Uniform Motion

In a given time:

$$speed_1 : speed_2 :: distance_1 : distance_2$$

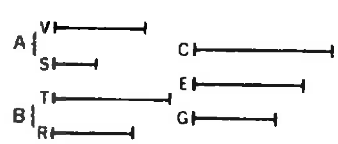
Over a given distance:

$$speed_1 : speed_2 :: elapsed\ time_2 : elapsed\ time_1$$

Therefore, *speed* in uniform motion varies directly with *distance* and inversely with *elapsed time*, the compound to two ratios

Proposition VI. *If two moveables are carried in equable motion, the ratio of their speeds will be compounded from the ratio of spaces run through and from the inverse ratio of the times.*

Proof: Let V and T represent the spaces, and S and R represent the times. Then the ratio of the speeds is represented by the ratio of C to G, compounded from the ratio of C to E (where C:E as V:T) and the ratio of E to G (where E:G as R:S).



Euclid's Elements

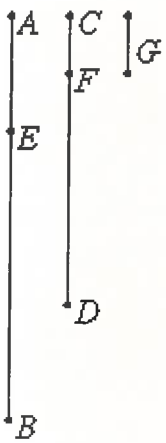
Book VII

Proposition 2

To find the greatest common measure of two given numbers not relatively prime.

Let *AB* and *CD* be the two given numbers not relatively prime.

It is required to find the greatest common measure of *AB* and *CD.*



If now *CD* measures *AB,* since it also measures itself, then *CD* is a common measure of *CD* and *AB.* And it is clear that it is also the greatest, for no greater number than *CD* measures *CD.*

But, if *CD* does not measure *AB,* then, when the less of the numbers *AB* and *CD* being continually subtracted from the greater, some number is left which measures the one before it.

For a unit is not left, otherwise *AB* and *CD* would be relatively prime, which is contrary to the hypothesis. VII.Def.12

VII.1

Therefore some number is left which measures the one before it.

Now let *CD,* measuring *BE,* leave *EA* less than itself, let *EA,* measuring *DF,* leave *FC* less than itself, and let *CF* measure *AE.*

Since then, *CF* measures *AE,* and *AE* measures *DF,* therefore *CF* also measures *DF.* But it measures itself, therefore it also measures the whole *CD.*

But *CD* measures *BE,* therefore *CF* also measures *BE.* And it also measures *EA,* therefore it measures the whole *BA.*

But it also measures *CD,* therefore *CF* measures *AB* and *CD.* Therefore *CF* is a common measure of *AB* and *CD.*

I say next that it is also the greatest.

If *CF* is not the greatest common measure of *AB* and *CD,* then some number *G,* which is greater than *CF,* measures the numbers *AB* and *CD.*

Now, since *G* measures *CD,* and *CD* measures *BE,* therefore *G* also measures *BE.* But it also measures the whole *BA,* therefore it measures the remainder *AE.*

But *AE* measures *DF,* therefore *G* also measures *DF.* And it measures the whole *DC,* therefore it also measures the remainder *CF,* that is, the greater measures the less, which is impossible.

Therefore no number which is greater than *CF* measures the numbers *AB* and *CD.* Therefore *CF* is the greatest common measure of *AB* and *CD.*

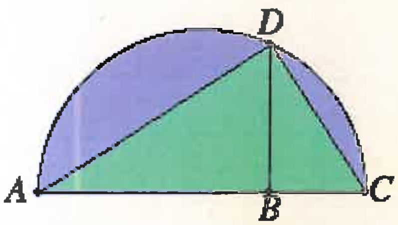
Corollary

Euclid's Elements
Book VI
Proposition 13

To find a mean proportional to two given straight lines.

Let *AB* and *BC* be the two given straight lines.

It is required to find a mean proportional to *AB* and *BC*.



Place them in a straight line, and describe the semicircle *ADC* on *AC*. Draw *BD* from the point *B* at right angles to the straight line *AC*, and join *AD* and *DC*. I.11

Since the angle *ADC* is an angle in a semicircle, it is right. III.31

And, since, in the right-angled triangle *ADC*, *BD* has been drawn from the right angle perpendicular to the base, therefore *BD* is a mean proportional between the segments of the base, *AB* and *BC*. VI.8.Cor

Therefore a mean proportional *BD* has been found to the two given straight lines *AB* and *BC*.

Q.E.F.

Guide

This construction of the mean proportional was used before in II.4 to find a square equal to a given rectangle. By proposition VI.17 coming up, the two constructions are equivalent. That is the mean proportional between two lines is the side of a square equal to the rectangle contained by the two lines. Algebraically, $a : x = x : b$ if and only if $ab = x^2$. Thus, x is the square root of ab.

When b is taken to have unit length, this construction gives the construction for the square root of a.

Use of this proposition

This construction is used in the proofs of propositions VI.25, X.27, and X.28.

Next proposition: VI.14

Previous: VI.12

Book VI introduction

Select from Book VI

Select book

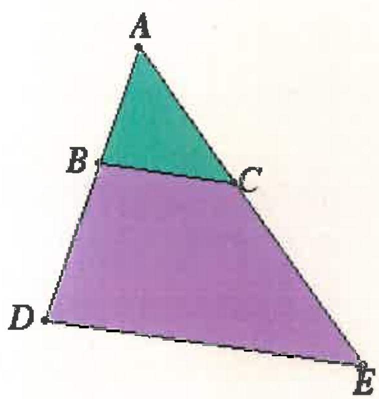
Select topic

Euclid's Elements
Book VI
Proposition 11

To find a third proportional to two given straight lines.

Let *AB* and *AC* be the two given straight lines, and let them be placed so as to contain any angle.

It is required to find a third proportional to *AB* and *AC*.



Produce them to the points *D* and *E*, and make *BD* equal to *AC*. Join *BC*, and draw *DE* through *D* parallel to it. I.3 I.31

Then since *BC* is parallel to a side *DE* of the triangle *ADE*, therefore, proportionally, *AB* is to *BD* as *AC* is to *CE*. VI.2

But *BD* equals *AC*, therefore *AB* is to *AC* as *AC* is to *CE*. V.7

Therefore a third proportional *CE* has been found to two given straight lines *AB* and *AC*.

Q.E.F.

Guide

If a and b are two magnitudes, then their third proportional is a magnitude c such that $a:b = b:c$. The third proportional is needed whenever a duplicate ratio is needed when the ratio itself is known. The duplicate ratio for $a:b$ is $a:c$.

Use of this proposition

This construction is used in propositions VI.19, VI.22, and a few propositions in Book X.

Next proposition: VI.12

Previous: VI.10

Book VI introduction

Select from Book VI

Select book

Select topic