THE NEWTONIAN REVOLUTION – Part One

Philosophy 167: Science Before Newton's *Principia*

Class 7

Galileo's *Two New Sciences*: Projectile Motion

October 14, 2014

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Philosophy 167: Science Before Newton's Principia

Assignment for October 14

Galileo's *Two New Sciences*: Projectile Motion

Reading:

Galileo, Two New Sciences, "The Third Day," p. [226] to end (i.e. pp. 175-216 in the Drake edition), reading Propositions III-VIII fully and then reading the Propositions, Corollaries, Scholia, and discussions, but skimming over or even ignoring all of the proofs.

---- "The Fourth Day," completely (i.e. pp. 217-260 in the Drake edition).

Questions to Focus On:

1. The Academician's Treatise from which Salviati is reading in "The Third Day" proceeds to develop and prove some 38 theorems on uniformly accelerated motion. Of what interest or value are these theorems to anyone other than theoretical mathematicians?

2. Which, if any, of the mathematical results of Galileo's theory of uniformly accelerated motions provide a basis for empirically testing the theory, given the technology available at the time?

3. What assumptions concerning motion are needed in Galileo's proof that the trajectory followed by a projectile in the absence of air resistance is a parabola?

4. In order to identify which parabola a given projectile follows, Galileo needs some way of specifying the relevant conditions at the start of its motion. How does he solve this problem?

5. At the end of the theory of projectile motion, Galileo provides three tables relating different features of the trajectory. How might one test these tables? How might the tables serve as a basis for testing the overall theory?

6. The theory of local motion presented in the last half of Two New Sciences includes two of the cornerstone "laws" of modern kinematics, the laws of free-fall and projectile motion. How compelling is the evidence that is presented in the book for these two laws?

Galileo's *Two New Sciences*: Projectile Motion

I. Issues Raised by Galileo's *Two New Sciences*

A. The Issues and Controversies Reviewed

1. Galileo was supposedly the "father" of modern experimental science by virtue of (1) his emphasis on the contrast between experiment and mere observation of nature and (2) his commitment to resolving issues empirically

   a. Have seen his ingenuity in designing experiments displayed in the *Dialogue* and *Two New Sciences* -- especially experiments putting the lie to Aristotelian doctrines

   b. But Galilean scholarship has suggested that many of the experiments he mentions he probably did not do

2. Worse, highly meaningful experiments on local motion, with careful measurements, were scarcely possible during his lifetime because of limited means for measuring time (and speed)

   a. Ideally would like to measure height or distance versus time, just as in astronomy, but total elapsed times are of the order of 5 sec or less, and need small fractions of these

   b. Thus the theory of local motion, in direct contrast to orbital astronomy, had few constraining data to build a theory around

3. Worse still, we can now see that Galileo, and presumably others, were laboring under a systematic conceptual error, conflating rolling on an inclined plane with falling

   a. Galileo's main way around the time problem was to "slow" time down by turning to inclined planes

   b. But there is a systematic error in his theory here, for a sphere rollling on its bottom has only 5/7 of the acceleration of a falling sphere, so any simple inference from inclined planes to vertical fall involves a fallacy

   c. Empirical results needed to force this distinction, but problems in getting good enough results -- e.g. good enough measures of distance in the first second -- to recognize the need

   d. Poorly behaved data whenever a mix of rolling and falling, as will happen except at low inclinations, and with good equipment

4. A further complication arises because Galileo is putting forward an idealized science -- i.e. a science about what would happen in the absence of air (and other medium) resistance effects

   a. Can try to minimize the effects of resistance, but no way to eliminate entirely, so that end up with need for special experimental domains in order to pursue meaningful evidence: experiments that require design, in contrast to mere interventions in natural processes

   b. For example, since resistance increases with velocity, all the more reason to use an inclined plane to keep velocities low

   c. In absence of suitable experiments, run the risk of developing an unfalsifiable theory, not really grounded empirically

d. And even with suitable experiments, discrepancies between theory and observation remain seriously ambiguous

5. All of this raises a host of issues about the contribution made by *Two New Sciences* in the development of 17th century science -- e.g.
   a. In what sense did Galileo discover and establish "laws" -- did he do more than guess (as Newton said of Kepler), and are they really our laws
   b. In what sense was he engaged in empirical science at all -- did he ultimately really differ in approach from, say, Descartes
6. Variety of interesting issues can be raised about Galileo, but from our point of view ultimately only two issues of concern
   a. How much was known (ca. 1650) about naturally accelerated local motion as consequence of *Two New Sciences*
   b. How did those involved think this knowledge was being secured -- i.e. what was their conception of empirical science, and what evidential reasoning were they following

B. Evidential Status of the "Law" of Free Fall

1. Galileo offered an impressive array of experiments falsifying Aristotle and legitimating his splitting of local motion into two distinct processes -- a basic process and induced resistance
   a. Simple vertical fall experiments enough to falsify Aristotle's claim that speed proportional to weight
   b. Variations of resistance with medium density and experiments with pendula justify parceling out variables, so that basic process depends only on such things as height and direction
   c. Thus an empirical basis for his idealization, which should then be thought of as a working hypothesis
   d. Shift away from naturally occurring process to one that occurs only in hypothetical or contrived circumstances a profound step
2. His fundamental claim -- the basic process involves uniform (vertical) acceleration in time -- is put forward as the simplest alternative to consider, a hypothesis
   a. Can give empirical grounds that motion is not uniform, and hence need some rule governing how speed changes
   b. Can show that proposed rule is at least physically plausible, and it is mathematically tractable
3. The principle of uniform acceleration entails a distinctive phenomenon -- incremental distances in a 1,3,5,... progression -- that had not been observed or reported before Galileo (in *Dialogue*)
   a. Distinctive both in the sense that a striking pattern (even if only to high approximation) and in the sense that not entailed by e.g. uniform acceleration in space or other salient alternatives
   b. Postulate relating speeds acquired in free fall to speeds acquired along an inclined plane provides an experimental framework in which 1,3,5,.... should be reasonably detectable

4. Inclined plane experiments yield results that are at least compatible with, and hence do not falsify, the theory
   a. Galileo claims much more in *Two New Sciences*: systematic experiments in which time measured for different lengths along plane across different inclinations, with agreement within e.g. 1/10 sec
   b. But Mersenne could not replicate even with a pendulum, Galileo presents no data for us to assess, and such precise results are unlikely without very good ways of measuring time
   c. Nevertheless real experimental results, done with care, should have shown enough agreement with predicted 1,3,5,... pattern to provide adequate grounds for not abandoning the theory!
   d. I.e. real experimental results, however obtained, should have come close enough to showing a 1,3,5,... pattern at low inclinations to give reason for preferring the theory to any other that had been proposed
   e. Failure to obtain the distinctive pattern at higher inclinations, however, raises worries
5. Moreover, the evidence presented for "the" law of free-fall in *Two New Sciences* has a clear lacuna, called attention to by Galileo, in the step from inclined plane results to claims about free fall
   a. Galileo's "postulate": speed acquired in fall through a given height is same regardless of inclination of path
   b. Lacuna that could be bridged by measuring and comparing accelerations -- i.e. distances in first second -- to determine whether one uniform underlying value, as theory claimed
   c. (Lacuna not just a matter of incomplete logic, as the difference in acceleration between rolling and falling shows)
6. By 1651 Riccioli bridges lacuna by measuring distance of fall in first second -- i.e. our g/2 -- to within 5 (or at least 10) percent and by observing 1,3,5,... progession in free fall
   a. Obtains more or less as good evidence for vertical fall as anyone had for inclined plane -- published results much too good to be true, but this just shows that actual results were surely good enough not to abandon theory
   b. Two step experiment -- first measure 1,3,5,... and with it what we call g/2, then predict and test times of fall from variety of heights -- also gives evidence that g reasonably uniform in vertical fall
   c. Nevertheless, his measured fall in the first second nearly 5 (if not 10) percent in error; so in fact the experiment was not so successful as it appeared to be at the time
   d. Precise measure in vertical fall difficult: a tenth of a second error in timing from a 145 ft height, perhaps from speed of sound delay, might account for this error, with any remaining from resistance effects; but why is the error uniform across all the results?
7. So, by 1650 the most that can be said is that the uniform (vertical) acceleration hypothesis had at least managed to survive some substantive empirical tests -- something no other theory had done

