a. Difficult to get the "right" results in the experiment even when know what they are supposed to be

b. Therefore terribly easy to get "wrong" results, potentially falsifying theory without knowing whether just a shortcoming in experimental design

c. Mersenne's problem (see Appendix) appears to have been imperfect rolling -- e.g. at angles too high – and not imprecise time measurement, but main point remains either way: experiment not straightforward to pull off, requiring careful design (and fairly small angles of inclination)

d. The special care and effort needed to get the experiment to yield the "right" result is not unique to this example: a general situation in science

5. Tom Settle's modern repetition of the experiment, following Galileo's instructions, was far more successful (Appendix)

a. With practice (and warm-up) measured times from water-clock within 1/10 sec of predicted times (as Galileo said), and most within ± 1/20 sec

b. Good agreement for 6.86 deg of inclination (vs. Galileo's announced maximum of 9.6 deg), as well as 3.7 deg

c. The question, then, is how much effort Galileo put into the development of this experiment

6. The experimental program thus could at least have shown that the observed results were compatible (to within observational errors) with the 1,3,5, ... progression -- i.e. the results did not clearly falsify the claim -- and they may have shown much more (e.g. evidence for Galileo's "postulate")

a. Quotation of data themselves would have helped us see just how strongly the results supported the claim

b. But no data quoted, either in *Two New Sciences* or in notebooks, and hence cannot assess this

c. Nor can assess whether he began to encounter confuting data at higher angles -- e.g. at 30 deg, for which t in theory 2 sec

d. {For reasons that will become evident below, if not already from Mersenne's data, the 1,3, 5,... progression will almost certainly cease to hold as the angle of inclination is increased}

D. The Earlier Inclined Plane Experiment -- Drake (*Galileo at Work*)

1. Drake has reconstructed an earlier inclined plane experiment, primarily from a notebook entry with a list of numbers that he interprets to be an observed instance of the 1,3,5,... pattern

a. Data points: 33, 130, 298, 526, 824, 1192, 1620, 2123 [2104], where latter is taken to be a correction

b. Compared with: 33, 132, 297, 528, 825, 1188, 1617, 2112

2. Outfitted inclined planes with frets that would yield slight sound when ball passed, and then moved frets until ball passed each in uniform times

a. Got around problem of measuring time by using musicians who could detect discrepancy to within 1/64 of a beat, with a half second beat

b. Musicians maintained beat, and frets were moved until e.g. each one passed in one beat -- e.g. around 4 sec

3. Only the last tabulated value, when the ball is moving rather fast, falls outside a 1/64 error of beat (see p. 89 of Drake *op. cit*.)

a. Drake's proposal is that experiment revealed the 1,3, 5,... progression to Galileo – phenomenon drawn from experiment

b. But equally, then, can be used to argue that it provides evidence for uniform acceleration down inclined plane

4. Depending on how musicians were used and the steadiness of their beat over time, this experiment could provide more telling support than the one claimed in *Two New Sciences*

a. If, indeed, Galileo obtained the 1,3,5,... progression from it, then cannot be accused of "cooking" the experiment

b. But worries arise if have the prediction first and trying to use this procedure to test it

5. Also, notice that this experiment, unlike the one in *Two New Sciences*, provides no empirical basis for choosing between the square rule and indefinitely many other rules that would yield the specific idealized numerical sequence given above

a. Square rule the simplest, but other curves can be fit through these data

b. Other experiment, by contrast, compares a variety of distances along same plane, thus showing in principle that the 1,3,5,... progression holds for numerous different initial increments, and the distances in the progression correlate with the sine of θ across different heights!

