

The Construction of Mathematical Identities in K-12 Mathematics Education

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Abstract

For many people, their first interactions with mathematics occur in the mathematical classroom, and have a far-reaching impact on their long-term relationship with the subject. Mathematical identity research is crucial for understanding how people engage with mathematics and, subsequently, how they perceive themselves as mathematical learners and doers. This dissertation broadly addresses the construction of mathematical identities for students and educators in K-12 mathematics classrooms. In particular, I address the following research question: *what are the factors that interact with each other to influence how K-12 students and educators take up specific mathematical identities?* In order to begin answering this question, I present the results of three empirical studies. The first is a case study of a Black girl as she navigates mathematics over the course of sixth grade. I provide evidence that her mathematical identity was socially constructed on cultural ideas about what it meant to be a mathematics student as a Black girl, which often came about as a result of social interactions she had. In the second study, I used data from a Classroom Teaching Experiment in a first-grade classroom to investigate how a teacher-researcher's responses to students' ideas are associated with how students were positioned into different mathematical micro-identities. I share evidence that the teacher-researcher's responses pointed to moment-to-moment micro-identities for several students. Lastly, the third study explores the narratives of two Black women paraeducators teaching mathematics to understand how their remembered histories contributed to the construction of their mathematical identities. I provide evidence that their K-12 mathematics experiences regulated the mathematics identities they enacted as present-day paraeducators.

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Chapter 1: Introduction

Rationale for Researching Mathematical Identity Construction

In the last few decades, mathematics education research has recognized identity as a major aspect that needs to be taken into account when investigating students' mathematical learning, and as informative for mathematics teaching practices. In this dissertation I drew from Martin (2006) to craft my own definition of mathematical identity. Martin (2006) defined mathematics identity as "the dispositions and deeply held beliefs that individuals develop, within their overall self-concept, about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives" (p. 206). I define mathematical identity as "how one perceives oneself as a doer and learner of mathematics, one's interpretations of others' perceptions of oneself as a mathematical learner, and/or how one relates to the mathematical content one engages with." Both mine and Martin's (2006) definition focus on one's self-perceptions about mathematics and one's beliefs about one's mathematical capabilities. Where my definition differs from Martin's (2006) is in the understanding that one's self-perception relative to mathematics also considers other people's beliefs about one's relationship to mathematics. Two decades ago, pioneers such as Boaler and Greeno (2000), Martin (2000), and Nasir and Hand (2008) accurately claimed identity was a significant, novel tool to analyze how people engage with mathematics content, and with the discipline as a whole. Much of this early research considered mathematical identities as stable and constructed over a long period of time, or what Wood (2013) categorized as a mathematical macro-identity. A mathematical macro-identity considers who a person is mathematically, and it is believed to be relatively unchangeable. Macro-identities are constant positions, they are who a person was, is, and will be in the future. Note that position here does not refer to one's physical location, but

one's social position relative to a particular context. Research has now demonstrated that macro-identity does not provide the most comprehensive representation of how people perceive themselves in the context of doing mathematics.

For instance, McCarthy and Moje (2002) disagreed with the concept of a stable macro-identity and claimed instead that identity is actually fluid and contextually dependent. Specifically, one's mathematical identity can be further broken down into a micro-identity, which Wood (2013) defined as an identity "enacted in a moment in time" (p. 778), with the ability to fluctuate. This is evident, for example, in Langer-Osuna's (2011) work addressing equity in a high school mathematics class, in which the author focused exclusively on a group of four students to examine which micro-identities students took up when collaborating on mathematics assignments. Langer-Osuna specifically looked at moment-to-moment interactions among the students to determine how these interactions influenced students' momentary mathematical micro-identities. Although her research examined identity in a mathematics context (i.e., how one is perceived as a mathematical learner) rather than mathematical identities themselves (e.g., being a mathematical learner), Langer-Osuna found that two of the students in her study, Brianna and Kofi, oscillated between different identities when collaborating. Brianna went from being a "good [leader] to bossy" (p. 212), while Kofi was initially a "quiet loner [who became a] smart ally" (p. 212). For my own research, I considered the contextual mathematical identities people were positioned into, either by themselves or by others.

Problem Statement

Knowledge about mathematical identities construction is crucial to gain insight into how people view themselves and others relative to their learning and doing of mathematics. Researching mathematical identity is important because it helps inform researchers and

practitioners about how significant the interactions one has with mathematics and others (van Langenhove & Harré, 1999) in a mathematical environment (Ball, 2018) are on one's engagement with the subject, as well as how identity and engagement interact. For students, particularly marginalized students in mathematics such as Black and female students, there is a plethora of research demonstrating how navigating the mathematical classroom can be a "painful" (Gholson & Martin, 2019, p. 396) experience. In understanding the effect of these interactions, researchers and practitioners must be better informed on how to support students, being mindful of how their marginalization can lead to vastly different journeys as mathematics students compared to students from dominant groups. Further, this research can expose sociomathematical norms (Yackel & Cobb, 1996) that position teachers as exclusive mathematical authority figures, and push for teaching practices that value students as knowledge-bearers as well. The myriad experiences one may have with mathematics may have momentous power in how one crafts a personal mathematical identity. For one, some may believe they cannot "do well in math" (Cribbs et al., 2021, p. 279), while for others, increasingly positive interpretations of what it means to do mathematics and who can do mathematics can lead to monumental declarations like that of Crystal in Chapter 3 in this dissertation, who declared herself a "math guru" (*June 4, 2021 Interview*, line 224).

In Chapter 5, I expand my research goal to consider educators, and their relationships to mathematics. More specifically, I focus on paraeducators¹ whom school districts rely on heavily, but who are often disconnected from the mathematics classroom and community. The National Resource Center for Paraeducators (NRCP) is an organization founded in 1979 with the sole intention of providing support for paraeducators, a role that came into existence over 50 years ago (Njuguna, 2016). Initially, paraeducators were limited to assisting with student monitoring or

¹ Paraeducators are also referred to as paraprofessionals, instructional assistants, teacher's aides, etc.

performing clerical tasks. Paraeducators were largely invisible as educators in this way, as the main role of teaching was for the main classroom teacher. As education changed over the years with teachers encountering more complex and pressing teaching situations, the role of the paraeducator changed as well. No longer do paraeducators essentially stand on the sidelines as nearly unseen in the classroom these days; now, paraeducators often work directly with students, some of whom are the most challenging to work with, and are expected to provide instructional support for them. According to Njuguna (2016), in some school districts paraeducators are also now being considered as primary targets for filling teacher shortages, which is happening without proper training. For mathematics particularly, a subject which has the reputation for often evoking negative affect, this lack of training and proper inclusion into the mathematics educator community can have profound negative effects on paraeducators and the construction of their own mathematical identities. If they have a negative mathematical identity, paraeducators may not be as confident engaging with mathematical content and, subsequently, less equipped to teach students, ultimately negatively impacting students' comprehension and, potentially, their ability to have a strong affect and mathematical identity themselves.

Theoretical Framework

In each of the stand-alone papers included in the dissertation I use different theoretical frameworks to ground my data; however, positioning theory undergirds them all. In the empirical chapters below I will expand on the specific frameworks to each. Here, I expand on the shared framing of positioning. I used van Lagenhove and Harré's (1999) positioning theory as my theoretical framework to investigate the construction of mathematical identities among various populations. While positioning has different connotations, such as physical, or describing one's professional role, positioning here refers to the ways in which people interact through discourse

and actions to arrange social relationships with others. For instance, a teacher may use specific language that positions her as a teacher to the student, and in response, the student may interpret this such that they subsequently interact using student-specific language or actions. Regarding mathematical identities, positioning theory is crucial to this work because it helps explain how people take up certain roles or positions in the mathematics classroom, and how they exist in shared storylines.

Langer-Osuna and Esmonde (2017) expanded van Lagenhove and Harré's (1999) concept by positing that positioning theory is a useful theoretical approach to conceptualize mathematical identities. They broadened positioning theory to focus specifically on mathematical identities. Positional theory or positional mathematical identities maintain that "mathematical identities develop through the claims people make about themselves and others in mathematical spaces" (Langer-Osuna & Esmonde, 2017, p. 639). People make agentic claims about their competence and sense of belonging in mathematics based on the social interactions they have with others. Langer-Osuna and Esmonde (2017) argued that although positional theory stresses individual agency, personal claims about one's mathematical identities are not without the influence of these interactions. In fact, individuals do not have the "capacity to solely determine their own identity" (p. 639). This is because these moment-to-moment interactions can "thicken" (Holland et al., 2001) over time. As such, it is possible for one to both position themselves and be positioned by others into particular mathematical identities. Positioning theory provides the necessary framework for investigating how people from different populations construct their mathematical identities, specifically, how their interactions with others inform the claims they make about the particular types of people when it comes to mathematics.

Research Questions

The broad research question I aim to explore through this dissertation is: *What are the factors which interact with each other to influence how K-12 mathematics students and educators take up specific mathematical identities?* In addition, each of the empirical chapters in this dissertation (Chapters 3-5) has its own research questions:

Study 1 (Chapter 3): What are the factors associated with a Nigerian-American girl's mathematical experiences, and how do they inform the construction of her mathematical identities?

Study 2 (Chapter 4): (RQ1) What do we learn about Wood's (2013) framework of mathematical micro-identities when we explore how it plays out in a new classroom context? and (RQ2) What are the connections between Wood's (2013) moment-to-moment positioning and enactment of mathematical micro-identities and how a student enacts influence?

Study 3 (Chapter 5): How do remembered histories of mathematics learning shape Black women paraeducators' identities?

Structure of the Dissertation

This dissertation is organized into six chapters. In this introductory chapter, I presented my rationale for studying mathematical identity construction in K-12 mathematics education, the problem statement, overarching theoretical framework anchoring this work, and research questions for each empirical study. Following this chapter, in Chapter 2 I focus on the different methodologies used in each of the three empirical studies comprising this dissertation, and the justification for each. The subsequent three chapters are stand-alone empirical studies. In Chapter 3, I present a case study on a sixth-grade Black female mathematics student, and how her mathematical identity was constructed and changed over the course of one school year. This

student's mathematical identity was heavily dependent on cultural ideas she internalized about what it meant to be a mathematics student as a Black girl. In Chapter 4, I present my second empirical study on how a teacher-researcher's responses were associated with various moment-to-moment micro-identities that she positioned several students into. In Chapter 5, I present my third empirical study which focuses on the narratives of two Black women paraeducators in early elementary mathematics classrooms, finding that their K-12 mathematics experiences largely contributed to the mathematics identities they took up as present-day paraeducators. Chapter 6 includes the discussion of my findings, the conclusions, and contributions to the literature. I also detail the limitations and implications of my research, while providing suggestions for future research.

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Chapter 2: Methodology

In the following chapters (3, 4 and 5), I provide details and specifics about the methodologies I used for each of the three empirical studies that comprise this dissertation. In this chapter, I begin with my positionality as a researcher and why this is significant relative to the participants in each study. Next, I provide context for each of the studies to describe the various methodologies used across all three studies, before delving deeper to provide a broad overview and justification for each. I close with limitations of each methodology and potential future research that could address these limitations.

Researcher's Positionality

I am a Black woman, a K-12 public school alumni, and former public school mathematics educator. As this dissertation is centered on the construction of mathematical identity, I begin with my identities that influenced my research because they have each contributed significantly to my work. As a child, I attended several public schools, each of them varying greatly in terms of demographics, resources, and location. While I excelled across all of the schools I attended, my confidence about my mathematical ability was largely related to the demographics of my school. I attended a large public school district in the North East United States where it was commonplace to see other Black girls around me, and even less common, be taught and led by Black women educators. I was in an advanced study program with other Black girls and other non-White students. And when certain mathematical topics were a bit more challenging to grasp, I might have questioned my intelligence, but my identity as a Black woman was never a factor related to that questioning.

Towards the conclusion of my K-12 academic journey, as a high school student, I moved to a district where I began to grasp what it meant to be minoritized, which was a jarring shift for

me. Not only were Black girls in the minority across my grade cohort, but I was also often the only Black girl in my mathematics classes. Moreover, in this new district, I was never again taught by another Black educator. My uniqueness as one of four Black girls in my higher-level mathematics courses was not something I celebrated; instead, it made me lean into perfectionism, afraid of making mistakes for fear of what it would look like to struggle as a Black girl. In McGee's (2015) work on fragile and robust mathematical identities, she characterized a student with a fragile mathematical identity as one who is extrinsically motivated to perform well in mathematics due to a hypersensitivity to stereotypes associated with their race. While a student may achieve highly, they do not necessarily have a positive affinity towards the subject. According to McGee's (2015) framework, I had an incredibly fragile mathematics identity; I did not particularly find joy in doing mathematics and I was constantly managing stereotype threat as a young Black woman. Nonetheless, I persisted and pursued a Bachelor's degree in mathematics, but these critical moments were momentous and my undergraduate mathematics experience was an extension of my high school mathematics experience. I was always one of a few, if not the only, Black woman in my mathematics courses, and perceived myself as 'not being good at math' or having a negative mathematics identity, even when my grades proved otherwise. Ironically, it was my experience as a mathematics educator in the same district I initially struggled in as a high school student that helped me become more aware of my Black womanhood in mathematics. This led to my interest in exploring mathematical identity and how one comes to see oneself as a mathematical learner and doer.

As a teacher, I did not yet have the terminology to describe my mathematics journey, but I began seeing myself in my Black girl students who were confronting the same scenarios as I did as a youth. This time, as an educator, I was empowered to step in and support all my students

but was especially cognizant of the Black girls in my classrooms, who were highly intelligent and capable, but were doubtful of their mathematics prowess. This dissertation is the result of these relationships with my first cohort of 7th grade Black girl students, who, like me, were and are capable of achieving highly in mathematics, but have and had long believed the lie that their identity as Black girls is and was a hindrance from doing so.

Context and Design

Across all three empirical studies, the goal of the dissertation was to get a sense of how different populations (students and educators) craft particular mathematical identities, either for themselves or for others, a stance Langer-Osuna and Esmonde (2017) posited can be true concurrently.