II. "The Third Day": Ramified Mathematical Development

A. Propositions VII-XXXVIII: The Mathematical Theory

1. Remainder of the "Third Day" devoted to 32 further propositions, with geometric proofs, establishing relationships among variables that we would now establish algebraically
    a. Propositions VIII-IX continue comparisons started in III-VI
    b. Propositions X-XXVI concern initial speeds and diverting from one initial motion to another; culminates in key Scholium following Prop. XXIII
    c. Propositions XXVII-XXXI concern minimal time trajectories
    d. Propositions XXXII-XXXVI concern time comparisons along different paths; culminates in second key Scholium
2. 16 of the Propositions called Theorems and 16, Problems, where latter give constructions (with ruler and compass) that determine unknown quantities -- e.g. height of equivalent plane
    a. From our point of view, all 32 propositions assert relations among variables of the sort expressed in algebraic equations
    b. E.g. Prop. XXI: let object fall vertically through AC, then divert along plane; space along plane in time through AC is between double and triple AC
3. Symbolic proof of Proposition XXX will help one appreciate task Galileo faced working within compass-and-ruler geometry
    a. Problem: angle of plane to reach given point at a distance along the horizontal fastest
    b. Now $s=1/2*a*\sin(\theta)*t^2$, and $d=s*\cos(\theta)$, where d is a fixed distance along the horizontal
    c. Hence $d=1/2*a*\sin(\theta)*\cos(\theta)*t^2$, so that minimum t to cover d when $\sin(\theta)*\cos(\theta)$ a maximum
    d. Calculus shows this to be when θ=45 deg (as does simple trigonometry) just as Galileo says
4. Many of the results involve acquiring an initial speed, say through vertical fall, then proceeding with this initial speed along some inclined plane -- diverting the motion
    a. Physically not really possible, for do not simply or automatically preserve speed when thus changing direction, though with smooth transition can come close in rolling
    b. Thus primarily expressing mathematical relationships, whether he was aware of this or not, given his scalar notion of speed
    c. Initial vertical fall a mathematical device to specify an initial velocity, something he could not just do numerically; looked on this way, legitimate and clever, as we shall see when we get to projectile motion
5. Also, many of the quoted results are physically wrong because of comparisons of rolling and falling motions, though results hold within each distinct regime of motion
6. These caveats aside, an impressive mathematical development from one definition and one postulate, just as Sagredo acknowledges (p. [266f]

a. Since nothing on local motion remotely comparable to it before, must have been very impressive at the time, illustrating the power of Archimedean approach to science
b. Lots of interesting mathematics and history of mathematics in the results we will be bypassing

B. A Key Result: "Conversion" into Horizontal Motion

1. Some of the most interesting results from our point of view are to be found in the Scholium on diverted motion following Prop. XXIII
   a. Section primarily concerned with consequences of "diverting" motion from vertical into planes, including horizontal
   b. But discussion includes Galileo's clearest remarks on what we now call the principle of inertia
2. A mathematical result: if accelerated motion converted to horizontal, then if s the distance traversed in the accelerated motion in time t, 2∗s the distance traversed in the horizontal motion in t
   a. Since measurement of speed more tractable in horizontal, offers a way of measuring maximum speed obtained during acceleration
   b. Of course, cannot divert in case of falling or sliding, but can do so, with care, in case of rolling
   c. Thus, an experimental program in the offing
3. As Galileo makes clear, reasoning here presupposes that no acceleration or deceleration at all along horizontal in the absence of air resistance etc.
   a. Reasoning presupposes that motion along horizontal "equable", which in turn entails "eternal"
   b. Behind this reasoning is claim that all acceleration or deceleration from nature alone involves the vertical
   c. Thus at least suggests a limited form of what we call the principle of inertia
4. Augment line of reasoning with claim that this natural acceleration associated with the vertical is always the same magnitude in ascent and descent, and obtain result that speed acquired in fall always just the amount needed to return to initial height
   a. Paired inclined planes of whatever slope will yield restored height (in absence of resistance etc.)
   b. Take reasoning to limit and have another argument for what one might call a horizontal principle of inertia
   c. But be careful here, for in the limit the surface of the earth is spherical, so that the continuing motion in question may be circular (in keeping with Galileo's conception of eternal motion)
   d. Another possible experimental program with rolling spheres
5. Galileo here brings out most clearly the central physics claim in his theory of local motion: only one natural mechanism of acceleration or deceleration -- i.e. of change of speed -- and it acts along the vertical in a completely uniform way
   a. Not just a mathematical theory giving theorems about uniformly accelerated motion, but a physical theory of (ideal) motion governed by natural processes near the surface of the earth

b. Can extend to motion constrained by an inclined plane (ignoring difference between rolling and falling); but note that he is unable to extend theory to pendular motion – curvilinear motion constrained to remain at some specified distance from a point

c. Only result for pendulums experimental: $T \propto \sqrt{\text{length}}$

C. Some Other Interesting (and Testable) Results

1. Many of the other mathematical results provide at least a basis for a test of the theory, if not a basis for an experimental program
    a. E.g. can test claim that 45 deg plane fastest for covering a given horizontal distance (though problems in maintaining rolling likely to yield exactly the wrong result)
    b. Many of the claims permit (potential) qualitative tests of the theory, and qualitative tests in general more tractable -- something Galileo appreciated
2. Probably the most interesting of these is that an inclined plane, though the shortest distance between two points, does not yield the shortest time between the points in natural motion (Prop. XXXVI)
    a. So long as slope of plane no greater than 45 deg, less time via two planes forming chords of same circle (assuming complete transfer of speed at point of intersection)
    b. Since proof depends on a large number of earlier results, this theorem in effect pulls a lot of them together
    c. Main idea: speed gained from D to B shortens time from B to C more than time lost in going from D to B
3. Scholium that follows extrapolates this result into an even more striking claim: circular arc faster than inclined plane even though distance traveled greater than along any sequence of planes
    a. Argument simply by taking reasoning on planes to limit
    b. Galileo wrongly asserts that circular arc the fastest, but correct in asserting that circular arc faster
    c. Problem of determining the fastest path becomes famous in 1691 with Jacob Bernoulli challenge -- answer, the cycloid
4. This consequence of the theory is quite unexpected and counterintuitive, and hence provides basis for a challenging test of theory
    a. At 90 deg arc the effects of rolling along plane and falling at beginning of circle will confirm the theory for the wrong reason -- reduced acceleration in rolling
    b. But at 45 deg, or maybe 30, with proper surfaces, maybe can maintain rolling of sphere in both cases, providing a true test
    c. But test very difficult to execute -- more likely to get confounded results than meaningful ones, raising the problem of how to assess actual results
5. Note logic of evidence here: an anomalous consequence of the theory which, if confirmed, shows that theory yielding new knowledge and hence not just recapitulating what is already known

a. A real chance for falsification -- bold conjecture leads to extraordinary claim that is amenable to being shown false

b. But also notice the danger of confirmation for unrecognized wrong reasons -- rolling along plane vs. falling on part of circle

c. Testing not necessarily so conclusive as one might think!

D. The Theory as a Question-Answering Device

1. Given the effort required just to understand the various propositions -- much less the proofs! -- natural to ask whether any of this display of mathematical facility of scientific interest

   a. A question often asked by scientists when faced with a highly ramified mathematical development of a theory -- is all of this worth the effort of understanding

   b. Brought home even more strikingly in the case of Galileo by the fact that much of his mathematical development can be telescoped into a single algebraic formula which can then be manipulated to yield the various results

2. One purpose served by such a ramified mathematical development has already been noted: generate a bunch of results that can serve as basis for testing the theory -- i.e. its basic claims

   a. Unexpected qualitative contrasts and distinctive phenomenal quantitative patterns a basis for tests to confirm -- or at least to falsify -- the theory

   b. Wider the variety of such results, the greater the supporting evidence for the theory if it "passes" all of the tests

   c. Also, given problems of experimentation in early stages of a science, the wider the variety of such results, the greater the chances of at least one meaningful test

   d. The predictive power of a theory: the range of (confirmed) predictions deducible from it

3. A second purpose served by putting a theory in the form of a small number of basic claims and a highly ramified mathematical development out of them is that the deductions can be turned around to yield explanations

   a. E.g. why do the incremental distances display essentially a 1,3,5,... progression: because the motion is one governed by uniform acceleration

   b. Explain a phenomenon by showing that it is a logical consequence of some basic mechanism -- i.e. by subsuming it under a law (deductive-nomological explanation)

   c. The explanatory power of a theory: a reflection of how much -- range and variety -- follows from how little initially assumed

   d. Explanatory power and predictive power in opposition to some extent, for generally can expand either at sacrifice of other insofar as the way in general to increase predictive power is to include more principles and qualifying complications in order to achieve closer agreement

   e. The fact that Galileo's theory of free-fall follows from only two principles gives it potentially great explanatory power

