

#### D. Newton's Results Contrasted with Huygens's

1. If Newton's  $X$  is taken to be the change in translational velocity, then his algebraic solution is in full accord with Huygens's for the simple case of a moving sphere impacting a sphere head-on
  - a. Thus Newton has solved the problem Huygens set himself, which is also the problem Descartes originally set
  - b. He has even derived this solution from principles, though not the principles Huygens had used
2. But Newton's rules are much more general than Huygens's, for Newton not focusing on a theoretically basic, mathematically tractable problem, but on a physically realistic problem
  - a. Allowing for rotational motion and oblique impact a major complication from a mathematical and theoretical point of view
  - b. Huygens is much more clearly in the style of Galileo, working out the theoretically basic, ideal case with mathematical rigor, plus a striking prediction to serve as a test
  - c. Newton, by contrast, is in the style of Descartes, with concern for mathematical niceties giving way to a preoccupation with physical concerns
3. Much the same point can be made about the principles that Newton assumes as axiomatic: they are stronger and more empirically loaded than Huygens's
  - a. No Torricelli principle, but in its place a least resistance principle that makes a Cartesian claim about changes in motion generally
  - b. Not even a relative motion principle, motivating the whole approach, but instead a strong physical claim that Huygens takes the trouble to derive from seemingly weaker principles -- though again a principle that makes a direct claim about changes in motion, albeit a claim in direct contradiction to Descartes
4. Like Huygens, Newton takes the trouble to show that his rules of impact entail that Descartes' conservation of total motion is false
  - a. Total motion, ignoring direction, can be greater or less than it was before impact
  - b. Newton's reasoning employs oblique encounters, and hence are different from -- indeed, much simpler, though more dependent on physical intuition, than Huygens's
5. Newton does not draw the conservation of  $B*v^2$  conclusion that Huygens does, but he does draw another important conclusion in line with Huygens: "the common center of two or more bodies changeth not its state of motion or rest by the reflection of those bodies one amongst another"
  - a. No proof offered, but a proof for "hard" bodies should not be that hard from the algebraic solution, and a proof from Huygens's solution for the limited problem is straightforward
  - b. In effect, then, no created motion of the overall system, so that Newton's results are in accord with Torricelli's principle
  - c. This inferred result will prove important later on

6. As noted in the class on Huygens, later in the 1670s Newton gave an algebraic solution for the head-on impact of spheres (see Appendix)
    - a. An isolated problem among several other problems in his Lucasian lectures on algebra
    - b. Important because, in addition to using the speed of separation principle, he there uses a version of his third law of motion -- the approach he ultimately generalizes in the *Principia*
- E. Newton's Reservations about the Potential for Evidence
1. Still a further contrast with Huygens is Newton's lack of interest in pursuing a Galilean type of experimental confirmation for his account of impact
    - a. No attempt to tie the theory to Galileo's account of free fall -- e.g. via Torricelli's principle -- nor any hint of developing evidence for it via the ballistic pendulum
    - b. And no results for intervening bodies -- indeed, no "salient" predictions derived from the theory at all -- that could be the basis for a qualitative experimental test of the theory
  2. The section of "Observations" at the end is present in one version of the paper, and not in another, suggesting that it was an add-on following some disappointing experiments
    - a. As Newton's work in optics at the time indicates, he was not at all reluctant to turn to experiments
    - b. And his theory of impact calls for some experimentation if only because it presupposes some empirically determined quantities
  3. The principal problem in experimenting with impact that Newton calls attention to is the failure to realize the conditions of impact assumed in the theory
    - a. Bodies are not perfectly hard, but deform under impact, with some loss in recoil, and also with a consequent sliding of the interfaces
    - b. Experiments also confounded by problems in restricting contact to a single point of tangency
  4. The other key problem concerns the effects of resisting media, not only in slowing the bodies down, but also in affecting what happens at the point of impact
    - a. (Interest in the effects of resisting media was growing in England during the 1660's, though it is unclear whether Newton was aware of this until the early 1670s)
    - b. Paragraph 1 of "Observations" lists all the confounding items that need to be taken into account in any experiments on impact (see Appendix)
  5. The point to notice is that Newton here has a very different conception of the role of idealizations in the development of a theory and consequently of the approach to be taken in developing empirical evidence for the theory -- i.e. different from Galileo's and Huygens's
    - a. Idealizations are not being adopted in order to permit the development of empirical evidence, but only to allow a mathematical solution for a first approximation
    - b. Hence, no interest in devising experiments in which confounding effects are minimized or controlled -- i.e. experiments in the style of Galileo and Huygens

- c. Instead an insistence on dealing with the physically real case, and disappointment when this proves unmanageable

### III. Newton on Circular Motion (vs. Descartes, Huygens)

#### A. Basic Theoretical Results, versus Huygens's

1. The “vellum manuscript” (Add. 3958, f. 45) that anticipates this one is a torn legal parchment with a lease on the front and calculations on the back
  - a. That manuscript thus likely dates from when Newton was home during the plague years, in 1665 or shortly thereafter
  - b. If so, then the manuscript on circular motion, written in Latin as if for publication, dates from after that
2. Unlike Huygens, Newton does not formulate the question about the *conatus a centro* in uniform circular motion as one about the (instantaneous) tension in a string
  - a. Through what space would the *conatus* impel a body in the time of one full revolution if it were not constrained
  - b. What the total sum of the incremental distances by which the body would have departed from the circle had it not been constrained during the time of one revolution -- i.e. the  $s$  corresponding to the uniform acceleration ( $r*\omega^2$ ) over one period of revolution
3. The answer:  $D*\pi^2$  (in full agreement with Huygens since  $v^2/r = r*\omega^2 = 2 D*\pi^2/P^2$  and  $s = a*t^2/2$ )
  - a. Newton's formulation amounts to:  $x/BD = ADEA^2/AD^2$
  - b. From the end of Book 3 of Euclid,  $BE/BA = BA/BD$  (Euclid's Proposition 36: the rectangle formed by BE and BD is equal to the square formed on BA)
  - c. But for an infinitesimal increment, BE approximates DE, and BA, DA, so that BD approximates  $AD^2/DE$ , and answer results by plugging in for BD above
4. Newton's interest in the problem not a mechanical one, like Huygens's, but one concerning the degree of *conatus*, just as Descartes had called for in *Principia* III, 59
  - a. Notion of total endeavor in a revolution strange to us, though does give a measure of acceleration in terms of distance covered versus time (squared)
  - b. Simply assumes uniform acceleration (and Galileo's rule for distance in uniform acceleration, taken from the *Dialogue*), whereas Huygens feels required to argue for it! -- that is, to argue that the induced tension in the string is commensurable with the tension in a hanging string
5. Newton does not develop the result in the manner of Huygens, for the issue that concerns him is not posed within the framework of Galilean mechanics
  - a. Huygens ties theory of centrifugal force to Galilean fall in order initially to obtain a measure of  $g$ , and subsequently to provide evidence for theory via measure of  $g$
  - b. Earlier, in Waste Book Newton had devised solution for circular motion via impact (Appendix);