Chapter 3

The first empirical study (Chapter 3) captures the first year of a three-year longitudinal case study of a middle school Black girl attending a predominantly White school. While I collected data for all three years, I only used data from her sixth-grade year to allow for a more diverse study into mathematical identity construction in this dissertation including a wider range group of people in the mathematical space (i.e., different aged-students in Chapter 4 and educators in Chapter 5). To learn about her mathematics experiences and how they impacted her mathematical identity development, I observed her in her mathematics classes and conducted same-day interviews after school. Given the huge shift in my own mathematical experience in middle school and my work as a middle school teacher, choosing to work with a middle school Black girl was intentional in my study design. Much of my interest was on what was going on in the mathematics classroom and how students might interpret these interactions, so conducting both classroom observations and interviews was important for this work. The observations

allowed me to get in-the-moment data about potential factors that might impact the student's experience (e.g., student-to-student and student-to-teacher dynamics, lesson context, specific mathematics topics, etc.). After observing the student in the classroom, I then honed in on moments and interactions I found interesting to learn where my interpretations and the student's converged and/or diverged.

Chapter 4

In the second study (Chapter 4), I used a subset of data from a larger NSF study² in which the researchers facilitated an eight-week, 16 lesson Classroom Teaching Experiment (CTE) with 22 first grade mathematics students to engage them in early algebraic thinking. By design, CTEs afford researchers the opportunity to test a hypothesis by designing learning environments in which students interact with specific content knowledge and practices and then inspect students' learning and development (Cobb & Steffe, 1983). All lessons in the CTE were video recorded. The lesson videos provided a plethora of data such as interactions between students and interactions between students and researchers during whole-group discussions. Although I was not a part of the data collection for this study, I quickly connected my background as a mathematics teacher to what I observed in the videos, as I witnessed how substantial these interactions were in guiding both the students and the researchers to make specific moves in the classroom, all of which had a profound effect on the development of students' mathematical identities.

Chapter 5

In the last study (Chapter 5), the two participants were part of a larger NSF study³ with 22 total participants focusing on providing professional development for paraeducators in early

² NSF DRK-12 1154355: Children's Understanding of Functions in Grades K-2.

³ NSF DRK-12 2101425: Doing the Math with Paraeducators: Enhancing and Expanding and Sustaining a Professional Development Model in PreK to Grade 3 Math Classrooms.

elementary mathematics classrooms. While collecting data for the larger study, it was often difficult to observe the paraeducators teaching mathematics, either due to being given menial tasks (i.e., managing student behavior, being pulled for other duties, etc.), inconsistencies in their schedules at their school, or because of challenging relationships with their partner teachers in who did not give opportunities to teach. Thus, relying on classroom observations proved to be insufficient for investigating the construction of their mathematics identity as it related to their educator identity. Furthermore, as I sought to delve into narratives that spanned the breadth of their mathematics journeys, interviews were the appropriate avenue for me to do so. In conducting the interviews with the paraeducators, our prior relationship due to our participation in the larger study and our common identities as Black women educators led to a deep trust on their part, which allowed for more revelatory insights that I might not have otherwise gained with classroom observations alone.

In the next section, I will focus on Chapters 3 and 5 only because both of these empirical studies involved race and its impact on the participants' mathematical identities.

Which Methodologies are Appropriate to Research the Impact of Race on Mathematical Identity Construction?

The research questions for Chapters 3 and 5 both have a focus on identity issues as they relate to participants' race. For Chapter 3 it is: *What are the factors associated with a Nigerian-American girl's mathematical experiences, and how do they inform the construction of her mathematical identities?* Meanwhile, the research question for Chapter 5 is: *How do remembered histories of mathematics learning shape Black women paraeducators' identities?*

Classroom Observations

Given the fact that the research question for Chapter 3 explicitly highlights race and gender, I intentionally chose classroom observations to analyze the interactions the participant had with her peers and with her teacher, and how her various social identities as a Black girl came into play in the mathematics classroom. Meyer et al. (2011) asserted that classroom observations permit researchers to study how these interactions affect students' development and proficiency in specific disciplines. In considering my research question, classroom observations were necessary for me to examine how the participant's identity as a Black girl further impacted these interactions, and more importantly, how she viewed herself as a mathematical learner. I was motivated to use observations as my methodology after reading Gholson and Martin's (2019) case study of a single student, which they did entirely through observations. In their study on Black girls' mathematical selves, classroom observations were a useful tool to investigate how "Black girlhood" (Gholson & Martin, 2014, p. 19) operated in the mathematics classroom, and subsequently, how it guided the student in enacting a particular mathematical identity. Classroom observations were a crucial first step in understanding how my study's participant's mathematical identity was constructed because they allowed me to exist concurrently in the same mathematical environment as her to observe these interactions. In addition to observations, I also conducted semi-structured interviews.

Interviews

As a qualitative method, interviews allow us "to see things we would not have seen through classroom observations" (Caddle, 2012, p. ii). These "things" include past moments, interactions, and experiences that inform how people navigate their current mathematical environments (Denzin & Lincoln, 2005), and make sense of their experiences. Prior research (Adams, 2013; Langer-Osuna & Esmonde, 2017; Williams, 2011) has demonstrated how present

narratives are significantly shaped by past narratives and experiences. As such, I used semi-structured interviews (i.e., asking participants open-ended questions) in Chapters 3 and 5 specifically because in recounting events and recalling memories, interviews can, according to Denzin and Lincoln (2005), help researchers probe interesting moments, and participants make sense of these experiences, which cannot always be achieved through classroom observations. For studies in which researchers seek to probe such moments, interviews are an appropriate method for gathering data, as they rely entirely on participants' responses. Because participants likely speak the most, interviews also position them as reliable knowledge bearers of their own histories, which is especially crucial for researching race. I contend that semi-structured interviews are optimal for researching race because the pre-set and open-ended questions can both guide the conversation towards answering the research questions, but also allow for flexibility to adjust the questions based on the participants' responses (Jamshed, 2014). This flexibility can reduce the rigidity of more formal interviews, which may make it difficult for participants to trust the interviewer in sharing potentially sensitive and weighty information.

Why are CTEs Appropriate for Researching Mathematical Identity Construction?

In Chapter 4, the researchers chose CTEs for data collection because they allow researchers to design a learning environment and observe learning and change within that designed environment. Rather than traditional classroom instruction which tends to focus on assessing student proficiency on the topic (Emerson & English, 2016), CTEs allow researchers to test a hypothesis by observing what a student can do and how they do it and subsequently making modifications to the experiment based on what students do. In this CTE, the researchers focused on whole-group discussions, and the teacher-researcher encouraged students to lead these discussions. In designing the CTE this way, there is a focus on interactions between

students and the teacher-researcher, which Langer-Osuna's (2015) research argued can reveal identity dynamics. CTEs are appropriate for analyzing how students construct their mathematical identities because the engineered environment can afford researchers more discretion (Ball, 2018) and flexibility than that of the traditional mathematics classroom, allowing researchers to explore what role interactions play in how people are positioned into certain mathematical identities.

Limitations and Future Research

Classroom Observations

In conducting classroom observations, one major limitation concerns the specific moment and occasion of the observation (Meyer et al., 2011), as the quality of the data is heavily dependent on the participant and context of the daily lesson. Meyer et al. (2011) posited that classrooms are dynamic and ever-changing, as are the interactions between students and teachers. This can lead to inconsistencies in the data such that it is difficult to make inferences about the role of these interactions in students' development. To address this, Meyer et al. (2011) suggested researchers develop classroom observation protocols that are flexible and capture changes in classroom dynamics.

Interviews

There are several potential limitations to employing interviews as a methodology, and they all depend on how structured the interview protocol is. Unstructured interviews, which may "[resemble] a conversation more than an interview" (Jamshed, 2014, p. 87) may lead to a variation of data quality, as the protocol may either be non-guided or casual, making it more challenging to answer the research question. Very structured, or more formal interviews, may favor the interviewer (Jamshed, 2014), which could lead to difficulty building trust between the

interviewer and interviewee, particularly with studies on race, which adds another layer of complexity. As mentioned above, a modification to these probable barriers is for researchers to opt for conducting semi-structured interviews, which strikes the balance between both unstructured and structured interviews in there being both directive and open ended questions that guide the can provide deeper insights into participants' experiences. Powell et al. (2005) also claimed that using interview protocols with specific, non-leading questions, which are not frequently used in everyday dialogue can help build trust between the interviewer and interviewee.

Classroom Teaching Experiments

There are also several limitations to CTEs as a methodology. One of these is that researchers might be constrained by teaching and learning practices that are normative to the particular classroom the experiment will be conducted in. While researchers come in with their own designs and ideas about executing their protocols, there are cultural norms and practices that have great influence on this execution. For instance, a classroom teacher may provide more prompting with her students during discussions, which could impede the teacher-researcher's exploration into understanding what a student knows about a concept. To address this, researchers could collaborate with teachers to define their roles during the experiment, being clear about the differences between a general classroom lesson and a CTE and their shifting role in sharing their classroom authority with the researcher. An additional limitation is that while researchers have a hypothesis they aim to test with CTEs, there are other phenomena that may arise that researchers did not plan for. This may lead to researchers having to adjust their experiment to make room for "the emergence of other potential pathways for learning and development" (Cobb et al., 2003, p. 10). To address this, researchers could pilot their design with

similarly aged participants to prepare for a variety of student responses, as well as design CTEs that are rigorous but that could also be modified should there be a divergence from the original hypothesis.

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Chapter 3: Exploring a Black Girl's Mathematical Identity: A Culturally Complex Narrative of Mathematical Success

Motivation and Introduction

My mathematical journey has been replete with the management of academic stress related to ideas about what it means to do and learn mathematics, and who is considered a mathematician. In wrestling with my own self-perception as a member of the mathematical community, my social identity as a Black girl and woman were paramount. Moreover, as a former mathematics teacher, my middle school students' experiences as Black girls in the mathematics classroom mirrored much of my own as a youth: being one of a few Black people, if not the only Black girl, and feeling invisible or excluded by peers. In her work on the critical literacy of race, Sealey-Ruiz (2021) included what she named the "archeology of self" (p. 287), a leading component of Racial Identity Development. The archeology of self is a "deep excavation and exploration of beliefs, biases, and ideas that shape how we engage in work" (p. 287). This study on mathematical identities construction is a result of who I am; it comes from my personal experience as a Black girl/woman, and as a Black mathematics teacher.

There is an abundance of deficit-based literature on Black students' academic prowess, particularly in mathematics, but much of this data focuses on achievement gaps, often between Black and White students. While this research is crucial for advocating for the need for more inclusive, equitable mathematics education for Black students, some have called for shifting away from this overemphasis to reimagine what academic success can look like for Black students (Gardner-Neblett et al., 2023). Further, while there is data on Black students' navigation of mathematics classrooms, the gaze is oftentimes on Black boys exclusively, leaving Black girls behind (Gholson, 2016). There are certainly overlaps in their racialized experiences, as Evans et

al. (2011) argued. Nevertheless, the multidimensionality of one's identity cannot be reduced to simply perceiving each aspect as merely additive. Black girls are both Black and female, but they occupy both spaces concurrently. Therefore, the racialized mathematical adventures of Black boys cannot simply be extrapolated to that of Black girls. Similarly, their gendered mathematical experiences will coincide with that of other girls but will diverge due to their racial identity. It is crucial to take an intersectional approach then (Crenshaw, 1989), focusing on the participants' specific journey as members of two minoritized groups because their identities occur altogether, all the time, and cannot be disentangled.

Because Black girls occupy two minoritized groups, their educational experiences are filled with complex issues, as their racial and gender groups increase the likelihood of them encountering discrimination. Joseph et al. (2019) argued that Black girls are dehumanized in society, and this dehumanization trickles down into all institutions, including education, and more specifically, in mathematics classrooms. Additionally, because mathematics classrooms are often led by White men, this can make Black girls not only susceptible to both racism and sexism but make it difficult for them and their teachers to imagine themselves as mathematics learners. Thus, in taking this inclusive lens, my study will allow me to comprehend the role that one student's identity as a Black girl played as she navigated mathematics. In this paper, I present a qualitative study of Crystal⁴. Crystal was a child of immigrant parents who hailed from Nigeria, and though she is American, her Nigerian culture played a significant role in her understanding about what it meant to be a mathematics student. I followed Crystal over the course of her sixth-grade year in mathematics classes to examine how her lived experiences as a Black girl affected her mathematical identity. The research question anchoring this study is: *What are the*

⁴ All names, including school names, are pseudonyms.

factors associated with a Nigerian-American girl's mathematical experiences, and how do they inform the construction of her mathematical identities?

Prior Research

As was mentioned above, there is a dearth of research into Black girls' identity in mathematics. In conducting this study, not only did I intend to add to a growing body of literature, I wanted this research to be authentic and specific. Crystal is a Black girl of Nigerian descent, so in this section, I focus on prior research on Black girls' identity in mathematics, as well as Black immigrant cultural values about education.

Black Girls' Identity in Mathematics

Research in the last couple decades has pushed back against conventional norms which concentrate primarily on students' mathematical content learning to incorporate the construct of identity. Proponents of this shift believe that knowledge of student identity relative to mathematics can help researchers gain a more holistic understanding of their students' experiences in mathematics classrooms (Martin, 2012; Nasir & Cooks, 2009; Nasir & Hand, 2008). Further, this research can help examine how students relate to mathematics, as well as how identity impacts their learning outcomes, as they are entangling and mutually informing. This scholarship on mathematics identity construction informed my own research as I attempted to understand how Crystal's multiple identities converged while in the mathematics classroom to shape her comprehension of mathematical content, as well as how the intersection of multiple identities created narratives for her about her inclusion in the mathematical community.