4. A third purpose served by a ramified mathematical development of a theory is that it provides a general question-answering device
    a. Given a question, theory identifies what combinations of information needed to determine answer and provides a way of then determining the answer (a la Bromberger: vicarious experiments): Galileo's solved Problems
    b. A crucial role of theories in science -- most of what you learn when you study sciences
    c. The form in which an indefinite amount of knowledge becomes contained in a fairly simple theory, fairly simple to learn
5. These three purposes, though somewhat distinct, are not unrelated
    a. Question-answering feature plays a critical role in the design of many experiments (as well as in engineering applications)
    b. Testing critical to be able to rely on question-answering when answering novel questions or ones with answers that cannot be readily checked empirically
    c. Explanatory power and predictive power both tied to question-answering power
6. The question-answering aspect of theories in physics more important than is sometimes noted -- look at e.g. Kepler's theory of planetary motion from this point of view
    a. In fact, a tradition dating back to Ptolemy in astronomy, Archimedes in mechanics
    b. Galileo can be thought of as bringing this tradition into a science of motion within mechanics

III. "The Fourth Day": Conceptual Development of the Theory

A. A Parabolic Path -- The Basic Idea

1. Galileo's proposed conceptualization of "projected" motion: suppose uniform motion along a plane which suddenly ends, so that vertical fall commences (with uniform acceleration)
    a. A basic case of a more general type: motion "compounded from two movements; that is, when it is moved equably and is also naturally accelerated." [268]
    b. Uniform horizontal plus uniformly accelerated vertical, "all impediments being put aside"
    c. Idea, then, is to combine the two theories from the "Third Day"
2. Note that Galileo here again seemingly committing himself to a limited version of what we call the principle of inertia -- for equable horizontal motion is taken to be without impediment
    a. I.e. horizontal motion would remain uniform and hence "perpetual" in nature because nothing in the basic natural process to change its speed (since resistance etc. excluded)
    b. Of course, once horizontal ends, then something in nature to change motion, namely propensity (*propensionem*) to fall to ground
    c. Hence a further assumption, open to challenge: uniform horizontal and uniformly accelerated vertical "propensities" continue unaltered when compounded -- no "cross-talk"!
3. Compounding two independent orthogonal motions was in some ways a radical move in the 17th century, however simple it is to us

a. As we noted earlier in the course, thoroughly anti-Aristotelian to think of two motions compounding to form a third motion: one motion always dominant

b. But can be seen in some classical geometry, especially post-Euclidean geometry -- e.g. Archimedes' spiral -- as well as in Ptolemy (compound circles) and Kepler (ellipse out of circle from radial motion)

4. Theorem 1: the resulting trajectory compounded from uniform horizontal and uniformly accelerated vertical motions is a semiparabolic line

   a. Galileo's proof employs the converse of lemma 1, which amounts to claim that y proportional to x^2 in parabolas

   b. Basic idea of proof straightforward: x proportional to t, y proportional to t^2, resulting in y proportional to x^2

5. Theorem 1 more limited than it may at first appear to be, for it says nothing about which parabola is described by the motion -- i.e. the dimensions of the parabola

   a. Theorem 1 the counterpart of Kepler's "law" stating that orbits are elliptical (conforming to area rule), without specifying what factors might determine which parameters

   b. Kepler never offers factors suitable for solving the "initial value" problem, but Galileo does

B. A Problem: Local Versus Circular Motion

1. Much as in the "Third Day" the basic claim immediately elicits a series of challenges to the very idea of conceptualizing projected motion in this way [273-274]

   a. Sagredo: surely not a parabola all the way to the center of the earth, and hence why a parabola even over first segment

   b. Simplicio: motion in a straight line along a horizontal is not uniformly removed from the surface of the earth, and hence will experience a deceleration as if going up an inclined plane!

   c. Simplicio: resistance effects cannot be removed, and they will "destroy" the two separate patterns of motion being compounded

   "All these difficulties make it highly improbable that anything demonstrated from such fickle assumptions can ever be verified in actual experiments" -- [274]

2. Galileo's response to the challenge on horizontal motion grants the point -- i.e. true horizontal motion not eternal -- but then invokes the authority of Archimedes to treat the earth's radius as infinite

   a. Idealization that is approximately correct for short horizontal distances, as evidenced by such practical procedures as parallel plumb lines in architecture

   b. Concedes that some correction may be needed when applying to various real motions -- thereby opening problems for the future

3. Galileo's response to the challenge about the path to the center of the earth: while reaching surface of earth, parabolic shape altered "only insensibly, whereas that shape is conceded to be enormously transformed in going on to end at the center" [275]

a. A famous problem to which Galileo had offered a solution that he had already learned to be mistaken (from Mersenne and Fermat) -- see Koyré's monograph

b. Concession drops claim that parabola even an idealization, for no longer said to hold exactly in absence of resistance

c. Raises a question for others: what trajectory does a projectile truly follow in the absence of resistance, with the earth's curvature taken into consideration

4. Galileo has a real problem here that he does not resolve, namely how to reconcile local motion near the surface of the earth with circular motion

    a. Response here simply withdraws claim that results would hold exactly without resistance -- hold only approximately, but still of practical value

    b. In process opens up a number of problems for others to address in future -- exact motions, relation between local motion and (celestial) circular motion, etc.

    c. These questions will receive increasing attention in subsequent years

5. Galileo's response to question about resistance is maybe his clearest statement that no science of resistance possible at all [275ff]

    "No firm science can be given of such events [*accidenti*] of heaviness, speed, and shape, which are variable in infinitely many ways. Hence to deal with such matters scientifically, it is necessary to abstract from them. We must find and demonstrate conclusions abstracted from the impediments, in order to make use of them in practice under those limitations that experience will teach us. And it will be of no little utility that materials and their shapes shall be selected which are least subject to impediments from the medium, as are things that are very heavy, and rounded."

    a. Practical defense: use scientific result as basic approximation and then fudge it as needed -- engineering defense

    b. Further argument: fudge not that large in cases of interest -- in particular, small enough that need not worry about how resistance separately undermines the individual patterns of the two compounded motions; so, combining separate components okay

    c. {Note reference to "supernatural" motion in cannon -- speeds greater than generable by nature, [278]}

C. The Problem of Compound Impetus: Which Parabola

1. Galileo's next concern is with the impetus and hence speed that occurs in compound motion

    a. Remember again that impetus relates to percussive effects on impact, which Galileo took to be proportional to weight and (mistakenly) speed

    b. Speed is the scalar magnitude of the velocity vector, always taken in the direction of the motion

2. Impetus and hence speed of two compounded uniform motions is simply the vector sum of the respective uniform impetuses and speeds

    a. For, spaces traversed in equal times can be used as a measure of speed and hence of impetus in the case of uniform motion

    b. And combined space is the vector sum of the individual spaces
    c. (Fortunately, Galileo did not resort to impact measurements here)
3. But cannot thus use spaces traversed as measures of speed or impetus when one component of the compound motion is not uniform
    a. For speed and hence impetus not proportional to space covered -- i.e. speed = $\sqrt{(2*s*a)}$ with uniform acceleration a
    b. Galileo is reluctant to appeal to some notion of instantaneous velocity, which was considered suspect (from Zeno's paradoxes)
    c. Galileo not mathematically permitted to express speed or impetus in units of length versus time
    d. He needs some geometric way to characterize compound impetus and speed, and cannot use our way, simply vectorially combining algebraic expressions for speed
4. A related problem Galileo faces is to be able to specify the specific parabola the projectile describes, given its initial horizontal motion and given the uniform vertical acceleration from nature
    a. Cannot just specify a value of initial velocity and then use algebra for the rest, as we would, for cannot specify such a value in units of distance/time
    b. Free fall rate, g, is set by nature, but uniform horizontal motion is arbitrary, so that the specific dimensions of the parabola depend on it – i.e. the scaling factor p in $x^2 = 4py$ (see Appendix)
    c. Galileo needs some measure of uniform horizontal motion that makes it (geometrically) commensurate with free fall
5. Salviati poses the problem in just this way when explaining Proposition IV [286, 288, and 289]
    a. Need unit of length, unit of time, unit of impetus, and unit of speed so that ratios possible
    b. Last unit should enable geometric comparisons and compounding of uniform horizontal and uniformly accelerated vertical motions

D. Galileo's Solution for Compound Impetus
1. Galileo correctly insists on some naturally governed phenomena as the basis for measurements generally, such as the pendulum for time [286]
    a. Need phenomena that are the same everywhere (and at all times) so that measurements made at different places (and times) are commensurate with one another
    b. I. e., need an "invariant" measure of speed and impetus
2. Galileo's proposal: use as measure of speed (and hence impetus) that gained in free fall from a specified height, so that *height* of fall from rest becomes a ***proxy*** for $speed^2$ and hence for *speed*
    a. He asserts that this is a natural and invariant relationship everywhere around entire surface of the earth, and hence meets the needed condition to be a universal measure
    b. This assertion has really been an implicit part of the physical theory throughout, but it has been made fully explicit for the first time here

c. Notice that Galileo's theory is what licenses a height to be a measure of speed -- a theory-mediated measure, using a proxy for speed (squared)!

