   a. But not in a position to infer distance from sun via motion rule alone, for directional orientation of each small arc not determined
   b. I.e. still had to make some assumption about trajectory
2. Asked what physical factor might conceivably cause an eccentric circle, as in revised "solar" theory, if sun controlling the velocity (via magnetic effect)
   a. Concluded that planets must have independent source of motion superposed on sun effect
   b. This representable by means of a small epicycle superposed on circle with sun at center, to produce a circle with sun off center (a classic Apollonian move, as discussed in *Apologia*)
   c. For orbit to be circle and area rule to be satisfied, motion in epicycle must not be uniform; but proceeded anyway
3. I.e. proceeded to use assumed circular trajectory and the area rule to determine heliocentric longitudes of Mars, comparing them with values obtained from the vicarious theory
   a. Result: good agreement in apsides and quadrants, but error in octants
   b. +8 min 21 sec for first octant, -8 min 1 sec for third (see figure from Wilson in Appendix) -- i.e. planet moving too rapidly in apsides, too slowly in quadrants
   c. Verify that these discrepancies not just from using area rule by confirming that comparable discrepancies emerge as well with the 1/r rule
   d. To satisfy area rule, then, orbit must be oval rather than circular, for need less area around the quadrants; same true with 1/r rule as well
4. Triangulation calculations of the sort described last week (using the modified earth-sun theory) confirmed that the orbit is an oval of some sort (see Appendix), but sensitivity to observational errors prevented him from concluding what specific oval
   a. These triangulations used heliocentric longitudes from the vicarious theory, earth-sun distances from the modified solar theory, and geocentric longitudes from Tycho's observations -- all elements that had not been available e.g. to Copernicus
   b. Calculations difficult, given need to control for observational errors; Kepler tried several ways
   c. Conclusion: orbit comes in around 800/152,500 parts, with estimated errors of 100 to 200 parts
5. Two upshots: (1) orbit some kind of oval; and (2) area rule survived a test, for it predicts an oval and the distance calculations, though inexact, confirm an oval
   a. (Moreover, the conclusion that it is some kind of an oval is theory-dependent to a sufficiently large extent that some would have seen it as piling hypothetical conclusions on top of hypothetical conclusions in just the way that elicits objections and complaints in courtrooms
   b. Maybe why Kepler hesitated for two years before finally abandoning the circle)
6. Did manage to get some valuable conclusions out of the triangulations (see Appendix):
   a. To within the bounds of uncertainty, the oval is bi-laterally symmetric with respect to the line of apsides (Chapter 51)

b. Only a line of apsides through the true sun has its end points maximally near and far from the true sun, and a line of apsides through the mean sun does not have its endpoints thus maximally near and far from it

c. Confirmation of the location of the aphelion, with aphelial distance = 166510, perihelial distance = 138173, eccentricity near 14169 (0.093 vs earlier 0.09265) for earth-sun orbit radius = 100000

7. Finally, notice that the combination of using true sun as a reference point and the area rule has eliminated almost all the error from a classic eccentric circular model for Mars! (see table)
   a. Why then is Kepler more famous for the ellipse than for these two steps?
   b. Suspect the answer is twofold:
      (1) The shift from using the circular geometries of the 2000 years of previous astronomy
      (2) No one worked through *Astronomia Nova* sufficiently to realize how little work the ellipse was doing for Mars

H. "Phase 5": Alternative Oval Trajectories

1. One might naturally expect Kepler at this point to adopt the classical oval from geometry, the ellipse, and see how it does in conjunction with the area rule in yielding heliocentric longitudes
   a. But not what he does, undoubtedly because saw no physical reason why orbit might be elliptical
   b. Not engaged in just finding a geometrically familiar trajectory that agrees within observational limits, but wants one with some physical basis
2. Thus tries same sort of epicyclet model as before, but now with uniform circular motion on epicycle
   a. This together with area rule yields a slightly egg-shaped oval, with bulge end at perihelion
   b. Uniform motion on epicycle less physically objectionable than former non-uniform motion
3. Difficulties in carrying through the area calculations led him to approximate this oval by an ellipse (his "auxiliary ellipse", as shown in the figure from Gingerich in Appendix)
   a. Result: good agreement again in apsides and quadrants, but error in octants -- errors essentially the reverse of before
   b. -8 min for first octant, +7 1/2 min for third (Fig 9) -- i.e. planet moving too slowly in apsides, too fast in quadrants
4. Upshot: auxiliary ellipse yields excessive correction versus circle, roughly twice the amount of correction needed; proceeds to check, for several chapters, whether approximations, including auxiliary ellipse, might be responsible for the excessive discrepancies
5. The auxiliary ellipse an idealization, but one not prompted by idea that nature will conform with mathematics; rather, purely to ease computation, which is recognized throughout to involve uncertainty owing to observational inaccuracies, and hence is at best only approximate to begin with

I. "Phase 6": The Elliptical Trajectory

1. Working under the assumption of the area rule, the discrepancies at the octants between the circle of "Phase 4" and the "auxiliary ellipse" of "Phase 5" are opposite, and almost exactly equal