When it comes to the construction of their mathematical identities, Black girls are unfortunately not the sole arbiters. The literature referenced above speaks to the plethora of stereotypes and beliefs Black girls must navigate due to their affinity grouping in two

marginalized oppressed groups in mathematics education. Evans et al.'s (2011) paper also sought to analyze academic self-concept in Black children; specifically, how adolescents navigate stereotypes of identity membership groups. Research into this is especially poignant for Black girls because of their multi-identity membership, which means they encounter when they possess the least amount of privilege, and they are more likely to be pigeonholed due to the specified identity group. In mathematics education, the privileged group tends to be White men, and this is evident in the ways in which researchers study students in mathematics. According to Martin (2013), much of the student-centered mathematics education research refers to White children as all children (Martin, 2009), thus completely leaving out Black girls' experiences.

Additionally, although mathematics is oftentimes portrayed as neutral (Battey & Leyva, 2016) or objective due to its universality, it is not free of bias, as there are expectations and norms about what it means to be a member of this community. Malloy's (1997) research on the rampant ethnocentrism in mathematics postulated that Black students are exposed to traditional ways of doing mathematics that may not always align with their cultural ways of knowing. I argue this is one way in which Black girls can also face obstacles in accessing mathematics, as they are outside of dominant culture, and are challenged with enculturating into the mathematics community. This outsider status may lead to Black girls encountering stereotypes about their mathematical proficiency, which may also lead to self-repudiations about their membership in the community (Malloy, 1997). Nonetheless, these stereotypes do not always serve as a hindrance to the mathematical identity construction, as these stereotypes can also be co-opted for success (McGee & Martin, 2011). They chronicled Black undergraduate mathematics students as they journeyed through their courses in hopes to gain insight into how positive agency in stereotype management contributed to academic achievement. The majority of their student

participants sought to strengthen their mathematical identities, seeking to succeed academically to disprove those who endorsed negative stereotypes.

This study is part of a larger research project focused on the experiences of Black girls in K-12 mathematics classrooms, and more specifically, during their middle school years. My research will provide a more diverse account of one student's mathematical journey, which will stray from traditional literature which tends to present singular narratives of Black students as a monolith. By narrowing in on Crystal, a member of an often-ignored group of students, my research will amplify Black girls' voices, normalizing the humane treatment of Black girls within mathematics classrooms (Joseph et al., 2019). Furthermore, it will also diversify the limited research on Black girls' mathematical identity by highlighting how Black immigrant cultural values take hold in students' self-perceptions relative to mathematics.

Black Immigrant Cultural Values About Education

In her work on the college choice process for Black immigrant children in the United States, Griffin et al. (2012) concentrated on these students' habitus, or the "worldview shap[ing]" the choices Black students from immigrant families make regarding college (Griffin et al., 2012, p. 97). Though each student has an individual habitus, culture undoubtedly influences the habitus one crafts for oneself, as it "predisposes [one] towards behaviors deemed [culturally] appropriate" (p. 97). Participants in Griffin et al.'s (2012) study described how their families emphasized that educational attainment was paramount, in many cases so that they could support their families. Consider Stacy, a Nigerian-born student, who claimed that education is a sort of "power...[that] you have to work hard to get," (p. 102) and once you do, "you're going to get a good job...and be "paid more" and "use some of that money to help your family" (p. 102). Likewise, Tiffany, a second-generation Eritrean immigrant asserted that "if you can come out of

[the United States] and...have a good education, have a good job, provide for your family, that's like the ultimate dream" (p. 102). While Griffin et al.'s (2012) study focused on undergraduate students, it still informs my research on Crystal, as these students' parents' cultural messages about education were the motivating factor driving students' academic behavior, which I argue begins prior to their matriculation into college.

Roubeni et al. (2015) carried out a qualitative study to analyze West African immigrant parents' migration narratives to identify how they connected their experiences of loss and educational ambitions for their children in the United States. The authors identified the most pertinent themes from their interviews and found that narratives about educational expectations were frequent across all interviews. Parents "expect[ed their children] to try hard and get [a] good education... And do very well" (p. 288). Another parent articulated that "the most important thing for [her] children is school. Because with education you can do anything" (p. 288). In reflecting on their decision to migrate to the United States, Roubeni et al. (2015) found that many parents were not only hopeful, but certain that if their children focused on their education, they would excel beyond their schooling, because the United States "is a country of opportunity" (p. 290). These parents expected their children to succeed academically because they believed that in doing so, education would be the means through which they would be propelled forward and obtain upward mobility. Roubeni et al.'s (2015) study provided evidence that these migration narratives were both common and powerful: common in that participants across various West African cultures overlapped in their elevation of academic prowess, and powerful in that they mitigated whatever loss the parents may have had as they looked forward to their children's ability to attain what they could not. This research is relevant because it is likely

that Crystal, my study's participant, encountered similar messaging as a child of Nigerian immigrants.

Theoretical Framework

I grounded my research on McGee's (2015) construct of fragile and robust identities, a framework the author used to survey the intersection of mathematical and racial identities relative to the mathematical experiences of Black undergraduates and modified it to consider gendered mathematical experiences. Further, I used Jaber and Hammer's (2016) concept of epistemic affect to expand on McGee's (2015) framework to more accurately explore Crystal's relationship with mathematics as a discipline. Lastly, I used Abes et al.'s (2007) Intersectional Model for Multiple Dimensions of Identity (I-MMDI) to ensure my analysis of Crystal took all of her identities into account to understand how she crafted her mathematical identity.

Fragile vs. Robust Mathematical Identities

Given the fact that mathematical identity construction involves the intersection of a plethora of factors including students' personal experiences, McGee (2015) argued that for Black students in particular, mathematics education researchers should also consider race as a dominant factor. I assert that in addition to race, researchers need to take an intersectional approach and consider other identity markers that could impact students' mathematical self-concept. Thus, in examining McGee's (2015) framework, I make space to consider gender as a factor.

In McGee's framework, a fragile identity is based on the "delicate" (2015, p. 604) relationship between a Black student's mathematical achievement and their constant racialization sustained in doing mathematics. I modified this definition to consider gendered dynamics while doing mathematics (text in italics indicate my additions to the gendered dimension). Contrastingly, a robust identity centers on the resilience and fortitude these students develop and

exhibit in spite of racialized *and gendered* mathematics experiences to maintain motivation inherent for mathematical achievement. There are three components to the fragile and robust identity framework: (a) “central motivations to succeed in mathematics,” (b) “the use of coping strategies in response to students’ racialized *and gendered* mathematical experiences,” and (c) “dispositions associated with one’s successful outcomes in mathematics” (p. 604). Central motivation refers to how Black students self-motivate as they do mathematics, either intrinsically or extrinsically. McGee contended that fragile and robust mathematical identities are fluid, as students can shift between the two depending on specific interactions and moments in their mathematical classrooms. Although McGee’s (2015) framework was centered on Black undergraduate students, I posit this can be applied to explore how Crystal’s mathematical identities came about because the underlying factors impacting the mathematical identities of the participants in McGee’s (2015) study were salient and held significant power, even in youth, for Black students.

Epistemic Affect

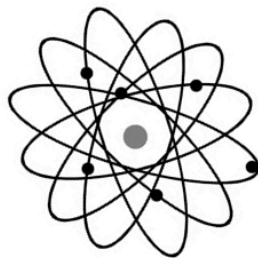
While the third component of McGee’s framework includes students’ disposition towards success in mathematics, I maintain that Jaber and Hammer’s (2016) concept of epistemic affect provides a more comprehensive perspective of students’ overall disposition towards mathematics. Epistemic affect considers the “feelings and emotions experienced within” (p. 189) mathematics, not just one’s attitudes about achieving success. Epistemic affect is thus a useful approach to consider how Crystal’s feelings about mathematics as a discipline ultimately influenced her mathematical identity construction.

Intersectional Model for Multiple Dimensions of Identity (I-MMDI)

The Intersectional Model for Multiple Dimensions of Identity (I-MMDI) was conceptualized by Abes et al. (2007) to examine students' intersecting identities and how these identities' salience changes across time and different contexts. The I-MMDI is a sense-making tool, as it helps students give meaning to the different experiences they have as a result of their identities. For Crystal specifically, the I-MMDI acknowledges the intersection of her identity as a first-generation Black girl of Nigerian descent and how this intersection contributed to her understanding of what it means to be a mathematics learner and doer. Figure 1 is a graphic connecting all three theories.

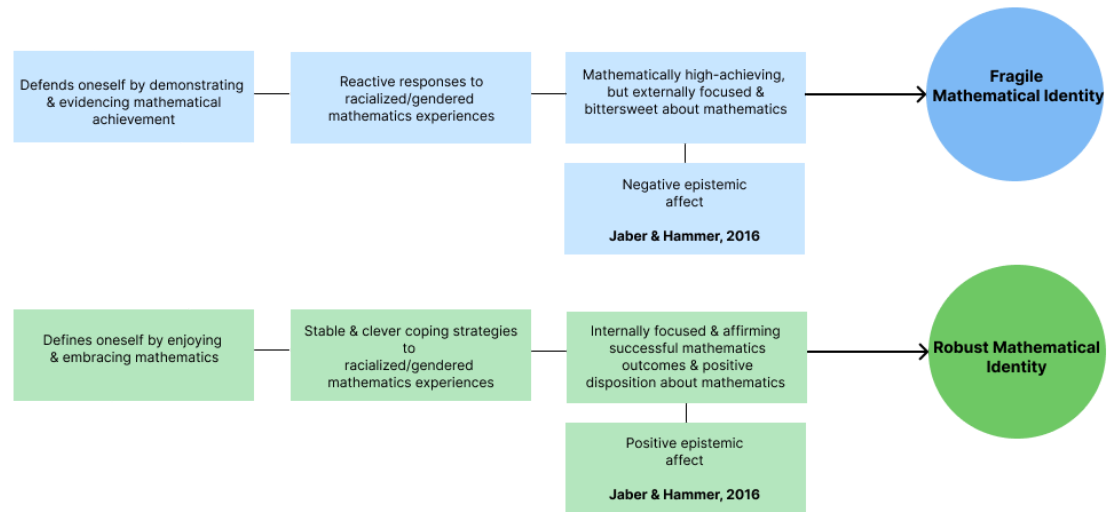
Figure 1.

Graphic connecting three theories used for theoretical framework.



Self-Perceptions of Multiple Identity Dimensions (Abes et al., 2007)

Meaning-Making Filter (Abes et al., 2007)



Methodology

I engaged in a year-long study of a middle school Black girl in her mathematics classroom at the Pinewood Middle School in the Northeastern United States, where I was previously employed. The study took place during the COVID-19 pandemic. Pinewood is a public suburban school of approximately 71% White, 13% Black, 6% Asian, 5% Hispanic, 4% multiracial and non-Hispanic, 0.2% Native American, and 0.1% Native Hawaiian and Pacific Islander students. Nearly 11% of Pinewood students are economically disadvantaged.

Positionality

As I mentioned above, I am a Black woman and a former middle school mathematics teacher who began this study into Black girls' mathematical identity construction in light of my own personal investigation into what my mathematical identities are, and how they came about. In my work as a teacher, I often considered how my self-perception as a mathematics learner would influence my work in teaching students, particularly how my practice would impact students' mathematical learning, and subsequently, the construction of their own mathematical identity. More specifically, given the small percentage of Black students enrolled at Pinewood, I was even more interested in how race would factor into this identity construction for my Black students who were marginalized as minoritized students and sought to ensure that in my role as their teacher, I did all I could to foster a positive, strong mathematical identity. While I cannot claim that Crystal's participation during the study was due to my own identity as a Black woman and/or as a mathematics teacher, Joseph et al.'s (2019) research confirms students' mathematical identities are affected by interactions with their teachers. I cannot claim to know the extent of this influence, but I assert they did have some influence, as I continue to have a relationship with Crystal post-study.

Participant

In selecting participants for this study, I distributed documents outlining the study to sixth grade teachers at Pinewood Middle School. Two students and their parents/guardians completed the consent and assent forms to participate. While two students originally signed up, one student was removed due to inconsistencies in attendance, leaving me with the single participant in this study, Crystal. Crystal is the younger sister of a student I had previously taught at Pinewood. Crystal identified as a Black girl of Nigerian descent, and she attended sixth grade through the full remote model up until mid April 2021⁵, when she switched to being fully in person, per State and District orders for all students to return to K-12 schools. Crystal's parents were Nigerian immigrants, while she was born in the United States. Crystal had high achievement in all of her classes and often discussed her affinity for English class, reading, and art. Crystal was the second youngest of four children, with two sisters and a brother, and she described having a tight knit relationship with her family.

Data Collection

There were three rounds of observations and interviews. A round consisted of at least three observations and three interviews: The first round of data collection was from February to March 2021, the second from April to May 2021, and the last was in June 2021, as the goal was to observe potential changes over time. There were a total of 11 observations and 10 individual interviews. A sample interview protocol is included in Appendix A.

I observed Crystal virtually, due to the COVID-19 pandemic, a total of five times and each of these observations lasted approximately 40 minutes. Virtual observations of Crystal's mathematics lessons occurred during the afternoon every four days through Google Meet to account for Pinewood's rotating schedule. As such, during some weeks, I observed Crystal two

⁵ This study took place during the COVID-19 pandemic.

times, and in the subsequent week, only once. These observations were not recorded. During these observations, I took field notes of how Crystal engaged and interacted with the lesson material, her teachers, and peers.

After the instructional model change in April 2021, I observed Crystal another six times in person and also took field notes. After each of the 11 lesson observations except for one (a scheduling mishap), I conducted online individual student interviews (i.e., a total of 10 interviews), also on Google Meet, to ask Crystal follow-up questions related to the lesson, as well as to pose questions about her identity. Individual interviews lasted up to thirty minutes and took place after school, mostly on the same day of my observations, barring any scheduling conflicts.