(1) Same height of fall, same speed -- a prerequisite

(2) Regardless of path, regardless of weight, regardless of shape of the movable: all those Galilean principles, and not just the principle of free fall, needed to license the use of height as a proxy for speed squared; so more thoroughly theory-mediated than one might notice

(3) Further claim: invariant over the surface of earth

3. Given some specified height as a unit of measure of speed and impetus, then measure of speed and impetus generally will be specified in terms of the square root (i.e. the mean proportional) of distance an object would have to fall to gain impetus in question

a. I.e. Given AB as basic measure of impetus, and want to know impetus acquired in fall through AC; measure is the length that is the mean proportional between AB and AC

b. Because, of course, speed = $\sqrt{(2*g*h)}$, so that ratio of speeds is as sqrt of ratios of lengths: AD = $\sqrt{(AB*AC)}$, so that AD/AC = $\sqrt{(AB/AC)}$

c. (Mean proportional determinable by ruler and compass; see Euclid, VI, 13)

4. Using vertical distance in this way provides a conveniently commensurate measure for comparing horizontal and vertical impetus and speed even when the vertical acceleration g -- the fall in one second -- is unknown

a. Horizontal distance traversed in time required for free fall from specified height = twice that height

b. Thus earlier theorem provides way of expressing uniform horizontal speed and impetus in terms of speed and impetus gained in free fall without as such requiring a specific value of g!

5. Note that Galileo might have tried to use impact measurements for impetus, as suggested by the discussion on [291-293]

a. Fortunately he did not, but even if he had, the rules of his mathematics would have forced him into some form of extensive representation of the measure, some form of proxy

b. This way not only gives him the latter, but also avoids problems of measuring percussive force, a parameter of great interest in military ballistics (see "Added Day")

E. Galileo's Determination of the Parabola

1. Suppose now we are given a projectile with uniform horizontal motion of a specified speed and impetus

a. Let the vertical distance p be that required for projectile to gain this speed and impetus in free fall -- i.e. the sublimity

b. But then the parabola is the one obtained by allowing the projectile to fall a further distance p while proceeding horizontally a distance 2p

c. This uniquely determines the parabolic trajectory in question

2. In effect the sublimity locates the directrix and focus of the parabola (see Galileo's figure for Proposition IV, or the Appendix)

    a. Directrix is horizontal through A, and focus is at D, where AD = 2p

    b. Point of tangency the point where horizontal impetus exactly equals the vertical impetus -- i.e. slope = 45 deg

3. Thus the parabola in question has three distinct parameters -- the sublimity, the altitude, and the amplitude, any two of which are sufficient to define the third and the parabola

    a. Corollary to Proposition V: $1/2 * amp = \sqrt{(p * alt)}$

    b. Can therefore infer p from amplitude and altitude, etc.

    c. Though recognize that Galileo does not do this algebraically, but under the tighter constraint that the length representing each be constructible via compass and ruler from the other two

4. Proposition IV then gives a geometric construction to determine compounded speed and impetus everywhere along the trajectory

    a. Algebraic equivalent: (*speed*/*initial speed*) = $\sqrt{(1+y/p)}$

    b. Therefore, (*impetus at impact*/*initial impetus*) proportional to $\sqrt{(1+h/p)}$ and thus greater than purely horizontal impetus

    c. Impact angle = $\arctan(\sqrt{(h/p)}) = \arctan(amp/(2p))$

5. Thus a totally unique determination of the projectile trajectory, given any two of the three quantities

    a. But this is just as it should be, for since g fixed, the parabolic trajectory depends only on the initial horizontal speed

    b. And p has the virtue of combining this initial horizontal speed and the acceleration g into the single parameter p needed to define the parabola (see Appendix)

    c. Only other thing special here is that corresponding parabola geometrically constructible, using compass and ruler, given any two of the quantities, represented of course as lengths

F. The Significance of Galileo's Approach

1. Galileo is thus able to define not only the path, but the speed and the direction everywhere along the path -- i.e. the trajectory, in the same sense as Kepler -- without having to resort to calculus

    a. Indeed, without having to resort to algebra (or analytic geometry, which was just beginning to take shape at the time)

    b. Constructive Euclidean geometry throughout: every quantity determinable via compass and ruler

2. More important, he manages to bypass both a theory of forces and a specific value of the acceleration of gravity in his choice of parameters determining the parabola

    a. Galileo's "sublimity" the perfect device for doing this since it absorbs the acceleration of gravity within it, in effect normalizing everything with respect to a unit of free fall

b. Galileo thus provides a purely "kinematic" account of projectile motion, in contrast to a "dynamic" account (to use phrasing that emerged in the nineteenth century)

c. Anyone inclined to sell Galileo short should give some thought to the remarkable ingenuity (and depth) of this solution!

3. Still more important from a methodological standpoint, he shows a thorough understanding of the constraints that have to be satisfied in coming up with measures of speed and impetus

a. We of course define speed in terms of distance divided by time, and hence we can bypass the problem he faced in this case

b. But we cannot bypass it generally, and his treatment of it is a model that others could follow

4. An extraordinarily impressive achievement at the time, though well within the reach of others, like Kepler and Descartes

a. Tartaglia had examined projectile motion empirically roughly 100 years before, but had not begun to reduce it to a mathematical account

b. And even Kepler's account is not strictly within the confines of Euclidean geometry – area rule, in particular, is non-Euclidean

5. By contrast Galileo was never able to construct such a theory of circular pendular motion

a. That is, he was unable to construct a kinematic account of pendular motion using uniform acceleration, for not uniformly accelerated at all, nor finitely compounded from uniformly accelerated motions: different conceptual obstacles

b. In one respect, then, not really the same category of phenomenon, and hence not so inappropriate that Galileo had trouble

IV. "The Fourth Day": The Theory of Projectile Motion

A. The Basic Claims Made by Galileo's Theory

1. So far have been considering only the manner in which Galileo conceptualized the problem of projectile motion; time now to turn to the theory itself and the empirical evidence for it

2. Basic claim: the semi-parabola is uniquely determined, given the sublimity p and the height h where the compound motion begins

a. The sublimity in effect compounds the initial horizontal speed and the uniform (universal) acceleration of gravity, g, into a single parameter that suffices to determine the shape of the semi-parabola

b. And the height then determines where the semi-parabola ends in impact with the ground

c. As Appendix indicates, h and p are then sufficient to determine all other quantities (geometrically as well as algebraically)

3. Further claim: by a symmetry argument, this semi-parabola can be extended into the full parabolic trajectory that occurs when a projectile is launched from the ground with an initial speed at some angle of projection θ

a. I.e. determine a full vertical parabola given angle of projection θ and the initial speed, stated in terms of a sublimity

b. If not stated in terms of a sublimity, then require specific value of vertical acceleration g to determine the parabola

c. Tan(θ) = 2∗*alt*/*amp* = 4∗*alt*/*range* = *range*/(4*p*)

d. {The mathematics required for oblique projection may have given Galileo trouble, necessitating his appeal to symmetry here without proof; Toricelli supplied the missing proof in the early 1640s, published in his *De Motu Gravium Naturaliter Descendentium et Projectorem*}

4. Galileo has two ways of specifying the full parabola without having to know the acceleration g:

   a. Given θ and *p*, with the latter representing the initial horizontal speed and impetus

   b. Or given range and θ, can infer *p* from above

   c. (All in the absence of resistance effects)

5. The analogy with Kepler is now complete: for a repeatable initial impetus, can measure angle of launch and range, from which the complete trajectory can be determined

   a. Can even recover time if measure time until projectile lands -- the correlate of the Period -- so that the complete trajectory in time is determined for a given initial angle and unknown, but uniformly repeatable impetus -- e.g. the impetus from a given amount of powder in a cannon

   b. Indeed, if measure time as well as range and initial angle, can infer acceleration of gravity g -- i.e. fall in first second -- (from *h*) and the unknown initial speed of projection (from *p* or from *a*)

   c. Thus a way of measuring fall in first second, if resistance effects are negligible

B. Mathematical Consequences of the Theory

1. The mathematical development of the theory of projectile motion, although somewhat more limited in terms of the number of results, is even more impressive in some of the entailed claims

2. Proposition VII and Corollary: for a given initial impetus, the maximum range is achieved when θ = 45 deg

   a. Something that had been observed in practice, and had been noted by Tartaglia

   b. As Galileo remarks, now being explained (and shown to be nomological)

   c. And explanation shows that it has nothing as such to do with air resistance effects, but instead comes from uniform horizontal and uniformly accelerated vertical compounded

   d. Notice how much evidential weight Galileo attaches to the result in these remarks

3. Proposition VIII: for a given initial impetus, the range varies symmetrically as θ varies on either side of 45 deg

   a. E.g. get the same range for θ=30 and θ=60 deg

   b. Empirical claim that Galileo says had not yet been observed in practice

   c. A potential confirming experiment, if can achieve repeatable initial impetuses

