of hypotheses about underlying mechanisms in the tract at all, and as it stands it is at least compatible with a vortex model
5. It is not so far removed from the conception found in Galileo's *Dialogue*, however -- a conception on which one is continually looking for empirical information that can be exploited as evidence to settle questions under dispute
   a. Wilson remarks that, partly through the influence of Barrow, Newton had come to maintain a sharp distinction between conjecture and experimentally established results (p. 139)
   b. On this view the fishing expedition Newton is here on amounts to exploiting Descartes' claim about the importance of the *conatus a centro* in all curvilinear motion to look for potentially compelling evidence for Copernicanism
6. If this is correct, then Newton had not yet begun to fashion a highly sophisticated conception of how to marshal empirical evidence in orbital mechanics at the time he wrote this tract
   a. Indeed, he was still more or less at the level of sophistication exhibited in Galileo's *Dialogue*, and far short of the much greater level Huygens had reached by this time
   b. But then the conception of science implicit here is far removed from that in the *Principia*

IV. Newton on Motion along a Cycloid (versus Huygens)

A. Newton's Two Part Proof of Isochronism

1. In his brief tract on the cycloidal pendulum, Newton provides derivations for three basic results announced by Huygens, but published only in 1673 in the *Horologium Oscillatorium*
   a. The isochronism of the cycloidal pendulum, and as a corollary that of the small arc circular pendulum