Data Selection

Due to limitations of online class observations (e.g., Crystal's choice regarding camera and microphone usage⁶), there was nominal data relevant to her identity in the classroom data. Conversely, much of the interview data related specifically to her identity, the focal point of this research. As a result, only the interview data were selected for analysis. I then transcribed each of the 10 interviews. In order to further pare down my dataset, I used a grounded approach (Glaser & Strauss, 1967) using ideas in formation (Charmaz, 2006) to identify emerging themes in the transcripts, highlighting those themes which repeated throughout all transcripts. Identity was an obvious theme, but others emerged throughout my review. I decided to select only the interviews in which identity emerged in our conversations at least once and from my own protocol. This led me to eliminate her March 3rd, 2021 interview, as there were no discussions about identity. Given this selection criteria, I also treated her February 10th, 2021 interview

⁶ Unless explicitly directed to, students were not required to have their cameras and microphones on in Crystal's virtual mathematics classroom, and she often kept both of these off.

differently: in this interview, I used a guided protocol, the Multidimensional Model of Black Identity (MMBI) (Sellers, 2013) which included statements related to identity. Since the questions were centrally focused on identity, I did not code this part of the interview, as I felt my questions influenced how Crystal answered. I coded only questions I developed on my own due to its consistency with other interviews with similar emergent themes. In total, there were nine interviews that were coded.

Data Analysis

I developed my coding scheme through a line-by-line review of my interview transcripts with an additional coder. We noted themes that emerged in the transcript lines, which were the following: (1) affect, (2) expectations, and (3) identity. These themes and their definitions are included below in Table 1. Note that these themes are not mutually exclusive.

Table 1.

Emergent themes and sub-themes, definitions, and transcript examples.

Themes and Sub-Themes	Definition	Transcript Example
Theme: Affect	“The conscious emotion that occurs in reaction to a thought or experience.” (Merriam-Webster Dictionary)	<i>“I don’t like asking for help.”</i> February 4, 2021, line 20
Sub-Theme: Epistemic affect	Feelings and emotions experienced with mathematics (Jaber &	<i>“I used-, I used to hate math.”</i> April 30, 2021, line 178

Hammer, 2015). These can be positive, negative or neutral.

Theme: Expectations	Social norms around what others consider “reasonable, due, necessary” or expected of someone. (taken from Merriam-Webster Dictionary’s definition of expect)	<i>“I’m just saying tha-, that everyone has to learn math.”</i> February 4, 2021, Line 85
Sub-Theme: Personal expectations	Personal beliefs about what is considered “reasonable, due, necessary” or expected (using Merriam-Webster Dictionary definition of “expect”)	<i>“Math is one of those things that you just have to do.”</i> February 4, 2021, line 71
Sub-Theme: Parental expectations	Expectancy messaging parents'/guardians' convey to children relative to their beliefs what is considered proper and necessary (Parsons et al. 1982).	<i>“And my parents kept-, kept on telling me, ‘make math your friend’.”</i> April 30, 2021, line 179

Sub-Theme: Societal expectations	Normative beliefs fostered by others in the social environment (Joseph et al. 2020).	<i>“They just portray [school] as a horrible place that you should never wanna go to.”</i> February 4, 2021, line 139
Theme: Identity	Social and/or personal constructions about self that can vary over time and contexts and that shape or are aspects of how humans understand the world around them and how they too are understood. (definition inspired by McCarthey & Moje, 2002)	<i>“I think of identity sort of like, personality or one’s personal life.”</i> May 10, 2021, Line 44
Sub-Theme: Racial/ethnic identity	The racial and ethnic identity group one ascribes to.	<i>“Like even though I’m Black...”</i> May 10, 2021, line 98 <i>“I’m Nigerian-American.”</i> May 10, 2021, line 100
Sub-Theme: Gender identity	The gender group one ascribes to.	<i>“I’m a 11 year old girl.”</i> February 4, 2021, line 96

Sub-Theme: Mathematical identity	How one perceives oneself as a doer and learner of mathematics, one's interpretations of others' perceptions of oneself as a mathematical learner, and/or how one relates to the mathematical content one engages with.	<i>"My parents always noticed that math was my-, was my weak spot."</i> February 4, 2021, line 24
Sub-Theme: Identity: other	All other utterances which were unrelated to any of the sub-themes for identity.	<i>"Uh, not exactly an artist or just someone who wants to draw."</i> May 10, 2021, line 53

I coded every line of the transcripts according to these three themes and used an utterance as my unit of analysis (Langer-Osuna, 2016), focusing only on Crystal's utterances. I define an utterance to be a single line of verbal discourse by a speaker, bound on each side (i.e., beginning and end of an utterance) by utterances by another speaker.

Once the dataset had been coded for the three themes, I returned to the nine transcripts to identify sub-themes (Saldaña, 2013) to gain a deeper understanding of how all of the themes materialized and oscillated over time. The sub-themes emerged as I explored Crystal's responses to questions that suggested that in order to clearly understand her mathematical experiences and the numerous factors which affected her identity construction, I needed to parse out the details as specifically as possible. The respective sub-themes are also shown in Table 1: (a) epistemic

affect (related to any aspect of doing mathematics), (b) personal expectations, (c) parental expectations, (d) societal expectations, (e) racial/ethnic identity, (f) gender identity, (g) mathematical identity, and (h) other. Other is defined as all utterances unrelated to race/ethnicity and gender, which are the focus of this study.

After refining the sub-themes, I shifted my attention towards identifying what may have prompted Crystal to answer my questions in particular ways, to learn more about Crystal's relationship to mathematics. For example, why might she have expressed a more positive or negative epistemic affect at different times (e.g., could this be due to whether or not she had an assessment that day)? More specifically, were there types of questions associated with specific sub-themes, and could any shifts in epistemic affect be explained? I considered each coded utterance in context to explore this, looking at how these themes and sub-themes throughout the interviews. I did this by first identifying the focus topics covered during our interviews and calculating the prevalence of sub-themes within topics to explore this issue in more depth, which are included in Table 2 below. I identified these topics by annotating Crystal's utterances to classify and summarize the overall focus of her discourse. For instance, at times when she was asked generally about a particular day in mathematics class, Crystal would focus on the specific lesson taught, and what she learned. In this case, I summarized that portion of her discourse as a *debrief of mathematics class*. Each interview covered a range of topics, and there was considerable overlap among the interviews in terms of topics discussed. Note that while there was overlap among the topics discussed, no interview had the exact same topics.

Table 2: *Focus topics across all interviews.*

Topic

Academic action
Academic expectations
Being a Black girl in mathematics
Beliefs about being a mathematics learner
Beliefs about being a student
Beliefs about mathematics
Beliefs about school
Debrief of mathematics class
Discussion about identity
Discussion about others' perception of her identity in mathematics
Discussion about self-perception of identity in mathematics
Interactions with others
Mathematical mindset
Parents' messages about being a mathematics learner
Parents' messages about student agency
Past mathematical experiences

These changes in how her utterances were coded for the themes and sub-themes were used to then construct my claims, which I explore in the next section.

Figure 2 displays a heat map comparing Crystal's utterances coded for the sub-themes, represented by the rows, and the topics, which are represented by the columns. The column widths are arbitrary and do not denote the length of interview time spent on them. Instances of 0% in the heat map are a result of the lack of overlap between the topic and sub-theme. Cell shading corresponds to the intersection of sub-themes and topics and are filled in with

percentages, which were calculated by dividing the count frequency of each of Crystal's utterances coded for each sub-theme relative to the interview topic and dividing it by the total number of her utterances relative to the specific topic. The lightest shade denotes lower percentages of these occurrences, while the darkest shade denotes higher percentages.

Figure 2.

Heat map comparing Crystal's utterances coded for each sub-theme.

Results

Figure 2 indicates that general epistemic affect, “feelings and emotions experienced within” (Jaber & Hammer, 2016, p. 189) and personal and social expectations were the most prevalent factors in the construction of Crystal’s mathematical identity, denoted by the darker shaded cells for these themes and sub-themes. Crystal’s feelings about mathematics fluctuated depending on the context surrounding her responses to interview questions. Her epistemic affect varied such that she conveyed a different emotion when discussing mathematics as a discipline both within and outside of the mathematics classroom, and relative to the relationships she had with her mathematics teachers. Crystal’s personal expectations were revealed most when discussing academics more broadly (academic expectations, beliefs about school), but mostly when discussing her beliefs about being a student.

Affect: Epistemic Affect is Contextually Situated.

Crystal conveyed generally negative affective views about mathematics throughout the duration of our interviews. A clear understanding of how her epistemic affect developed and shifted throughout her sixth-grade year may help explain how she perceived herself as a mathematical learner.

Negative Affect

Crystal’s interviews provide evidence that her epistemic affect about mathematics was largely negative when speaking about her past mathematical experiences (20.69% of utterances were coded for negative epistemic affect), her personal convictions about the subject (12.7%), or mindset about doing mathematics (15.22%).

Across all nine interviews, Crystal had 56 utterances about epistemic affect, and of those, more than half (29 or approximately 52%) were negative. In just the first interview on February

4, there were seven utterances related to epistemic affect, and Crystal spoke negatively three times, or 42.9% of these utterances, when reflecting on her elementary mathematics experience. She frequently voiced her displeasure at being singled out by her teachers about her academic struggles: “To tell the truth, [in] elementary school, I actually didn’t like advocating, to be honest... I don’t like asking for help... Because I...my parents always noticed that math...was my weak spot. And...I didn’t really like it whenever-, after a class, they always pointed out the fact that I wasn’t dealing well with the math question. And then that just made me feel sad” (*February 4, 2021 Interview*, line 24).

In the same interview, when asked about her decision to work independently when given the option to work in a group during mathematics class, Crystal also said: “I don’t know. Whenever...I’m with someone else and then I’m doing something, I kinda feel sort of uncomfortable. It’s sort of like when I’m doi-, like when I’m answering a math problem and like, my parents are nearby, I feel sort of uncomfortable, because I don’t like it when I make mistakes in front of others” (line 45).

This feeling of vulnerability in her mathematics classroom contributed greatly to Crystal’s general uneasiness towards doing mathematics, such that she described feeling emotional pain, which she hid from others, particularly during assessments. Crystal recalled having to put on a “cool face” when taking a quiz. When asked about her emotions going into the quiz, Crystal responded: “A bunch of mixed feelings...I was putting on a cool face and except a lot was going on back there. I wasn’t panicking, but I also was panicking at the same time” (*February 10, 2021 Interview*, line 63). She further explained that she “[didn’t] even know, why [she] want[ed] to” put on a cool face. Crystal elaborated: “I’m not sure if I would call it panicking. I mostly just call it worrying. Like, part of me was like, ‘OK, I can do this’, but the

other half of me was like, ‘oh, no, I’m not ready, I can’t do this’” (*February 10, 2021 Interview*, line 69).

Neutral Affect

When speaking about mathematics as a discipline outside of the mathematics classroom, however, Crystal also displayed neutral affect towards the subject, with 13 out of 56 utterances about epistemic affect, or approximately 23%, coded for neutral affect. She believed learning and excelling in mathematics was critical for anyone’s long term success, but also mentioned at least once in each of the first three interviews that the subject is compulsory in nature: “math is just one of those subjects that I don’t like, except it just has to be done” (*February 4, 2021 Interview*, line 33). In this sense, although she also acknowledged her general negative feelings about mathematics, this was not the ultimate emotion Crystal had; she was able to acknowledge the subject’s usefulness. She also explained how her parents used previous mathematics experiences to challenge her to develop more of a positive affect about mathematics, or at least overlook her emotions to concentrate on the greater goal of performing well. 17.24% of her utterances about her mathematics history were coded for neutral epistemic affect. Crystal described an instance where her mother told her that she “[didn’t] have to like math to be good at it” (*February 4, 2021 Interview*, line 33). I posit that Crystal’s realization that mathematics was obligatory was associated in part to her internalizing her parents’ expectations about her mathematical achievement and its relation to her general success in school, as well as after school. This balancing act between these two frames of mind may have served as a mitigation tool for her epistemic affect, because Crystal also communicated a neutral stance at times. In the same interview, Crystal declared that in order for one “to excel in their life [they needed to know

math]. If you don't know math, then what are you supposed to do?" (*February 4, 2021 Interview*, line 87).

Here, Crystal displayed a more dutiful attitude, neither expressing a disparaging nor cheerful disposition. There were 18 utterances coded for negative epistemic affect during the first three interviews, which had 69 total utterances related to affect. By the second round of interviews, this was reduced by 50% with only nine instances coded for the same. Crystal continued to highlight her parents' thoughts about mathematics, and how they encouraged her to neutralize her initial thoughts: "I used to hate math. And in other words, basically just finding excuses not to do it. And my parents kept on telling me, 'make math your friend. Don't...keep on running away from it'" (*April 30, 2021 Interview*, lines 178-179). I recognized a change in language in that she described previous feelings about mathematics and asked her to explain what changed: "Mmm...well, I mean, I guess it's-, maybe it's just a matter of maturity, probably?...Or just-, or you know, just listening to my parents" (*April 30, 2021 Interview*, line 182 & line 184).

Positive Affect

As we progressed through the interviews, there were instances of positive epistemic affect. 24% of Crystals' utterances for epistemic affect were positive, which is compelling in consideration of how she began the interviews. This positive affect materialized in relation to discussions about her mathematics teachers, Mrs. Monteiro and Mrs. Cook, again demonstrating affect is contextually bound. Crystal described a mathematics anxiety video shown in Mrs. Monteiro's class which helped her realize that she "[wasn't] BAD (emphasis added) at math," despite her constantly "telling [herself], 'I THINK I am.'" (*February 23, 2021 Interview*, line 218). In showing this video, Mrs. Monteiro helped to provide the enculturation into the

mathematics community that Malloy (1997) wrote about by implicitly reframing Crystal's angle on her mathematical experience. Midway through the year, Crystal expressed hesitancy in switching from Mrs. Monteiro to Mrs. Cook as a result of transitioning from a fully remote learning model to a full in-person one. She had a compelling breakthrough - she asserted that "Mrs. Monteiro is part of the reason why math is so fun" (*April 11, 2021 interview*, line 86). Her hesitancy about her new teacher was assuaged as she stated her new teacher "[is]...really fun" (*April 30, 2021 interview*, line 57).