4. Proposition X: impetus at impact is impetus resulting in vertical fall from a height equal to the sublimity + the altitude -- thus relating the speed (and impact effects) of a projectile to that of same projectile in free fall
5. Propositions XI and XII relate altitude and amplitude for a given initial impetus, yielding a table of (relative) amplitudes and hence ranges as a function of θ and a table of (relative) altitudes as a function of θ
   a. Tables 1 and 2, [304], with amplitudes normalized to an amplitude of 10000, and altitudes normalized to an altitude of 5000, respectively for θ = 45 deg
   b. Tables can be used to determine relative magnitudes under the assumption of same impetus, but different θ
6. Proposition XIV relating altitude and sublimity for a given range, yielding a table of (relative) altitudes and sublimities as functions of θ
   a. Table 3, [307], with distances normalized to an amplitude of 10000 and hence a range of 20000.
   b. Tabulated values slightly inexact -- Galileo does not bother to carry enough significant figures; better calculations would preserve the strict symmetry that the theory entails

C. The Form and Content of Galileo's Table

1. As the tables and the accompanying Propositions attest, the mathematical theory allows a large number of problems to be solved, extending beyond those that can only be solved geometrically
   a. Problems of the form, given certain quantities, determine others
   b. I.e. the theory again yields a reasonably rich question-answering device
2. Since tables are given in the form of relative quantities, will in general need to know something in order to obtain a specific result from them
   a. Values tabulated represent such things as *range*(θ)/ *range*(45) for a fixed impetus and *altitude*(θ)/ *range*(θ) and *sublimity*(θ)/*range*(θ) for a fixed impetus
   b. Hence, can perform calculations as soon as know e.g. range(45) for the impetus in question -- something that will have the effect of pinning down the impetus in a preferred way
3. The obvious question is why not table with absolute, rather than relative quantities
   a. Answer: doesn't have precise values of g -- i.e. fall in first sec -- or initial horizontal speed, v_0
   b. The precise value of g, as we have seen, is a little hard to come by, but poses little problem when put alongside that of determining the precise value of muzzle velocity
   c. If could ignore resistance effects, could infer muzzle velocity (and g) from actual trajectories
   d. Question: conduct repeated experiments to see if obtain uniform value of g via measurement of range, θ and time, along with values of muzzle velocity; if not, then theory offers a basis for reaching some conclusions about resistance effects
4. One practical virtue of giving relative values in the tables is that error associated with resistance effects tends in practical applications to be canceled out to some extent

a. *True-range*(θ)= *theory-range*(θ) – *resist-loss*(θ)

b. *True-range*(45) = *theory-range*(45) – *resist-loss*(45)

c. Suppose true ranges in both of above cases 10 percent lower than the theoretical ranges -- or, more generally, that *resist-loss* distance is proportional to theo*ry-range*

d. Then *true-range*(θ)/*true-range*(45) would be exactly equal to *theory-range*(θ)/*theory-range*(45)

5. The point generalizes still further: the relative values in the table tend to be more empirically correct than the theory itself is, once resistance effects intrude

a. For effects of resistance tend to cancel out in the relative values as a consequence of their having the same direction of effect in both the numerator and the denominator

b. The percent errors in the table and in calibrated calculations based on the table will be smaller than the errors that would be obtained from using true values of g and muzzle velocity!

6. This is an instance of a general practice in engineering for achieving predictions: use *calibration* of a theory in order to compensate for effects not taken into account, thereby achieving better predictions than the theory itself can yield

D. "Real World" Deviations from the Theory

1. In sum, the theory of projectile motion makes some predictions, raising the issue whether additional empirical evidence accrues to the overall theory from any direct evidence for it

a. The theory of projectile motion is derived from the theory of naturally accelerated motion, supplemented by two further assumptions

(1) A principle of horizontal inertia, at least over short distances

(2) A claim that separate horizontal and vertical patterns remain in effect when compounded

b. Consequently, any evidence of projectile motion is evidence for the main thesis of the overall theory -- uniform vertical acceleration

2. In real world resistance deceleration depends on a viscous effect and a kinetic (pressure) effect

$$R = C_1 * r * v/mass + C_2 * (r^2) * (v^2)/mass$$

where in air $C_1 \approx 0.00031$ kg/m∗sec and $C_2 \approx 0.87$ kg/m^3, where deceleration always in opposite direction from motion; note: this, as we shall see, is only an engineering approximation

a. Depends on mass, size, and velocity, with v^2 term dominating once velocity large enough for any given size and mass

b. Effect appreciable for 10 gm pebble, 1 cm in radius: see figure in Appendix (from French)

c. Result shows that some real care is needed in designing experiments in which air resistance effects are minimized

3. Consider an equivalent combination -- a 1 kg sphere of 10 cm radius -- and compare deceleration to g (9.81 m/sec^2)

a. With v = 10 m/sec, R around 0.9 m/sec^2, less than 10 percent of g, so that effect limited

b. With v = 30 m/sec, R around 8 m/sec^2, roughly comparable to g and hence a quite pronounced effect (near terminal velocity)

c. With v = 100 m/sec, R around 87 m/sec^2, overwhelming the effect of g -- Galileo's "supernatural" case

d. An increase of the mass to 10 kg, keeping everything else the same, reduces the effect in the case of 10 m/sec to less than a 1 percent and, in the case of 30 m/sec, to less than 10 percent

4. Two other factors complicate the situation with projectile motion even further

a. Trouble in measuring or effecting reproducible values of initial velocity, needed to compare e.g. different angles

b. v^2 resistance term has the effect of undercutting the claim that the two components of motion remain in effect independent when compounded, for this term produces cross-talk between the two -- i.e. Simplicio's hunch was right, can no longer just superpose orthogonal components

5. Experimental evidence for theory and hence for underlying claims not going to be easy to obtain from projectile motions

a. Need quite dense spheres, with moderate repeatable velocities, and hence probably require special laboratory set-ups

b. E.g., use inclined plane to produce controlled rolling across horizontal, and then measure amplitude for different heights and initial velocities -- "ski-jump" (see below)

(1) A way of testing not just parabola, but which parabola insofar as $a = 2*\sqrt{(h*p)}$

(2) Could have exposed rolling vs. falling, from wrong parabola, for

$$a_{obs}/a_{pred}=\sqrt{(g_{roll}/g_{fall})}=\sqrt{(5/7)}$$

c. Little chance from artillery or other salient ballistic phenomena of interest, for velocities too high

E. Practical and Scientific Concerns Contrasted

1. Galileo, who regards a science of resistance as impossible [p. 275f!], offers no experimental results in the text, yet clearly thinks his idealized theory is the best one can hope for

a. His view is that air resistance has a small effect in some cases, a larger effect in others, and a dominant effect with "supernatural" velocities, where he openly concedes that the true trajectory will not be a parabola

b. Generally his claims about how small the effects will be are greatly exaggerated, though he is right that there are regimes in which they are small enough to allow meaningful empirical tests of the theory

2. The Tables are of some value for practical "technological" purposes, though the domain in question is probably more limited, or at least the accuracy is more limited, than he thought

a. The relative values given in the tables end up masking errors from resistance effects via calibration, using a measurement for a case in which resistance effects are of course present

   b. Predictions from the tables will be good so long as the reference or calibrating measurement has a typical percentage of resistance effect
   c. Thanks to this, tables may yield decent predictions, say from 30 to 60 deg, even when resistance effects are highly pronounced
3. The numbers in the Tables are thus presented in just the wrong form for purposes of bringing empirical data to bear on the theory!
   a. They understate discrepancies, whether from resistance or from any other uniform error, and hence allow experimental results to appear to provide stronger confirmation for the theory than they really do
   b. They **mask** any systematic discrepancies that could be used to argue that the theory is false, or is open to refinement
   c. Better off for purposes of marshaling empirical evidence if discrepancies exposed as completely and clearly as possible, if only to pursue improved, more telling experiments
4. This is a general feature of "engineering" oriented idealized theories and models that are first calibrated before being applied, where the calibration serves to compensate for neglected effects
   a. The fact that they work in the practical realm provides some evidential support for the theory
   b. But generally less support than it appears to, for more often than not the calibrated theory works to the extent it does too much for the wrong reason
   c. And almost nothing is learned from meticulously comparing theory with observation in these cases, except limits of their practical value
5. Galileo shows little sign of seeing this, but others we will be studying saw the difference quite clearly
   a. In 1670's Collins, among others, attempted to provide better tables for artillery, building on Galileo's theory
   b. Clear by then that Galileo's tables were proving to be of limited military value
   c. A continuing question: what is the trajectory a projectile follows in air
   d. A question of importance in the 19th and 20th centuries -- Babbage and Eckart, the designers of digital computers in the two centuries, focused on the ballistics problem