6. In sum, I am prepared to grant that Galileo did perform some sort of inclined plane experiments, obtaining the 1,3,5,... progression to sufficiently high accuracy for small angles to convince him legitimately that it is a real phenomenon

a. How long it took others to become proficient in this experiment is unclear -- or whether they even tried to become proficient

b. Also unclear whether Galileo discovered that get the "right" result only so long as angle small and then suppressed this finding

c. (Would like to know more about the history of inclined plane experiments, for reasons that are about to become evident)

E. A Crucial Lacuna in the Evidential Argument

1. The truly crucial lacuna in Galileo's evidential reasoning using the inclined plane -- the lacuna that yields a true, systematic error -- lies in the Postulate

a. Says ball on an inclined plane will acquire the same speed in falling through a height h whether it falls vertically or along an inclined plane

   b. But unless the ball can somehow be made to fall along the plane entirely friction free -- i.e. be made to slide, with no frictional resistance at the point of contact with the plane -- this is false
2. Rolling down an inclined plane is different from sliding friction free down an inclined plane
   a. Rolling of a spherical ball in a groove: a = 5/7∗g∗sin(θ), so that

$$v_{end} = \sqrt{(10/7*g*h)},\ t_{end} = \sqrt{(14/5*h/g)}/\sin(\theta),\ \text{and}\ l_{end} = 5/14*g*t^2*\sin(\theta)$$

   b. Friction-free and medium resistance-free slide or fall has a = g∗sin(θ), so that $v_{end} = \sqrt{(2gh)}$, $t_{end} = \sqrt{(2gh)}/\sin(\theta)$ and $l_{end} = 1/2*gt^2\sin(\theta)$
3. In other words, the rate of acceleration for a sphere rolling on its bottom is 5/7 that of an object completely unencumbered by friction, and hence the velocity acquired by a rolling ball through a height h will be 84.5 percent of that acquired by the falling ball
   a. A 28 percent difference in accelerations, a 15 percent difference in velocities, with consequent differences in times etc.
   b. Something Galileo was totally unaware of throughout
4. Given his descriptions of the inclined plane and the bronze ball, his inclined plane experiments almost certainly involved rolling
   a. Extraordinarily difficult to get any object to slide down an inclined plane friction-free, especially a round object, for the least friction will be sufficient to initiate rolling
   b. Comparably difficult to get a round ball to roll vertically downwards
   c. Can maintain rolling on inclined plane so long as inclination limited and ball does not bounce at all, whether using a v-gutter or a curved groove (see appendix for contrast between them)
   d. Once the inclination becomes high enough, however, the ball intermittently bounces from the surface, accelerating at the full rate rather than 5/7 of it briefly before resuming rolling
   e. Result will then be a somewhat random mixture of two different rates of acceleration
5. The reason Galileo never understood this conceptual distinction was, of course, that no empirical observations ever forced it on him
   a. An example in which empirical contrast needed to draw distinction in the first place, after which theorizing can respond
   b. Huygens appears to have been the first to appreciate the difference, in the early 1690s, when he had become concerned with what became known as the principle of conservation of *vis viva* and applied considerations from his work on the center of oscillation, but did not publish (acknowledge Nico Bertoloni Meli, in response to my challenge)
   c. Became theoretically evident with Euler's work of 1765, and to some extent even with his watershed monograph of 1750; question is whether anyone noted it publicly earlier
   d. Quite a comment about experimental practice if 1765 is the first publication where the difference is fully evident, and no prior experimental result announcing a conflict