Expectations: Influence of Intersecting Expectations

Crystal's personal expectations related to her mathematical experience often intersected with her interpretation of societal and parental expectations. In Figure 2, much of Crystal's utterances centered on her beliefs and personal expectations: 62.5% were related to beliefs about being a student, 30% were about her beliefs about school, 27.27% were about academic expectations and 27.08% were about others' perceptions about her mathematics identity. Even while she spoke a lot about her own beliefs, Crystal frequently referenced a lack of confidence about her own skills and societal messages about what it meant to be a student (62.5%) or a mathematics learner (9.38%). When asked to consider whether or not the idea of one being a "math person"⁷ (*February 4, 2021 Interview*, line 162) was valid, she struggled to come up with a clear answer: "To be honest, I don't know how I feel about that quote. Part of me thinks that it's true...I just loved reading, and I could just sit down with a book and read for hours. Except for my brother, he was all about math and stuff, except he wasn't very good in...ELA. Yeah, but my other that is it may NOT be true, because like, a person that...has never loved math...can come to love it. So does that make them a math person? Because being a math person probably means that you were a math person...like before, like, right when you were born." Crystal used

⁷ There is a common fixed belief in society that people are either a "math person" or not (Boaler, 2015).

an analogy to compare herself and her affinity to English Language Arts (ELA) with her brother, who enjoyed mathematics more than ELA. She initially concluded that one could be labeled according to the subject they leaned toward most but reversed this thought in questioning whether or not one could share their partiality with more than one subject.

In reflecting on her fifth-grade mathematics experience, she also claimed she formerly believed that doing mathematics required perfection. “I didn’t really like fifth grade [inaudible] is that oftentimes I would, I would get in this habit of pouting during math class. And sometimes even crying...I thought I was dumb, I was getting things wrong. I thought everyone knows this but I don’t. So, what’s wrong with me?” (*April 30, 2021 Interview*, line 101). Crystal spoke at length about her personal expectations and beliefs about school and being a student, and much of this seemed to be rooted in perfectionism. Crystal’s perfectionism continued into sixth grade, and this is unmistakably visible in her numerous utterances expressing how others perceived her mathematics identity (27.08%), particularly regarding advocating for help, as it reveals her struggles to others. In Crystal’s perspective, mathematics students do not make mistakes when working, and to do so is a negative mark on one’s competence: “Like, some people accept their mistakes, except I’m just one of those people that don’t like them...Bottom line, I’m the kind of person that likes to be right” (*April 30, 2021 Interview*).

Crystal’s expectations were so closely linked to socialized ones that it became difficult at times to identify the point of intersection. These social expectations included ideas that mathematics is inflexible or only accessible for people naturally adept at doing mathematics in addition to her acknowledgement, but not endorsement, of negative messaging about Black students’ mathematical competency. Further, these social expectations were often relayed to her by her parents, which also proved to be challenging to delineate.

When she discussed societal expectations in any context, the overwhelming majority of utterances were about her parents' beliefs (100% of utterances about *parents' beliefs about being a mathematics learner*, 66.67% of utterances about *parents' beliefs about student agency*). Crystal's expectations about being a mathematics student were largely rooted in perfectionism, and she was largely aware of the societal expectations for Black female mathematics students imparted to her by her parents. She had a keen awareness of societal expectations about mathematics which imply that mathematics learners are not typically perceived to be Black or female. Crystal postulated that the barriers Black women encountered in the mathematical field had everything to do with inequity through racism, mainly. Consider the following exchange: "But maybe, a lot of Black women, much more than you think, WANT to get into the math field. Except they just aren't given a chance. It's for obvious reasons. It's because they're Black. Maybe they want to get into the math field, except...all these White people...that are part of the math field are like, 'you can't do this. You can't be in the math field. Get out of here!'" (February 23, 2021 Interview, line 237).

Crystal's parents wielded momentous power in her understanding of what it meant to be a Black girl learning mathematics. Of all discourse focusing on this topic, 20.83% were coded for parental expectations. She recalled how her parents "always told [her] Black people are really, really smart" (February 23, 2021 Interview, line 204) At the same time, Crystal also confidently acknowledged their consistent conveyance about pervasive, detrimental societal messages about being Black, effortlessly diagnosing racism as the root issue, declaring a sort of acceptance and understanding of the discrimination she would face in the future: "I'm pretty sure there's one thing that I might guarantee the future may be a teensy weensy bit harder, since the discrimination against Blacks hasn't completely disappeared from the surface of the earth yet.

And... my father keeps on telling us that we have to make the most of it, while we're young, because we're already in a disadvantage due to the color of our skin" (*June 8, 2021 Interview*, line 47). Figure 2 shows that Crystal's parents' expectations were a huge influence for her, particularly in her understanding of social expectations on her as a Black girl mathematics student. I posit that Crystal's personal expectations were likely a conglomeration of her parents' expectations, which were embedded in societal expectations about being a Black girl mathematics student.

Crystal conceded earning perfect marks in all her classes and making her parents proud is "80% my parents" (*June 3, 2021 Interview*, line 44), and "20% me" (*June 3, 2021 Interview*, line 46). Although Crystal made great strides in changing her perspective about being a mathematics learner throughout the course of the interviews, she did not deviate from perfectionism. She strikingly recalled a conversation with her father that many Black parents/guardians have with their own children: "we have to work twice as hard as a White person might work" (*June 8, 2021 Interview*, line 49). This statement underscored how critical the expectations of others were for Crystal.

Identity: Salience of Racial/Ethnic Identity.

In conversations about her beliefs about school, 100% of her utterances were coded as Identity: other.⁸ Outside of this however, in looking at Figure 2, it is evident that Crystal's racial and ethnic identities as a Black Nigerian-American girl governed much of her interviews. Gender proved to be insignificant, as there was only one overlap: only 8.82% of utterances centered on discussions about identity were coded for gender identity. Conversely, 23.53% of the same utterances were coded for racial/ethnic identity. Even when we discussed being a Black girl in

⁸ The Identity: other sub-theme referred to any utterances about identity more broadly that did not focus on race, ethnicity, gender, or mathematics. For instance, in her first interview, Crystal described others viewing her as "friendly" (*February 4, 2021 Interview*, line 153).

mathematics, Crystal did not talk solely about how her identity as a girl impacted her. Notably, 0% of utterances on this topic were coded for gender. Nonetheless, when discussing the same topic, it was her racial and ethnic identity that were most pertinent, as evidenced by the fact that 37.5% of these utterances were coded for racial/ethnic identity.

Unsurprisingly, discussion topics about mathematics were mostly coded for mathematical identity (i.e., *beliefs about being a mathematics learner* (43.75%), *beliefs about mathematics* (3.17%), *discussion about others' perception of her mathematics identity* (12.5%), *discussion about self-perception of mathematics identity* (31.91%), and *past mathematical experiences* (17.24%)). What is notable however, is that there were very little overlap between these topics regarding the racial/ethnic and mathematical identity sub-themes, and in some instances, there was no overlap at all (i.e., *beliefs about being a mathematics learner*, *beliefs about mathematics*, *discussion about others' perception of her mathematics identity*, *parents' beliefs about being a mathematics learner*). 20% of utterances related to the latter discussion topic, *parents' beliefs about being a mathematics learner*, was coded for racial/ethnic identity. 0% if these same utterances were coded for mathematical identity. This data demonstrates that Crystal's racial and ethnic identity as a Black Nigerian-American girl might not have been as salient in her personal relationship to mathematics. Nevertheless, she acknowledged these sub-themes were salient for her parents, and their expectations, as I established in the previous section, regulated her own expectations about doing and learning mathematics.

Discussion and Conclusion

The goal of this paper was to answer the following questions: *What are the factors associated with a Nigerian-American girl's mathematical experiences, and how do they inform the construction of her mathematical identities?* Through my analysis in this paper, I built upon

McGee's (2015) framework on fragile versus robust mathematical identity by extending it to include epistemic affect (Jaber & Hammer, 2016). I provided evidence of how Crystal's epistemic affect and expectations about mathematics intersected and informed one another as she crafted her mathematical identity. Her social identities as a Black Nigerian-American girl underpinned her epistemic affect and expectations. Together, these factors ultimately pointed to Crystal having a fragile mathematical identity.

Fragile versus Robust Mathematical Identity and Epistemic Affect

Component A - Central Motivations to Succeed in Mathematics

The first component of McGee's (2015) framework is central motivations to succeed in mathematics. Consider the fact that much of Crystal's interviews pointed to her Nigerian parents' expectations for her. At the start of the interviews, she interpreted her parents' preferences for her as perfectionism, and sought to achieve highly to comply with their academic standards. In achieving highly in mathematics, Crystal was defending herself against societal expectations about mathematical success as a Black girl. In doing so, she refused to advocate for necessary help at times for fear it would denote to others that she was not perfect when it came to mathematics. Even while doing mathematics was difficult or painful, Crystal put up a façade to appear otherwise as a protective measure. She was clear about her negative epistemic affect about mathematics, and putting on this cool face protected her from the gaze of others when her confidence about her own mathematical competence was low. Putting on a cool face insulated Crystal from the distress of dealing with a complex subject like math, but it also was a reaction to her understanding of what it meant to be a Black girl studying mathematics.

Component B - The Use of Coping Strategies in Response to Students' Racialized and Gendered Mathematical Experiences

The second component of McGee's (2015) framework is the use of coping strategies in response to students' racialized mathematical experiences. I expanded this component to also consider students' gendered mathematical experiences. In their research on how Black mathematics and engineering undergraduate students manage academic stereotype threat, McGee and Martin (2011) implied that a hyper awareness of racial stereotypes can negatively impact students' academic achievement. However, their research does not align with my own data, as Crystal's self-reported mathematical achievement remained high throughout the academic year. According to McGee's (2015) framework, Black students who employ effective coping strategies in the face of racialized experiences tend to fare better, and have more robust mathematical identities. Earlier in her last interview, Crystal asserted that deficit-based societal expectations and mindsets about Black girls and women do not impact her at all, likely due to the positive messaging and examples imparted from her parents. While Crystal was aware of negative stereotypes about Black students and was largely motivated to achieve highly in mathematics to fight this stereotype threat, Crystal's identity as the child of immigrants from a cultural community that privileges Black excellence seemed to shield her from the harm of these negative stereotypes.

Component C - Dispositions Associated with One's Successful Outcomes in Mathematics

The third component of McGee's (2015) framework is dispositions associated with one's successful outcomes in mathematics. Over time, Crystal began to perceive help differently; in fact, she concluded it was good. Even in this shift, Crystal might have done so in order to meet her parents' goal of having a more positive affect towards mathematics, being successful in the subject, and making them proud. From Crystal's viewpoint, the way by which she could do this was to pursue perfectionism as a mathematics student. Though Crystal's personal expectations

about mathematics remained intertwined with that of her parents and general society as the interviews continued, it became apparent that she was more receptive to her parents' expectations. Regardless, her motivation to achieve highly in mathematics was not an internal motivation, she was strictly extrinsically focused. Crystal's holding onto norms about perfectionism as a protective measure against deficit-based expectations about Black mathematics students demonstrates that her leading motivation to excel highly in mathematics was not due to the personal reward of success.

Rather, in assimilating to her parents' expectations, Crystal's mathematical behaviors and attitudes reflected a fixation on defending herself against others' perceptions of her mathematical competence. Ironically, rather than focusing on proving those who endorse negative beliefs about doing mathematics while Black, Crystal focused on proving her parents right in her own endorsement of their expectations. As such, based on McGee's (2015) framework, I argue that overall Crystal had a fragile mathematical identity.

Intersecting Identities

Underlying Crystal's interviews was the major theme of what life is often like for Black immigrants or children of immigrants. Her experience as a mathematics student was full of comments about why mathematics was necessary, even if it was unenjoyable, and all of this was connected to her internalization of her parents' expectations for her as immigrants themselves. As was evidenced above, much of their messages to her were about the inequity she would encounter in education and life as a Black girl, which Crystal interpreted as truth; truth in the sense that she believed there are societal disadvantages due to her being Black, rather than a personal endorsement of these judgments. Crystal was extrinsically motivated to achieve highly in mathematics, most notably by her parents. Her parents' desire for her to excel certainly

worked to galvanize her, as she acknowledged and agreed that their standards for her were linked to their comprehension of race and racism in the United States. Even while Crystal accepted that she would face inequity as a result of her gender, she believed even this was incomparable to the adversity she might encounter due to her race. Her goals of attaining a perfect academic record lend support to the notion that she primarily sought to make her parents proud in her achievement, and being proud of herself was secondary.

This messaging is not unique to Crystal and her parents. Rather, it is common rhetoric for students like her, who hail from immigrant communities or are immigrants themselves. I argue that while students' achieving academic success is a goal most parents/guardians have for their children, for immigrants and their children, it is vital for future success in a new, and often intimidating world. Although the participants in Griffin et al.'s (2012) study came from immigrant families, they were immigrants to the United States themselves. It is important to note, however, that my participant, Crystal, existed in a sort of liminal space in that in her self-identification as Nigerian-American, she was both American and a child of Nigerian immigrants. According to Katz (2004), Crystal is in the "middle," which means she had to interpret and negotiate between the two different cultures simultaneously as she constructed a mathematical identity. Consequently, the literature on how Black Americans and Black immigrants navigate the educational field cannot be extrapolated onto Crystal entirely. Furthermore, Crystal's encounter with United States schooling must also take into account her identity as a Black girl.

Limitations and Future Directions for Research

There are several potential limitations which limit this study's generalizability. This paper explored a qualitative study of just one student who attended a predominately White suburban

school. Our interviews did not explicitly refer to Whiteness and its likely influence on Crystal's mathematical experience. Crystal was one of a few Black students in her classroom, her mathematics teachers were both White females, and the majority of her peers were White, which reflected her town's demographics as well. Future research looking to explore Black girls' mathematical identities could investigate racial dynamics with other students and mathematics teachers to observe the impact on Black girls' mathematical self-concepts. Additionally, future research could consider exploring how students from similar backgrounds as Crystal, i.e., from an immigrant family, in more homogenous school settings construct their mathematical identities.