F. Galileo’s Suppressed "Ski-Jump" Experiment

1. Drake (as well as Hahn and Damerow *et al*) have argued from a sheet in Galileo's notebooks that he indeed did perform the "ski-jump" experiment (folio 116v, which Drake assigns to 1608 (p. 129f))
   a. That is, let a sphere acquire its horizontal speed along a table from descending along an inclined plane and then measure the distance -- i.e. amplitude -- covered horizontally versus the height of further descent after it leaves the table
   b. As shown in the figure in the Appendix, taken from Galileo’s notebooks and cleaned up by Hahn
2. My table in the Appendix compares what Galileo measured against what his theory would have predicted: [(*theory* – *observed*)/*theory*]

   a. His numbers in notebook indicate a fairly consistent, slightly greater than 17 percent discrepancy between observed and predicted amplitudes (as a fraction of the theoretical amplitudes)
   b. If the horizontal speed was acquired by the ball rolling on its bottom down an inclined plane, as Drake naturally proposes, then the rolling-falling discrepancy is 15.5 percent, i.e. $1-\sqrt{(5/7)}$
   c. Hahn (see Appendix) has argued that the greater than 15.5 percent discrepancies fit well with Galileo having used an inclined-plane groove in this experiment in which the sphere does not roll on its bottom, but on tangent points between the bottom and its middle
   d. In particular, if the width of the groove is 4/9 the diameter of the rolling sphere, the discrepancy between theory and observation should be 18.3%
3. At the very least, then, we have reason to think that Galileo really did do this experiment and got the results he quoted; yet he decided not to publish those results, presumably because they were incompatible with his theory
   a. Drake thinks Galileo attributed this to a loss of speed in the transition from inclined plane to horizontal, but he might as well have attributed at least part of it to the effects of air resistance
   b. But in a situation in which there were too many other things to attribute the discrepancy to, and he chose not to follow up the result with a series of further complementary experiments, nor did he publish the idea of the experiment, giving others a chance to follow it up
   c. A result that remains unpublished becomes merely a part of Galileo's biography, and not part of science
4. Still, we should notice that this is just the sort of further experiment, beyond the inclined-plane experiment described in "The Third Day," to complement it and provide a cross-check on it
   a. The deep lesson here, which Galileo seems not to have appreciated, is that getting well-established results from experiments requires more than single experiments in isolation -- the very practice Kepler had followed in pursuing cross-checks for results in *Astronomia Nova*
   b. It requires a host of complementary experiments, cross-checking one another in order to provide safeguards against being misled by tacit, unrecognized assumptions in a single experiment of just the sort that occurred with the rolling-falling lacuna in his inclined-plane experiments
   c. In particular, when discrepancies appear, follow-on experiments exploring those discrepancies are the best hope for sorting out what is going on
   d. Especially experiments like this one that eliminate time as something that has to be measured!
5. The fact that Galileo appears not to have pursued such further experiments says something about the limitations of his conception of using experiments to establish conclusions
   a. Insofar as Galileo's sublimity does make the experiment obvious, what is more interesting to me is whether others subsequently performed it and elected not to report the results
   b. That would be quite a comment on experimental practice during the 17th century

G. Galileo's "Laws" at the End of "The Fourth Day"

1. Upshot: comparatively little public direct evidence for the theory of projectile motion at the time Galileo put it forward
   a. The maximum range at 45 deg result a plus for the theory
   b. To the extent that the Tables work in practical situations, some evidence, though limited
   c. Perhaps a little more evidence from symmetry around 45 deg, provided that it is observed
2. Much of the evidence for Galileo's projectile theory must therefore come from evidence for the individual underlying claims
   a. Uniform acceleration: the law of free fall
   b. A universal value of vertical acceleration
   c. Continuing horizontal motion over relevant distances
   d. No cross-talk between two components of motion when compounded
3. We have already found that the evidence for uniform vertical acceleration was limited and problematic at the time
   a. Inclined plane 1,3,5,... pattern, and Riccioli's observation of the same in free fall
   b. But experimental data highly suspect and not independently confirmed, forcing us to conclusion that at most the experiments had failed to falsify the claim
   c. Riccioli's efforts open the way to improved experiments, however, including much more careful inclined plane measurements
4. Little evidence had yet been developed on the universality of g, though it might be forthcoming once experiments improve
   a. Once rolling versus falling exposed, inclined plane may provide good results (for will appreciate need for one type of motion)
   b. Uniformity of measured value of g would be strong evidence for uniform acceleration
5. Thus little overall empirical evidence for Galileo's theory: it is not clearly false, but it is far from empirically established
   a. And issues about nomologicality, *ceteris paribus* conditions, range of laws, whether exact or only approximate, and character of approximation if the latter -- issues raised by Galileo himself in the "Fourth Day" -- have scarcely been empirically addressed at all
   b. The contrast with Kepler's laws here is quite sharp
6. Much more reason to continue with the theory at the time than to think it true, once and for all
   a. The rich mathematical development offers potential for experiments, especially as technology improves
   b. Also, it is the only game in town -- the only theory that offers even the beginning of a possibility for bringing empirical data systematically to bear
   c. And its mathematical form and extent set a standard that any alternative to it had to meet

V. Remarks on Galileo's Contribution to Modern Science

A. Observation, Theory, and Experiment

1. Galileo saw and expressed more clearly than anyone the limitations of simple observation of nature and hence the need for experiments
   a. Intrinsic limits on what we can observe -- e.g. from time and acuity -- and limits arising because events in nature are the combination of multiple processes, the observation of any one of which is then confounded by the others
   b. These limits can be pushed back through experimentation -- observation in specially contrived circumstances -- and through the use of special equipment -- e.g. the telescope
   c. Highly contrived experiments often needed to isolate one natural process from others, as in freedom from air resistance, and even then may have to extrapolate to what would happen in the limit
2. He also saw that experiments, to a much greater extent than mere observation, are closely tied to theories
   a. His experiments themselves were tied to specific theoretical predictions which the theory itself usually makes salient, typically because other theories not likely to make the same prediction
   b. The raw data -- e.g. distance covered -- will typically have to be interpreted using the theory -- e.g. as speed
   c. The theory to which the experiment is addressed is often an idealization, so that e.g. whether data falsify it can be a matter of interpretation, dictated in part by the theory itself
   d. Background theoretical assumptions about confounding effects will be required to justify the claim that the effects are being controlled for and the experiment is not being confounded
   e. And background theoretical assumptions may be needed to relate what is actually measured (e.g. amount of water escaping) to what it is being taken to measure (viz. time)
3. Any special equipment used will reflect background theories about what it does and does not provide
   a. Optics for the telescope, water flow and pendulum for time, percussion effects for impetus etc.
   b. Theories tie observations and measurements to quantities of interest, and license interpretations and claims of accuracy
4. Galileo saw experimentation as serving several roles
   a. Falsification of theories above all else -- a Popperian view
   b. Cross-roads experiments to justify initial assumptions in developing a theory
   c. Confirmation of theory via successful salient prediction, or more modestly by failure to falsify
      (1) Quantitative patterns such as the 1,3,5,... progression and double-height result, $a=2*\sqrt{(h*p)}$
      (2) Qualitative results such as simple incline not the fastest between two points and all chords to bottom traversed in same time, or even 1,3,5,… progression of high approximation

d. Though far less so than Mersenne and Riccioli, measurement of quantities of interest, which the theory makes salient and implies should have stable measurable values

5. Galileo was one of the first to realize that experimentation becomes much more potent in the context of theories with rich mathematical development
   a. Mathematics ties variables to one another, licensing multiple experiments and measurements bearing on a single claim
   b. When pushed hard enough, mathematics may yield special, even remarkable, predictions that allow more telling tests
   c. Question-answering theories help in the design and interpretation of special equipment
6. Finally, Galileo recognized not just the need for skill and ingenuity in the conceptual design of experiments, but also in their detailed implementation
   a. E.g. vellum in the grooves of the inclined plane -- such tricks needed for experiment not to be confounded by unwanted elements
   b. Making an experiment work at all -- or better, with more exacting results -- often takes exceptional skill of a special sort, and exceptional pertinacity