F. The Effect of the Lacuna on Propositions III-VI

1. The need for a rolling versus falling distinction enters only in comparisons between the two, and not in any comparisons within either regime itself
   a. Thus, for example, Proposition II and the 1,3,5,... corollary hold perfectly well for all rolling spherical balls (though not for cylindrical tubes) with respect to one another, and for all falling objects, independent of shape
   b. Therefore, the main consequence of the lack of the distinction is to widen the lacuna in the evidential reasoning in which observations for rolling balls are taken to provide evidence for what happens with falling objects
2. Proposition III, which says that the times of descent from the same height along inclined planes and vertical are proportional to the distance traversed -- i.e. time is inversely proportional to the sine of the angle of inclination -- is false if object is rolling on plane
   a. But result holds for different inclined planes so long as ball rolling (and same geometrical shape)
   b. Similarly for Propositions IV and V, giving comparative times of descent for different planes
   c. And all three propositions would hold in a regime of pure sliding or falling
3. A possible experiment here to reveal the need to distinguish rolling and falling: drop a ball and start a ball rolling down an inclined plane at the same time, with the heights chosen so that theory predicts both balls will reach the ground at the same time
   a. According to Proposition III, the 12 braccia plane inclined at 30 deg would be matched by a 24 braccia height; but fall would happen in less than 2 seconds, and rolling on a 30 deg incline might be difficult to maintain
   b. Instead use 2 braccia height incline, which a 72 braccia height (14 story building) would correspond to; in ideal circumstances, rolling would take 3.46 sec and fall, 2.93 sec, a noticeable difference; but in real world air resistance on falling sphere and brief bouncing will move these numbers closer to one another, making the difference difficult to detect
   c. A conclusive test thus not so easy
4. The beautiful result in Proposition VI, all chords from top of circle or to bottom have the same time of descent, similarly holds only for rolling or falling separately
   a. Here again have a result that would seem to be the basis for a critical test of the theory that might not require excessively precise measurements
   b. But carrying through such a test, and thereby revealing the difference between rolling and falling, very difficult
5. Propositions III-VI outwardly provide not only a means for calculating and predicting what will happen, but also a basis for designing experiments
   a. This was clearly Galileo's intent

b. Indeed, notice Galileo’s whole approach here: put forward a hypothesis, develop a mathematical theory yielding striking predictions that are amenable to quasi-qualitative tests!

c. But the experiments were almost certainly too difficult to set up in a way that would yield meaningful results at the time

G. Mersenne's Efforts and the Lacuna: 1633-1647

1. Mersenne, perhaps provoked by a remark in Galileo's *Dialogue*, sees a different way of bridging any lacuna in the evidence for claims about free-fall: measure the distance of fall in the first second -- in effect g/2 -- the constant of proportionality in $s \propto t^2$

   a. Galileo's remark: objects fall 4 cubits in first sec, which Mersenne knew to be way too small

   b. Galileo himself calls attention to a lacuna in the argument for the Postulate in the original edition of *Two New Sciences* [207]

   c. If stable value regardless of height, and if it yields reasonable results for total elapsed times, then direct evidence for claim that free fall uniformly accelerated

2. Fr. Marin Mersenne (1588-1648) a professor of natural philosophy at the University of Paris, a close friend of Gassendi and Descartes, and a long time correspondent and admirer of Galileo's

   a. Deeply committed to experimentation, and hence naturally tried to reproduce Galileo's experiments, as well as to conduct many further ones on his own, in the process discovering such things as the non-isochronism of circular pendula

   b. Relevant publications: *Les Méchanique de Galilée* (1634), *Harmonie Universelle* (1636), *Les nouevelles pensées de Galilée* (1639), *Cogitata Physico-Mathematica*, *Phenomena Ballistica* (1644), and *Reflexiones Physico-Mathematica* (1647)

   c. Huge intellectual correspondence: 17 volumes already published

3. Mersenne made the advance of using a 3 and 1/2 ft pendulum, with period near 1 sec (1 sec at 60 deg arc), to measure the distance objects would fall in varying times

   a. Experiment performed 50 times, yielding 3 (Paris) ft in 1/2 sec, 12 in 1 sec, 48 in 2 sec, 108 in 3 sec, 147 in 3.5 sec (*Harmonie Universelle*, p. 138)

   b. Note that listed values were perfect; actual values were 110 ft in 3 sec and 146.5 ft in 3.5 sec

   c. Still, would appear to be fairly compelling evidence for the 1,3,5,... progression in free fall

   d. A few years later (in 1640s) dropped objects from 300 ft dome of St. Peter's Basilica in Rome, finding times not 5 sec, as implied by above, but between 5 and 6 seconds

   e. Ultimately concluded that Galileo's principle of free fall is only a rough approximation, even in absence of air resistance

4. Moreover, his earlier positive result is not all that good in retrospect, for implied acceleration is way too small -- around 788 cm/sec/sec, versus correct value of 981

   a. Indeed, value closer to that for rolling spherical ball (701 cm/sec/sec) than for falling object