Another limitation to this study is the fact that McGee's (2015) framework was created to accommodate college mathematics students, and Crystal was a middle schooler. As such, the framework does not entirely encapsulate Crystal's adolescent experience. Further, there is no data on Crystal's mathematical achievement to provide more extensive insight into her mathematical experience. Therefore, the data cannot necessarily be extrapolated onto all Black female middle school students, and future research should center the mathematical experiences of Black girls across different ethnic groups to discover the diversity of thought into how they navigate the mathematical community differently from each other, and from Black boys. McGee's (2015) research considered how these students' management of stereotype threat impacted their mathematical experiences, achievement and identity. While there are some overlaps between McGee's (2015) participants and Crystal, particularly with their perfectionism, encountering racialized mathematics and persisting in spite of such has different implications for an adult preparing to potentially enter the workforce than a child, who has many years of schooling ahead of her.

Implications for Practice

Whereas at the onset of our interviews she shied away from being labeled as such, Crystal began to envision herself as someone with a relatively strong mathematical identity. She also recollected positive interactions with her mathematics teachers, which certainly influenced this change in her disposition towards the subject. This pivotal change provided evidence that, like Joseph et. al's (2020) findings, interactions with others, with content, and societal expectations all contribute to the mathematical experience for students. For Black girls, positive interactions with mathematics teachers are vital for uplifting and normalizing them in the mathematics classroom (Joseph et al., 2019) and contributing to the positive development of their mathematical identity.

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Appendix A

Sample interview protocol used for this study.

1. Question: How would you define mathematics?
Goal: To learn about what students think math is.
2. Question: Do you think math is useful? Why?
Goal: To learn about whether or not students think there is a purpose to learning/applying math.
3. Question: Do you know what identity means? If so, can you define it in your own words?
Goal: To learn about what students know about identity.
4. Question: What is your identity? Do you have more than one?
Goal: To learn what students know about intersectionality.
5. Question: Which identity is most important to you? Why?
Goal: To learn about which identity is most salient for students.
6. Question: Do you think different identities have different advantages/disadvantages in school? If so, which ones?
Goal: To learn what students know about privilege regarding different identities.
7. Question: Which identity do you think others notice first when they see/think about you?
8. Question: Which identity matters most when you're in math class? Why?
Goal: To learn about students' ability to gauge how they're perceived by others.
9. Question: Do you think that some children in your class are better at math than others?
How do you know?
10. Question: Is there such a thing as being a math person or being naturally good at math?
Goal: To learn about students' fixed vs. growth mindsets regarding math.

11. Question: Would you want to be known as good at math? If so, what would have to change for this to happen for you?

Goal: To learn about whether or not students see themselves as capable of doing math. To learn about the supports they believe they need to make this happen.

Chapter 4: Understanding Influence in Relation to Mathematical Micro-Identities: Moment-to-Moment Positioning and Mathematical Authority

Introduction and Motivation

This study was inspired by Engle and Conant's (2002) foundational paper on productive disciplinary engagement that focused on one case study of a group of students' discussions about orcas. Engle and Conant's (2002) research uncovered how one student in particular was able to wield his authority among his peers during these discussions. Similar to Engle and Conant (2002) we built on a prior investigation (Brizuela et al., 2023) that was focused on understanding one student's mathematical authority by considering how this student and their⁹ peers were positioned into micro-identities throughout one lesson. Using Wood's (2013) framing of micro-identities we explore the following research questions:

(RQ1) What do we learn about Wood's (2013) framework of mathematical micro-identities when we explore how it plays out in a new classroom context?

(RQ2) What are the connections between Wood's (2013) moment-to-moment positioning and enactment of mathematical micro-identities and how a student enacts influence?

Mathematical identity is "a person's beliefs, attitudes, emotions, and dispositions about mathematics and their resulting motivation and approach to learning and using mathematics knowledge" (Froschl & Sprung, 2016, p. 320). Micro-identities are "enacted in a moment of time" (Wood, 2013, p. 778) and can fluctuate depending on the context.

Recently, mathematics education literature has broadened to include discussions about students' identities and their effect on how students navigate various experiences within the mathematics classroom (Cribbs et al., 2021; Philip & Gupta, 2020). This paper considers the

⁹ Here and henceforth, we refer to these students using gender neutral pronouns because as in Brizuela et al., (2023), gender is not a focus of our study.

positioning and enactment of mathematical micro-identities of a group of students whose mathematical authority dynamics were previously described by Brizuela et al. (2023). We focused on a small slice of rich data to show what is happening on a microlevel in terms of identity.

In Brizuela et al.'s (2023) paper on the construction of students' mathematical authority, the authors examined first grade mathematics students' discourse to explore how they were positioned by their peers as mathematical authorities, and how this positioning was constructed over time. The purpose behind their research was to examine students as the authority bearers through analyses of students' mathematical contributions during whole-group discussions. With Engle et al.'s (2014) theory of influence undergirding the analysis, their focus was on positioning, and not identity, but the results reveal one particular student's, Tia's, identity as a mathematical authority. Here we argue that Brizuela et al.'s (2023) results reveal Tia's macro-identity as a mathematical authority among their peers. Macro-identities are "relatively stable, long-term constructions of who a person is" (Wood, 2013, p. 776). While Tia's identity as a mathematical authority may not be as long-term relatively speaking because it does not span multiple years of schooling, for instance, it is evident that through Tia's role as a discussant and Tia's peers' responses to their contributions, that this is how Tia was thought of in this context.

In response to Wood's (2013) call to better understand micro-identities, we used the same dataset as Brizuela et al. (2023) and investigated a different dimension; namely, we considered the nature of the positioning and enactment of moment-to-moment mathematical micro-identities as one student, Tia, is positioned as a mathematical authority (Wood, 2013).

Theoretical Framework

Conceptually speaking, identity is fluid and ever-changing and it influences how both teachers and students relate to mathematics and to one another. Much of the research on mathematical identity centers on macro-identities. We use Wood's (2013) concept of mathematical micro-identity that suggests that mathematics learners can take up identities and be positioned into identities which are "enacted in a moment of time" (p. 778), and that fluctuate depending on the context, at times over the course of a lesson. We aim to better understand the positioning and enactment of mathematical micro-identities in a context in which a student is positioned into a macro-identity as a mathematical authority. The motivation for this focus is to better understand the dynamics of this context and the context itself because a key part of understanding identity is understanding how individuals see themselves, how others perceive them (Langer-Osuna & Esmonde, 2017), and the context in which all of this is occurring (McCarthy & Moje, 2002).

In this paper, we use interactional positioning theory (van Langenhove & Harré, 1999) to attend to the ways in which a teacher-researcher and students contributed mathematically and responded to mathematical contributions during a mathematics lesson of a Classroom Teaching Experiment (CTE). To study how mathematical micro-identities are constructed, Wood also used interactional positioning theory (van Langenhove & Harré, 1999), focusing on how people are labeled or identified through interactions with one another, as well as how these labels shift as interactions shift. While position has different connotations, such as being physical or describing one's role in terms of practice, positioning here refers to the ways in which people interact through discourse and actions to structure social dynamics. Enactment involves how one presents themselves in what they do or say. Positioning, enactment, and identity are not equivalent terms; rather, positioning and enactment involve the interactions that lead to a particular placement, or

identity. van Langenhove and Harré (1999) described interactions as frequently occurring verbal or non-verbal communication between people. Further, the positions people take on as a result of their interactions may come about through their own labeling, or through that of others. Because mathematical identity is associated not just with how students position themselves, but how they are positioned by others (Langer-Osuna & Esmonde, 2017), our analysis includes both how individuals are positioned into identities and whether and how they enact those identities.

Prior Research

Engle et al. (2014) explored how influence was developed and applied through a case study of fifth grade science students' interactions during small group discussions that they drew from Engle and Conant's (2002) work investigating productive disciplinary engagement. Engle et al. (2014) defined influence as the "degree to which an individual becomes socially positioned as having convinced others to take up his or her ideas" (p. 250). Engle et al. (2014) observed how different students in the group gained and negotiated authority amongst their peers in a discussion about the classification of orcas as dolphins or whales. They focused specifically on one student, Brian, who gained undeserved influence despite the low quality of his contributions. To probe how influence was negotiated in Engle and Conant's (2002) study, Engle et al. (2014) used proposal negotiation units (PNUs), a unit for modeling the components of influence. The first phase of a PNU is the proposal bid, where an idea is proposed. The second is when the proposal is either taken up, changed, ignored, or rejected. In analyzing the eight episodes described in Engle and Conant (2002), Engle et al. (2014) developed a five-component framework for influence theory that Brizuela et al. (2023) used as a foundation for their analysis of student interactions.

In the same way that Engle et al. (2014) built on Engle and Conant (2002), we built on Brizuela et al. (2023). Specifically, we analyzed a subset of Brizuela et al.'s (2023) data, one lesson, which was deemed critical by the authors; we analyzed it by building on Engle et al.'s approach (2014) to studying influence and used Wood's (2013) definitions to understand the teacher-researcher's positioning of students into mathematical micro-identities and the enactment of those micro-identities.

Brizuela et al. (2023) analyzed the ideas proposed by the students who had the most airtime over the course of 12 lessons in a CTE. They found that one student, Tia, had more airtime, made more contributions, proposed more ideas, and that Tia's peers trusted them as a source of mathematical authority. Brizuela et al. (2023) discussed the links between authority and trust but did not address identity, how Tia or their peers were positioned into mathematical micro-identities, or how these identities were enacted. Thus, our goal is to further investigate these dynamics, with a specific aim to explore the positioning and enactment of mathematical micro-identities.

To attend to the emergence of Tia's macro-identity as a mathematical authority we illustrated PNUs, similar to Engle et al. (2014) and Brizuela et al. (2023). To explore the process by which that macro-identity emerged from specific and smaller "moment(s) in time," we identified the positioning and enactment of mathematical micro-identities, similar to Wood (2013).

Wood (2013) posited that mathematical micro-identities can shift frequently, depending on the context, making them variable instead of static. Wood (2013) observed three mathematical micro-identities among the student participants in her study: the *mathematical explainer* (a student explaining their reasoning in response to teacher inquiry), the *mathematical student* (a

student capable of learning), and the *mathematical menial worker* (a less than mathematically capable student). In Wood's (2013) work mathematical explainer and mathematical student fall under a broader umbrella of mathematical capability. Moreover, Wood argued that students shift between them at any given time.

In this study we elaborate on the findings of Brizuela et al. (2023) who described the emergence of one student as a mathematical authority, and described how this student, and their peers, were also positioned into different micro-identities. To answer our research questions we analyzed Lesson 12 from Brizuela et al. (2023), which included a "pivotal moment" (p. 7) in which one student's, Tia's, mathematical authority became apparent through another student's, Azaiah's, trust in Tia's (incorrect) mathematical idea. We focused on the subset of five students who were identified as the focal students in Brizuela et al. (2023); this includes Tia and Azaiah.

Methodology

Participants

The participants in this study were the same as those in Brizuela et al.'s (2023) study, mirroring the design in Engle et al. (2014) who built on Engle and Conant's (2002) study. Our participants included a teacher-researcher, a university professor in education at the time, and 22 first grade students who attended a public school in the Northeast United States. At the time of the study, 1% of the students at this school identified as White, 47.3% identified as African American, and 46.8% as Latine. The year that these data were collected, 52% of third grade students were considered "warning/failing" on a state mandated standardized test in mathematics, and an additional 40% were considered "needing improvement." As mentioned above, we focused on a subset of five of these students who were identified as the focal students in Brizuela et al. (2023).

4.1 The Classroom Teaching Experiment

The CTE was designed to engage students in early algebraic thinking. The 16 CTE lessons involved reviewing geometric pattern sequences to introduce the idea of co-varying relationships and then reasoning about functions in the form of $y=mx$ and in the form of $y=x+b$. Lesson 12 from the CTE, which is the focus of this study, is included in the Appendix.

All CTE lessons involved a problem scenario, which the teacher-researcher presented to the students before a whole-group discussion to explore their understandings. When reasoning about functions, students were challenged to identify a verbal rule or expression to represent the relationship. In order to capture students' ideas, the teacher-researcher typically used chart paper or a white board. The CTE lessons were supplemental to students' mathematics instruction with their regular classroom teacher.

4.2 Data Sources and Selection

The data analyzed in this study come from the videorecording of one lesson (#12) which was designated by Brizuela et al. (2023) as a significant lesson because of a moment when Tia's influence seems to coalesce into mathematical authority. The goal of our data selection was "not representativeness but rather the...immersion in the details of a complex moment" (Nemirovsky et al., 2013, p. 385). We used microgenetic analysis to understand the moment-to-moment micro-identities at play as one student, Tia, was established as a mathematical authority in this context. In this lesson, students discussed how to represent the ages of two sisters, Janice and Keisha (see Brizuela et al., 2015). In the situation being discussed, Keisha is three years older than Janice.

The analysis of these data involved identifying (1) mathematical micro-identities according to Wood (2013) and (2) PNUs according to Brizuela et al. (2023).

4.3 Analysis of Students' Mathematical Micro-Identities

To continue with a microgenetic approach that details a complex moment, instead of representing many moments (Nemirovsky et al., 2013), we analyzed the whole group discussion from Lesson 12, keeping track of any moments when students were (a) positioned into and (b) enacted mathematical micro-identities and then the PNUs according to Brizuela et al. (2023).

Table 1 includes Wood's (2013) micro-identities: (1) *Mathematical explainer*; (2) *Mathematical student*; and (3) *Mathematical menial worker*¹⁰.

Table 1. *Mathematical micro-identities (Wood, 2013).*

Mathematical Micro-identity	Definition
Mathematical explainer	This student is “responsible for articulating an argument for [their] solution to the task” (Wood, 2013, p. 789).
Mathematical student	This student is “mathematically capable” (p. 790).
Mathematical menial worker	This student is “less than mathematically capable” (p. 792).