B. The Theory of Local Motion: Scope and Content

1. What Galileo has left us with is a *fragment* of a physical theory about natural motion near the surface of the earth in the absence of air resistance and various frictional effects
   a. Fragment covers free fall, motion constrained by an inclined plane, and motion of a projectile over limited distances once it is launched
   b. Offers answers to questions that had never been answered before, as advertised at the outset [190]
   c. Puts forward three laws still with us -- law of free fall, law of projectile motion, and a not quite right law of inclined planes -- as well as his initial form of pathwise independence
2. Theory built around eight fundamental claims about how natural mechanisms affect motion near the surface of the earth in the absence of resistance etc., the last of which is mere conjecture on his part
   a. The propensity to fall vertically to the earth is the only natural mechanism altering such motion
   b. This propensity becomes expressed in the form of uniform acceleration (in time)
   c. The speed acquired over a given height is independent of the path followed
   d. The speed acquired is the same for all bodies falling from the same height, and hence independent of weight (and shape, etc.)
   e. The speed acquired is just sufficient to raise the body back to its initial height
   f. Naturally acquired motion remains uniform and in a straight line along the horizontal (at least for distances small in comparison with the radius of the earth)
   g. A body projected horizontally describes to high approximation a semi-parabola (at least for distances small in comparison with the radius of the earth), with the dimensions of the semi-

parabola dictated by the height from which its initial horizontal speed would be acquired naturally; and, by symmetry, a body projected at an acute angle upwards describes a corresponding full parabola (with the same qualification)

   h. The distance of fall in the first second (i.e. g) is the same everywhere around the earth

3. The theory Galileo has offered is only a fragment because it fails to cover all the processes governed by this propensity to fall
   a. In particular, fails to cover pendular motion -- motion under vertical acceleration with object constrained to remain a fixed distance from some point (versus constrained by plane to a particular direction)
   b. But just a fragment also because it wrongly conflates rolling with slipping and falling, and hence requires supplementation
   c. And because no answer to projectile trajectory when curvature of earth taken into account
   d. {I.e. lots of clean-up work to be done}
4. Comparatively little evidence was available at the time in support of the theory -- i.e. in support of the fundamental claims
   a. Sufficient agreement with 1,3,5,... pattern in inclined planes and (thanks to Riccioli) free fall, and successful explanation of maximum range at 45 deg give reasons not to reject
   b. But much too limited measurements of acceleration rates -- i.e. fall in first sec -- many independent measurements of which would have provided stronger evidence; and far too limited a range of experiments
   c. {Again, lots of clean-up work to be done}
5. Yet a rich mathematical development, yielding a large range of potential experiments for the future
   a. Sufficiently wide range of results from so few fundamental assumptions that theory promises to have high explanatory power
   b. Successes with three types of motion give reason for expecting it ultimately to cover pendular motion too
6. Therefore the community had good reasons to take the theory seriously and to continue work in developing and assessing it
   a. Find ways of bringing experimental results to bear -- e.g. as Mersenne and Riccioli did -- including results determining g and thus verifying its uniformity and ubiquity
   b. Extend the mathematical development to cover other types of motion governed by the propensity to fall and to answer such further questions as, what is the shortest time path?

C. Galileo's Research in its Historical Context

1. The research reported in Days 3 and 4 of *Two New Sciences*, which was almost entirely conducted in Padua and hence before 1610, was not so insular and unique as my presentation of it has suggested
   a. Have already emphasized the tradition in Italian mechanics leading up to it

b. Tartaglia, in particular, had worked on artillery, though not remotely with the mathematical success of Galileo

c. And others elsewhere both before and after worked on projectile motion in particular

2. Among the most notable of these, though he never published, was Galileo's contemporary, Thomas Harriott (ca. 1560-1621), sometimes called "the English Galileo"

a. His work on projectiles appears to have recognized from the outset that the observed trajectory is not symmetric in the manner of a parabola, but with descent more sharply vertical than ascent

b. He nevertheless pursued a mathematical theory akin to Galileo's, though the kind of trajectories he worked on were less mathematically tractable

c. (See Schemmel's two volumes, the second of which contains the manuscripts)

3. In virtually the immediate wake of *Two New Sciences*, Torricelli published his *De Motu Gravium Naturaliter Descendentium et Projectorum* of 1644, rederiving and extending Galileo's results

a. Have already mentioned both his independent derivation of pathwise independence from the principle named after him and his proof of upward projection at an angle completes the parabola

b. Openly acknowledged the idealized character of the theory of projectile motion

c. Subsequent work generally proceeded more from his book than from *Two New Sciences*

4. Several figures engaged in further development of Galileo's theory of motion between 1642 and 1684, especially Huygens, while others, whether proceeding from Galileo or striking out in new directions, tackled the artillery problem

a. Have already mentioned John Collins in England, but also Robert Anderson

b. Apparently with little success in providing military with a mathematical approach of any use to them, for they kept relying on trial-and-error

c. See the published version of Rupert Hall's dissertation (1952) for details on ballistics

d. And while he didn't work on the projectile problem, Gassendi did much in the early 1640s to support controversial Galilean claims

5. A more useful mathematical theory of projectile motion was not the only problem Galileo bequeathed to those following him:

a. The problem of the truly isochronous trajectory in fall, and with it a theory of isochronous pendular motion

b. The problem of the true trajectory of fastest descent, the brachistochrone problem

c. The problem of the trajectory of a projectile in the absence of a resisting medium, but with the curvature of the earth taken into account, and related problem of descent to the center of earth

d. A question about how far vertically the claim of uniform acceleration remains valid

D. Galilean Conceptions of Empirical Science

1. Whatever else is to be said, Galileo thought that empirical science could reach conclusions much stronger than mere conjecture

a. See quote from (1605) letter on supernova of 1604 in Appendix
b. ***Beyond conjecture*** through marshalling many detailed observations
c. Question: what exactly, on Galileo's view, is the epistemic status of the conclusions that reach beyond mere conjecture?

2. At the outset of the discussion of naturally accelerated motion Galileo recommends his theory on the grounds that it is simple and in agreement with experimental observations
    a. I.e. a conception of empirical science under which it is aiming for the simplest theory that saves *experimental* phenomena -- i.e. phenomena in specially contrived circumstances
    b. But then he proceeds to offer very few experimental phenomena with which the theory is known to be in agreement
    c. And when he does offer them, his discussions of the agreement sought between theory and experiment end up suggesting three different conceptions of empirical science -- i.e. three different conceptions of how to marshal empirical evidence
3. A common thread to all three conceptions is that the theory of local motion is not to be regarded as "perfectible" in Kepler's sense -- not to be asked ultimately to make predictions exactly in accord with phenomena of motion observed in nature
    a. At the beginning the theory is presented as an idealization -- what would happen exactly if no resistance or friction effects were present
    b. And in the case of projectile motion, the theory is presented as holding only approximately over limited ranges, even in the absence of resistance
4. One conception of science: seeking detailed agreement, within observational accuracy, between predictions and experimental observations in which confounding effects suitably controlled
    a. Suggested in description of inclined plane [212f], where he describes a detailed experimental program of measurements of location versus time, and asserts that predictions always within "the tenth part of a pulse-beat"
    b. A conception of evidence like Kepler's to a point, but without imposing any requirements on underlying physics
    c. (No problem of selecting among competing alternatives)
5. Second conception: seeking sufficient agreement between predictions and experimental observations -- especially predictions of distinctive phenomenal patterns and salient qualitative contrasts -- to show that theory not empirically false
    a. Suggested in Galileo's descriptions of other experiments, such as the ones for free-fall, and perhaps what he actually did with the inclined plane, and what Mersenne and Riccioli unquestionably did throughout
    b. A Popperian conception of evidence (bold conjectures followed by efforts to refute), but equally a conception more accepting of the limits of experiment

6. Third conception: seeking sufficient agreement between predictions and empirical phenomena of interest to allow theory to serve practical purposes, both in prediction and explanation
    a. Suggested in Galileo's descriptions of the sort of accord to be expected in the case of projectiles and in his discussions (as well as the format) of his tables for projectile motion
    b. An "engineering" conception of evidence in which the crucial issue is whether the theory enhances the ability to understand (i.e. explain) and predict
7. These three conceptions involve fundamentally different evidential logics -- something that Galileo seems to show no cognizance of
    a. On the first conception, systematic discrepancies are a source of concern until they are shown to result from confounding effects in the experiment, after which attention turns to trying to find still better experiments or experiments that allow corrections to be introduced to compensate for the systematic errors produced by the confounding effects
    b. On the second conception, systematic discrepancies can be dismissed (without detailed arguments) so long as they are small enough that experimental results are not out-and-out incompatible with the theory
    c. On the third conception, systematic discrepancies are something to be adjusted for outside the theory -- e.g. through calibration -- and are of no evidential concern except when faced with competing theories that have a promise of being superior from a practical standpoint
8. {In general the best way to get at evidential logic is to consider what the response would have been to discrepancies between observation and theory!}