To code for Wood's (2013) micro-identities, at each turn of talk, we asked, “who is being spoken to?” If it was clear that someone was being spoken to, we asked, “how is this person being positioned? Are they being asked to explain (i.e., *mathematical explainer*), to contribute

¹⁰ Although we used Wood's (2013) original micro-identities, we believe it is important for researchers to consider taking an asset-based approach to categorize students' mathematical identities. For example, the “menial worker” label could be reconsidered from an asset-based perspective, e.g., “emerging learner” or “developing learner”.

mathematically without explaining (i.e., *mathematical student*), or to contribute in a non-mathematical way (i.e., *mathematical menial worker*)?” After asking this, we coded for one of the micro-identities above. We then considered how this person responded. Did they enact the micro-identity in which they were positioned? In other words, did they explain a mathematical idea? Did they share a mathematical response without explaining it? Or did they contribute in a non-mathematical way?

After coding the transcript, we identified the frequency of Wood’s (2013) micro-identities for the five focal students.

4.4 Analysis of Proposing Mathematical Ideas and Responses to Those Ideas (PNUs)

To identify mathematical ideas, we used Brizuela et al.’s (2023) definition, an idea is “a thought or suggestion which can be articulated orally or in writing, [and this is done using] the smallest possible unit” (p. 14). This means that differing and/or conflicting ideas could be proposed by the same student, in the same moment, or among other children in the same classroom. The types of ideas proposed may vary, as they may be contextually dependent on the activities or tasks at hand. Further, ideas could also be used to create new ideas, forgotten about, or re-introduced later. A speaker could also propose several ideas, but responses to these ideas could treat them as one single idea.

Our access to small group data was inconsistent so we decided to focus, like Brizuela et al. (2023) did, on whole group discussions. Once ideas were flagged, the research team coded the various ways in which students’ ideas were responded to using the codes from Brizuela et al. (2023) (see Table 2) that were based on Engle et al.’s (2014) PNUs.

Table 2. *Codes developed by Brizuela et al. (2023) to study influence.*

PNU part	Code Description	Example from Brizuela et al. (2023)
Bid	<i>An idea is proposed</i> when a student or adult shares something (orally, gesturing, writing) they are thinking. It does not have to be an idea that has not previously been shared, nor does it have to be shared by students alone. It can also be stated as a question.	Student: Mm . . . so if Mary had two, then John would have zero (p. 88).
Response	<i>An idea is accepted</i> after a child proposed an idea and someone responded indicating that they agree with that child's idea. Sometimes it is subtle, and other times it is lengthy and clearly articulated. The acceptance does not need to happen immediately after the child proposed the idea.	Student: The problems saying that when the girl was on the sidewalk she found two pennies . . . So she went back home to put it in her jar. To save it. Teacher/researcher: To save it. Good (enthusiastically). Does anyone else have another way of saying it? [students raise their hands] (p. 88).

An idea is responded to (neutrally) after a child proposed an idea and someone responded to it without accepting or rejecting it. The response might simply be an acknowledgement, or it might be a push for more information (e.g., justification). The response does not need to happen immediately after the child proposed an idea. This is often done by the teacher-researcher. The code is typically not used in conjunction with other codes because, for instance, if an idea is accepted then it cannot also be responded to neutrally.

Teacher – researcher: Z plus Z? Ok (said flatly). Well, we’re gonna talk about/(moving on to different topic) (p. 88).

An idea is explicitly rejected after a child proposed an idea, and someone responds, stating that they disagree with the child’s idea. The rejection does not need to happen immediately after the child proposed an idea.

Student: Because, if she still has that same two, then – then it's three because he has one and you’re skipping two numbers. And so, you’re skipping one number. So, I disagree (p. 88).

<i>An idea is not taken up</i> after a child proposed an idea, and someone avoids taking up the child's idea by either not acknowledging it or making a statement that suggests they are questioning the accuracy of the child's idea. It often involves a transition towards talking about a different idea as well. We do not refer to this code as a child's idea is not accepted because that could suggest it was rejected, and we would be inferring that the child's idea was intentionally not taken up when that is not necessarily the case in these instances. This code includes possible implicit rejections.	Student: So, if – that means someone must've gave Mary one more and then the other person gave Mary another one. Teacher – researcher: Yeah (in passing – not an affirmation). So, if John has one, Leo, how many does Mary have (p. 88)?
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It is important to note here that several key aspects of the exchanges between the students and teacher-researcher that were visible in the lesson video recordings were not as clear in the transcripts. For instance, inflections and tones of voice were not captured in the transcripts, which could potentially make a difference when coding an utterance as a neutral response, an implicit acceptance, or a subtle challenge to a student's proposed idea. This was addressed by having coders watch the videos while reading the transcripts.

To record the positioning and enactment of Wood's (2013) micro-identities and the PNUs according to Brizuela et al. (2023), we coded the Lesson 12 transcript line-by-line; our unit of analysis was an utterance, using Langer-Osuna's (2015) definition:

What was stated by a speaker when the speaker held the floor, regardless of whether the utterance was interrupted or was a completed turn. In the case of an interrupted utterance, if the speaker gained the floor again to complete the turn, the two turns were each coded as separate utterances. (p. 59)

To confirm the reliability of the coding, two members of the research team reviewed the analysis. If the two coders disagreed on a code from either Table 1 or 2, they discussed the code until agreement was met, and if needed they involved another member of the research team for third-party resolution (Syed & Nelson, 2015).

Results

Our analysis was aimed at answering two research questions: (RQ1) *What do we learn about Wood's (2013) framework of mathematical micro-identities when we explore how it plays out in a new classroom context?* (RQ2) *What are the connections between Wood's (2013) moment-to-moment positioning and enactment of mathematical micro-identities and how a student enacts influence?*

Several results came from our analysis. To organize this results section we summarize five key findings, and note in which subsection of the results we address each finding. We report on:

- (1) frequencies of positioning and enactment of mathematical micro-identities (subsection 5.1);
- (2) specialized mathematical micro-identities that add nuance to Wood's (2013; subsection 5.2);
- (3) how mathematical micro-identities and specialized mathematical micro-identities fluctuated in moment-to-moment discussion (subsection 5.3),

specifically positioning and enactment of mathematical micro-identities changed within and across a single PNU (subsection 5.4);

(4) how the menial mathematical worker micro-identity was uncommon in our data (subsection 5.5);

(5) how positioning and enactment of mathematical micro-identities were connected to the bid and response parts of PNUs (see Table 2), specifically enactment was connected to proposing ideas, while positioning was connected to responding to ideas (subsection 5.6). Moreover, the specialized mathematical micro-identities of correct and incorrect were connected to specific responses to ideas (accept and reject, respectively; subsection 5.7).

5.1 Frequencies of positioning and enactment of mathematical micro-identities

We begin by presenting frequencies to the mathematical micro-identities for the five focal students (see Table 3).

Table 3. *The frequencies of each micro-identity for each student in Lesson 12 based on positioning and enactment.*

Positioned as	Tia	Zyla	Nyla	Azaiah	Leo
Mathematical student	5	4	5	0	7
Mathematical explainer	10	11	14	4	9
Mathematical menial worker	0	0	1	0	1
Enacted	Tia	Zyla	Nyla	Azaiah	Leo

Mathematical student	0	0	1	1	2
Mathematical explainer	10	8	19	5	9
Mathematical menial worker	0	0	0	0	0

According to Table 3, Tia and Nyla were positioned as a *mathematical student* the most (5 instances each). Nyla was positioned as a *mathematical explainer* the most (14 instances), and only Nyla and Leo were positioned as *mathematical menial workers* (one instance each). Nyla enacted *mathematical explainer* the most (19 instances); on five occasions she enacted *mathematical explainer* when she was not positioned as a *mathematical explainer*, suggesting she initiated some of her mathematical explaining without prompting. There were no instances in which students enacted *mathematical menial worker*. According to Table 3, overall students were positioned and enacted *mathematical explainer* more than the other two mathematical micro-identities, suggesting much of the classroom discourse was focused on explaining and justifying mathematical ideas.

5.2 Wood's (2013) can be nuanced into specialized mathematical micro-identities

As expressed in our research question, we sought to investigate the positioning and enactment of mathematical micro-identities. This analysis resulted in building on Wood's (2013) framework because we identified specialized micro-identities. Specifically, we observed that when students were positioned as *mathematical students* or *mathematical explainers*, according to Wood's (2013) definitions, they were sometimes simultaneously positioned into something more nuanced, such as *correct* or *incorrect*, as *discussing* or evaluating another person's idea, or

as *collaborating* with another person. We did not observe specializations with the mathematical menial worker micro-identity.

The specializations are shown in Table 4. The micro-identities from Table 2 could be further specialized using each of the specializations in Table 4.

Table 4. *Mathematics micro-identity specializations.*

Mathematics micro-identity specializations	
Correct	When someone agrees, using words and/or gestures, with another person's mathematics idea or claim
Incorrect	When someone disagrees, using words and/or gestures, with another person's mathematics idea or claim
Collaborating	When people work together on proposing or responding to a mathematics ideas
Discussing	When someone evaluates another person's mathematics idea or claim

When a speaker explicitly stated their agreement or disagreement with another, we called this *correct* or *incorrect*. This was distinguished from *discussing*, in which a speaker explained their agreement or disagreement without necessarily stating so. We also observed moments when speakers collaborated by talking through their mathematical thinking or someone else's mathematical thinking aloud; this was referred to as *collaborating*. Additionally, in this lesson,

we observed students *collaborating* through what the students themselves described as copying¹¹; we considered this to be the *collaborating* specialization of *mathematical student* or *mathematical explainer*.

Next, we review the frequencies of specialized mathematical micro-identities that we identified in Lesson 12. Table 5 shows that in Lesson 12, we observed two instances of the teacher-researcher positioning a student as both a *mathematical student* and a *mathematical explainer*. We observed three instances of the teacher-researcher positioning a student as a *collaborating mathematical explainer* or the student enacting *collaborating mathematical explainer*. We only observed Tia and Leo be positioned into or enact this micro-identity.

Table 5. *The frequencies of each specialized micro-identity in Lesson 12 based on positioning and enactment.*

Specialized Micro-Identity	Count
Correct mathematical explainer	1
Correct mathematical student	17
Incorrect mathematical explainer	3
Incorrect mathematical student	4
Incorrect mathematical menial worker	0

¹¹ In Lesson 12 several students accused one another of “copying” the others’ work. The teacher-researcher encouraged them to see copying as a compliment because they were collaborating in order to solve the mathematics problem.

Discussing mathematical student	7
Discussing mathematical explainer	13
Collaborating mathematical student	15
Collaborating mathematical explainer	3

As seen in Table 5 above, over the course of one lesson, Lesson 12, we observed numerous instances of specialized versions of Wood’s micro-identities times. Specifically, we observed 18 instances of positioning or enacting of *correct mathematical student* and *mathematical explainer*; and seven instances of positioning or enacting of *incorrect mathematical student* and *mathematical explainer*. We observed 20 instances of students being positioned as or enacting *discussing mathematical explainer*.

5.3 The nature of positioning and enactment of (specialized) mathematical micro-identities is consistent with Wood (2013)

One of Wood’s (2013) key findings was that mathematical micro-identities fluctuate, often. We observed this trend with the positioning and enactment of the mathematical micro-identities *and* with the ways in which these micro-identities were specialized. That is, sometimes the student was positioned into the same micro-identity, but sometimes the micro-identity fluctuated and/or the specialized aspect of the micro-identity fluctuated. We also observed that enactment and positioning do not necessarily match (i.e., at times a student was positioned into one micro-identity but enacted another). As with Wood’s (2013) findings,

positioning and enactment of (specialized) mathematical micro-identities were brief and changed frequently over the course of a single lesson.

The positioning and enactment of micro-identities even varied within one PNU. For example, the teacher-researcher positioned Tia as *mathematical student* or *explainer*; Tia then enacted *mathematical explainer*. In response the teacher positioned Tia as a *mathematical student*, but Tia again enacted *mathematical explainer*. Then, in response, the teacher-researcher positioned Tia as a *collaborating mathematical explainer*. This all occurred over the course of two PNUs. We share the details of this scenario below.

5.4 Fluctuations in micro-identities occur within and across PNUs

The following excerpt involves the class discussing how to represent the ages of two sisters, Janice and Keisha (see Brizuela et al., 2015; Brizuela et al., 2023). We share this excerpt to illustrate the positioning and enactment of (specialized) mathematical micro-identities *in situ*, highlighting the moment-to-moment nature of these interactions and mapping them to PNUs to begin to make connections across these analyses and address RQ2.

To start the discussion, the teacher-researcher fielded ideas from the entire class and called on Zyla and then Tia. In asking all the students for their ideas about how to solve the problem, the teacher-researcher positioned them all as *mathematical students*. But in selecting Tia specifically, she positioned Tia into the *mathematical explainer*, *mathematical student*, and *collaborating mathematical explainer* micro-identities.

First the teacher said, “Questions or ideas about what the problem is telling us about Janice and Keisha. Tia?” In this moment, she positioned Tia as a *mathematical explainer* and *mathematical student*. Tia responded by enacting *mathematical explainer*; Tia said, “She could

be—she could be any number, because—because Keisha is three—is three years older than Janice,” simultaneously proposing an idea.

Tia continued, “if she’s twenty and...she’s twenty, Janice can be—Janice can be... nineteen. And then—and if—and if... and if Keisha’s three—three years older than—than Janice, then... then she’s twenty-three.” In this contribution, Tia continued enacting *mathematical explainer* and proposed an idea. The teacher-researcher questioned, “If she’s twenty, then she’s twenty-three? If Janice is twenty, Keisha’s twenty-three?” Her question was no longer directed at Tia and is a repeat of Tia’s explanation. In doing so, she responded to Tia’s idea neutrally. Moreover, the teacher-researcher did not explicitly ask for further explanation, so this was coded as her positioning Tia into the *mathematical student* micro-identity and no longer *mathematical explainer*. Nonetheless, Tia interpreted this utterance as a push to explain further and took up the *mathematical explainer* micro-identity again.