E. A Conjecture About How Galileo Proceeded

1. My own sense of what Galileo did is that he took one basic insight -- the 1,3,5,... pattern -- and ran with it as far as it would carry him
    a. I don't care whether he first happened upon the insight theoretically or experimentally
    b. He saw both the theoretical and the evidential potential suggested by the pattern
    c. Then systematically explored by means of cross-roads experiments which variables make a difference to this phenomenon
2. To his credit, he developed the most elaborate and extensive mathematical theory that he could off of this basic insight
    a. Saw the empirical advantages of doing so -- the potential for developing evidence in the future -- as well as the aesthetic and conceptual advantages
    b. As such, ended up teaching the scientific world a basic lesson: in areas of basic physics the evidential criterion must ultimately be a combination of simplicity, completeness, and exactness
3. Equally to his credit, he saw how easy it was to mount empirical evidence against the theory, especially because of its idealization away from real-world factors and therefore did all he could to shield it from being too readily rejected

a. His putting forward the theory as an idealization, in contrast to Kepler's doing so, is clearly an attempt to protect the theory from falsification
b. A sound tenet of scientific practice, especially in the early stages of theory construction: don't abandon a theory because of discrepancies with observation unless in a position to use these discrepancies to advantage in forming a new theory
c. For theory is an indispensable lens to learning from experiment, so that better to have some theory that permits continued inquiry than no theory at all
d. (This is an alternative construal of what Koyré calls Galileo's "Platonism")

4. In this effort to shield the theory from being prematurely rejected Galileo commits his worst mistake: in not making clear the extent of agreement between theory and experiment, he saddles the world with a seriously distorted picture of scientific evidence
    a. Basic picture: once you have a reasonably correct theory, the evidence for it will simply fall into line straightforwardly
    b. Thus no real difference in the evidence problem faced in the early stages of theory construction and in an advanced science; the only problem is happening upon the truth
5. And as a corollary to this mistake, he imposes on the world an element of confusion about the logic of evidential reasoning in science that we are still laboring under
    a. Confusion from conflating three quite different conceptions of science and evidential reasoning, as if they all had the same underlying logic, when they don't at all
    b. A confusion that surfaces repeatedly when scientists begin debating the relative merits of different theories
    c. The world outside science might have been better off if Galileo had been as difficult to read as Kepler was
6. Perhaps the best way to summarize my view of Galileo's new theory of motion is that it was far more local in two respects than is usually recognized when viewed retrospectively
    a. His science of motion was truly local, confined not only to motions near the surface of the earth, but ones that were often of limited extent
        (1) That was perfectly consistent with the tradition of the "mechanics" of machines like the lever which formed its context
        (2) This in contrast to the view that Galileo was formulating a "kinematics" of motion of universal scope
    b. The experiments Galileo presents as evidence, most of which are single, isolated experiments, deliver only limited evidence to the claims he takes them to be establishing
        (1) Specifically because they were not supported by numerous complementary and follow-on experiments of the sort the "ski-jump" experiment would have been had it succeeded
        (2) As such, his claims were backed-up by only highly localized evidence

c. These two together make the legend of Galileo's achievements somewhat bloated versus what he truly achieved

7. That said, the legend began in the 1650s when the eight fundamental Galilean claims listed above became widely accepted largely on the basis of Galileo's authority and the experiments he cited plus those of Mersenne and Riccioli, plus Gassendi's from the late 1630s (to be discussed next time)
   a. That doubtlessly helped promote the idea that individual experiments can be more definitive than they almost ever are
   b. The idea that limited experiments can be decisive is what Shapin and Schaffer decry in their well-known book on pneumatic experiments in the 1660s, *Leviathan and the Air-Pump*
   c. That Galileo's experimental results seemingly established the claims he proposed they did was no less far from being self-evident or self-explanatory, then and now, than those they criticize
   d. However much I disagree with the bottom-line of their book – *scientific truth is a social construct* – I wholeheartedly agree with the theme singled out from it: *the inadequacy of the* [*historiographical*] *method which regards experimentally produced matters of fact as self-evident and self-explanatory*
   e. Hopefully our short study of Galileo's efforts in mechanics has provided convincing evidence for it

<u>Select Sources</u>

Galilei, Galileo, *Discourses and Mathematical Demonstrations Concerning Two New Sciences Pertaining to Mechanics and Local Motions*, tr. Stillman Drake, including "History of Free Fall: Aristotle to Galileo," Wall & Emerson, 1989.

--------, *Mathematical Discourses and Demonstrations Touching Two New Sciences, pertaining to the Mechanicks and Local Motion*, tr. Thomas Salusbury, London, 1665.

--------, *Dialogues Concerning Two new Sciences*, tr. Henry Crew and Alfonso de Salvio, Prometheus Books, 1991.

--------, *Discorsi e Demonstrazioni Matematiche intorno à due nuoue scienze attenenti alla mecanica e i movimenti locali*, Leiden, 1638, available via internet at *www.imss.fi.it*.

--------, *Galileo's Early Notebooks: The Physical Questions, A Translation from the Latin with Historical and Palelological Commentary*, William A. Wallace, University of Notre Dame Press, 1977.

--------, *Galileo's Notes on Motion, Arranged in Probable Order of Composition and Presented in Reduced Facsilime*, Stillman Drake, Istituto e Museo do Storia della Scienza, Monografia n. 3, 1979.

--------, *On Motion and On Mechanics*, tr. Stillman Drake and Israel E. Drabkin, *University of Wisconsin Press, 1960*.

Drake, Stillman, *Galileo at Work: His Scientific Biography*, University of Chicago Press, 1978; reissued by Dover.

Fantoli, Annibale, *Galileo: For Copernicanism and for the Church*, 3rd English Ed., tr. George V. Coyne, S.J., Vatican Observatory Publications (distributed by University of Notre Dame Press), 2003.

Renn, Jürgen (ed.), *Galileo in Context*, Cambridge University Press, 2001.

Koyré, Alexandre, *A Documentary History of the Problem of Fall from Kepler to Newton*, *Transactions of the American Philosophical Society*, vol. 45, part 4, 1955.

Hahn, Alexander J., "The Pendulum Swings Again: A Mathematical Reassessment of Galileo's Experiments with Inclined Planes," *Archive for History of Exact Sciences*, vol. 56, 2002, pp. 339-361.

Damerov, V., Freudenthal, G., McLaughlin, P., and Renn, J., *Exploring the Limits of Preclassical Mechanics: A Study of Conceptual Developments in Early Modern Science: Free Fall and Compounded Motion in the Work of Descartes, Galileo, and Beeckman*, Springer-Verlag, 1992.

Schemmel, Matthias, *The English Galileo: Thomas Harriot's Work on Motions as an Example of Preclassical Mechanics*, 2 vols., Springer, 2008.

Torricelli, Evangelista, *De Motu Gravium Naturaliter Descendentium et Projectorum*, excerpts tr. Judith Nelson and G. E. Smith, Florence, 1644.

Mersenne, Marin, *Harmonie Universelle, The Books on Instruments*, tr. Roger E. Chapman, Springer, 1957.

--------, *Novarum observationum physic-mathematicarum*, vol. 3, Paris, 1647.

Hall, A. Rupert, *Ballistics in the Seventeenth Century*, Cambridge University Press, 1952.

Drake, Stillman, and Drabkin, I. E. (tr.), *Mechanics in Sixteenth-Century Italy: Selections from Tartaglia, Benedetti, Guido Ubaldo, & Galileo*, University of Wisconsin Press, 1969.

Bertoloni Meli, Domenico, *Thinking with Objects: The Transformation of Mechanics in the Seventeenth Century*, Johns Hopkins University Press, 2006.

Palmieri, Paolo, *Reenacting Galileo's Experiments: Rediscovering the Techniques of Seventeenth-Century Science*, Edward Mellen Press, 2008.

Boschiero, Luciano, *Experiment and Natural Philosophy in Seventeenth-Century Tuscany: The History of the Accademia del Cimento*, Springer, 2007.

Shapin, Steven and Schaffer, Simon, *Leviathan and the Air Pump: Hobbes, Boyle, and the Experimental Life*, Princeton University Press, 1985.

Dugas, René, *A History of Mechanics*, tr. J. R. Maddox, Central Book Company, 1957; reissued by Dover.

--------, *Mechanics in the Seventeenth Century (from the Scholastic Antecedents to Classical Thought)*, tr. Freda Jacquot, Central Book Company, 1958.

French, A. P., *Newtonian Mechanics*, Norton, 1971.

MacMillan, William Duncan, *Statics and the Dynamics of a Particle*, Dover, 1958.

De Mestre, Neville, *The Mathematics of Projectiles in Sport*, Cambridge University Press, 1990.

Speiser, David, *Discovering the Principles of Mechanics, 1600-1800*, ed. Kim Williams and Sandro Caparrini, Birkhäuser, 2008.

Credits for Appendix

Slides 4, 6-11, 14-16, 19, 23-26: Galileo (1989)

Slides 12, 13: Galileo (1638)

Slide 17: French (1971)

Slide 28: Galileo (1979)

Slides 30-33: Hahn (2002)

Slide 36: Fantoli (2003)

Slide 39: Shapin and Schaffer (1985)