Tia went on to explain their thinking, and the teacher joined, positioning Tia as a *collaborating mathematical explainer* and responding to Tia’s idea neutrally. To illustrate the *collaborating* specialization, we share a transcript because *collaborating* typically involved two or more people collectively talking through an idea. The following exchange shows when the teacher-researcher’s positioning and Tia’s enactment shifted to the *collaborating mathematical explainer* micro-identity.

Tia: If Janice is—if Janice is nineteen, then Keisha, if she’s three years older than her—

Teacher: If she’s nineteen, twenty...

Tia: Twenty-one, twenty-two, twenty...

Teacher: ...twenty-two. She's—so if Keisha—if Janice is nineteen, Keisha's twenty-two?
Ok. So, I heard two times already, “she could be any number.” What other ways of showing any number do you know?

In this discussion we observed the teacher position all students as *mathematical students*, and then Tia, specifically, as a *mathematical explainer*. Tia's response involved enacting *mathematical explainer* and initiating a PNU by proposing an idea. The teacher responds to Tia's idea neutrally, which is the second part of the PNU. In doing so, she simultaneously positions her as a *mathematical student*. These kinds of exchanges in which a student proposes an idea and someone responds, shows how the positioning and enactment of mathematical micro-identities maps to the two parts of the PNU. The example also illustrates *collaborating mathematical explainer*.

5.5 Comparing Ours and Wood's (2013) Findings on Menial Mathematical Workers

We observed only two instances in which students were positioned as *menial mathematical workers*. One instance was with Nyah, and the other was with Leo. The infrequency of *menial mathematical workers* is one way our findings seemed to differ from Wood's (2013). To explore if this observation was unique to Lesson 12, we conducted an ad hoc analysis, which we describe below.

We reviewed our entire Lesson 1-12 dataset and observed that the only other occurrences of this micro-identity were when the teacher-researcher positioned both Nyah and Zyla as *menial mathematical workers* in Lessons 9 and 11, respectively. This occurred twice for Nyah and once for Zyla. Thus, we analyzed how the teacher-researcher positioned Nyah in Lesson 9 and Zyla in Lesson 11 as we had done with the five students in Lesson 12. That is, we identified Wood's

(2013) mathematical micro-identities but focused only on Nyah in Lesson 9 and Zyla in Lesson 11.

We found that the positioning of both Nyah and Zyla fluctuated throughout the lessons. Specifically, we observed that the teacher-researcher's positioning of Nyah shifted from *mathematical explainer* to *mathematical menial worker* to *mathematical student* and *mathematical explainer* within Lesson 9. In terms of Zyla, we observed that the teacher-researcher fluctuated in her positioning of Zyla from *mathematical explainer* to a *mathematical menial worker* to a *mathematical student* and back to *mathematical explainer* within Lesson 11. This fluctuation in positions confirms Wood's (2013) stance that mathematical identities are fluid and contextually-situated.

Thus far, we have addressed RQ1 by describing how our findings build on and confirm aspects of Wood's (2013) framework of mathematical micro-identities. In the following section, we shift our focus and address RQ2. In doing so, we describe the connections between Wood's (2013) moment-to-moment positioning and enactment of micro-identities and how one student enacted influence.

5.6 Enactment and positioning of micro-identities occurred when ideas were proposed and responded to

Through our analysis, we identified a relationship between how ideas were proposed and responded to (see Table 1) and how individuals were positioned into Wood's (2013) mathematical micro-identities (see Table 2). Exchanges began with the teacher-researcher positioning a student or students as Wood's (2013) *mathematical students* or *mathematical explainers* by asking a question. The question was usually an open question, suggesting that she was seeking an explanation, and thus students were positioned as *mathematical explainers*. After

the teacher-researcher posed the question, a student would enact Wood's (2013) micro-identity of *mathematical student* or *mathematical explainer*. This occurred through a bid or a proposal, the first part of a PNU. The teacher-researcher then responded to the students' idea, the second part of the PNU, with additional positioning into Wood's (2013) micro-identities. That is, when ideas were proposed and responded to, students enacted and were positioned into mathematical micro-identities, respectively. This scenario is illustrated in the example above in section 5.4.

5.7 The incorrect/correct specializations map to how ideas were responded to

During these moments of enactment and positioning, ideas were a currency exchanged between the teacher-researcher and students because these ideas were evaluated. In Brizuela et al.'s (2023) analysis the ideas were accepted, rejected, or responded to neutrally. In our analysis, the specialized micro-identities of *correct* and *incorrect* emerged from these moments of evaluating ideas. Not surprisingly, we observed a link between the ways in which ideas were evaluated and the PNUs because correct ideas were more likely to be accepted, while incorrect ideas were more likely to either be responded to neutrally or rejected.

The following example illustrates how Zyla and Tia are positioned as *correct mathematical explainers* and *collaborating or discussing mathematical explainers* and how their ideas are evaluated.

In this particular exchange, Zyla proposed an idea when she recalled a similar problem from the previous week in which a second grader suggested using a variable to represent the story problem; in this moment, Zyla enacted and was positioned as a *mathematical explainer*. The teacher-researcher responded neutrally to her proposal when she said, "Oh! Could you remind us (that idea)—(from) last week, right?" Rather than accepting or rejecting the idea, she

simply moves the conversation forward. She is also positioning Zyla as a *mathematical explainer*.

In response, Zyla proposed an idea, to represent the girls' ages, $J+3$, which the teacher-researcher acknowledged was correct, positioning Zyla now as a *correct mathematical explainer* and accepting the idea; this shows the link between the *correct* specialization and accepting an idea. The teacher-researcher then reminded the class that another student, Nyah, had a similar idea as Zyla, positioning Nyah and Zyla as either *collaborating* or *discussing mathematical explainers* (at this point, her intent is unclear—did they build on one another or should they engage in a discussion about the ideas?). In response, Tia showed disagreement. The exchange is as follows:

Teacher: And so Zyla is saying, well I'm using that idea and I'm saying then Keisha has to be J plus three. Which I think is the same thing that—that you were saying uh Nyah. It's like what you were saying. A little differently. Right? [Nyah nods] Ok. [Tia shakes her head] You are disagreeing Tia.

When the teacher-researcher acknowledged Tia's disagreement, she positioned her as a *discussing mathematical explainer*, by looping her into a discussion. Tia then enacted the *discussing mathematical explainer* micro-identity by justifying why they disagreed and by proposing a different idea than Zyla's.

This next part of the exchange illustrates the link between accepting and rejecting ideas and correct and incorrect positioning, *and* that a student can position and respond to ideas—it is not always the teacher-researcher.

First, Tia proposes the new idea. She referred to Zyla's proposed idea: $j + 3$ and said, "I think it's L—because J, K, L." By proposing this new idea and justifying it, she is enacting

discussing mathematical explainer. The teacher responds neutrally and does not ask for more explanation, positioning Tia as a *discussing mathematical student*. Nonetheless, Tia enacts *mathematical explainer* and proposes an idea. She said, “J, K, L, M, N, O, P.” The teacher neither rejects or accepts an idea but instead responds neutrally and positions Tia, Nyah, and Zyla as *discussing mathematical explainers*. The teacher-researcher said, “I think, uh you’re counting from J, K, L. And I think Nyah is doing K, L, M. You’re just starting at different places. So I can’t say that one is better than the other. Ok?” She thanks the students for sharing their ideas.

Then, without being positioned, Azaiah enacts the *mathematical student* micro-identity by disagreeing with Tia’s idea, rejecting it and positioning Tia as an *incorrect mathematical student*. This exchange illustrates the link between rejecting an idea and positioning someone as *incorrect*.

As Azaiah explained why he disagreed, Azaiah took up the *mathematical explainer* position. Eventually, Azaiah concluded that Tia “was the right one” regarding the previous idea Tia proposed, accepting Tia’s idea and positioning her as a *correct mathematical explainer*. That is even though Azaiah had earlier positioned Tia as an *incorrect mathematical student*, Azaiah reversed the positioning and considered Tia a *correct mathematical explainer*. This moment when he accepts her idea and positions her as *correct*, again highlights this aforementioned link between the *correct* and *incorrect* specializations and accepting and rejecting ideas parts of the PNU.

Discussion and Conclusions

We sought to investigate the nature of the positioning and enactment of mathematical micro-identities in a context in which a student enacts influence as described by Brizuela et al. (2023). To operationalize and study influence, we relied on Brizuela et al.’s approach and used

PNUs. This focused our attention on the relationship between PNUs and mathematical micro-identities, which we discuss more in detail here.

We observed that positioning and enactment of micro-identities occur as parts of the PNUs. At times enactment corresponded to a proposal, and positioning corresponded to the response to that proposal, but this was not always the case. We also observed fluctuations in how students were positioned and enacted and to further detail these fluctuations we nuanced Wood's (2013) three mathematical micro-identities. That is, the specialized mathematical micro-identities only further emphasize the fluctuating nature of the kind of positioning and enactment of identity that Wood is attending to.

Specialized micro-identities are subsets of the broader micro-identities and are more nuanced, either describing a particular action the person is engaged in while enacting a type of micro-identity, or an action that someone expects them to enact. These subtleties seem critical in understanding when and how value is placed on individuals and/or individuals' ideas, which was a key goal of Brizuela et al.'s (2023) study.

Consider the specialization of the *mathematical explainer* micro-identity, the *discussing mathematical explainer*—when students are positioned into this micro-identity, they are expected to evaluate their peers' ideas and explain their evaluation. These moments of evaluation are moments of valuation. That is, when ideas are evaluated, they are given more or less value. Additionally, the ideas are attributed to an individual who is simultaneously positioned into a micro-identity as their ideas, which now have value, are solicited, proposed, and responded to. The specialized mathematical micro-identities made further connections to Brizuela et al.'s (2023) findings because they highlighted that the positioning of these micro-identities was linked to specific aspects of the PNUs from Brizuela et al.'s (2023) analysis. For example, when Tia

was positioned by Azaiah as an incorrect mathematical student their idea was explicitly rejected. On the contrary when Tia was positioned as a correct mathematical explainer by Azaiah, their idea was accepted. This connection suggests that PNUs are comprised of moments in which students are positioned into and enact Wood's (2013) (specialized) mathematical micro-identities.

One way the analysis of mathematical micro-identities differed from Brizuela et al.'s (2023) study is that in coding for mathematical micro-identities we distinguish between positioning and enactment. PNUs also have two parts—bids and responses. We argue the bids or proposals often parallel enactment. However, while there are several different micro-identities and specializations of them that an individual can enact, there is only one option for a bid—an idea is proposed. This single code does not capture the different ways in which an individual might propose an idea, or more broadly, which mathematical micro-identity they might enact.

Related to this, we also observed that at times students are positioned into one mathematical micro-identity, but they enact another. For instance, a student may be positioned as a *mathematical student* and may subsequently enact the *mathematical explainer* micro-identity by providing a justification for their reasoning. The analysis of micro-identities captures the agency of these moments when students use an opportunity to enact as an opportunity to shift the way they are identifying. By distinguishing positioning from enacting, we shed light on these moments. This could be a new direction of research focused on mathematical micro-identities and mathematical authority by considering both how students were positioned, as well as whether the students used their agency to enact these same micro-identities.

Based on the findings of Brizuela et al. (2023) we know that these micro-identities are small units of something much larger, the construction of a macro-identity, and while the analysis

presented in this paper cannot explicitly connect the positioning and enactment of students' mathematical micro-identities to a specific macro-identity, we are reminded of thickening a term introduced in prior literature (Langer-Osuna, 2011; Turner et al., 2013; Wenger, 1998; Wortham, 2006; Yamakawa et al., 2009) that helps explain how this micro- to macro-identity positioning and enactment might unfold. Thickening is the solidifying and forming of an identity over time. We argue that the positioning and enactment of mathematical micro-identities, particularly involving Tia, thickened over time into the macro-identity of mathematical authority, and the results of our analysis begin to shed light on how and why this process might unfold.

Lastly, we discuss how the context of the CTE shaped the ways in which students were positioned into mathematical micro-identities, and in turn, likely the ways in which they enacted these micro-identities as well. In a standard mathematics classroom, the teacher faces a plethora of constraints that affect their teaching, students' learning, and, ultimately, the mathematics identity development of the students. These constraints can include curricular choices, student behavior, access to technology and other resources, etc. Conversely, for teacher-researchers conducting CTEs, there may be considerably fewer constraints, as the experiments are specially designed to allow the teacher-researcher more freedom and focused instruction to explore. In entering the classroom as an outsider, teacher-researchers can come into the mathematics classroom with their own lessons, tools, and may experience less challenging student behavior, as the main teacher is there as an additional support. Without these constraints, the teacher-researcher in Brizuela et al. (2023) and the study reported here had more flexibility to execute their intentional design and implementation of the CTE. As such, then, we argue that Azaiah's evaluation of his classmates' mathematical contributions was specific to the CTE and

did not tell the entire story about mathematical authority and who is positioned into which mathematical micro-identities in that classroom.

There are two other reasons the CTE context is noteworthy. First, because it is potentially an explanation for the infrequent observations of the mathematical menial worker micro-identity. That is, because these lessons were part of a CTE, the teacher-researcher was free from the pressures of regular classroom teaching and thus likely more willing to dedicate time to letting ideas (incorrect or correct) flow freely, without the pressure of losing instructional time. More specifically in regard to positioning students' into mathematical micro-identities, the teacher-researcher likely did not need to be as intentional with where she sourced mathematical ideas as a regular classroom teacher might be. Second, we also observed that the teacher was the person who most often positioned students into micro-identities. Unlike Wood (2013) we rarely observed students positioning each other into micro-identities. This could be a result of the CTE context because we observed students in the whole group discussion part of this lesson, instead of small groups where they may have had more agency in positioning their peers.

Implications for Future Research

While this paper investigated the nature of positioning and enactment of mathematical micro-identities, future research could consider exploring the same in a traditional mathematics classroom to observe how the difference in methodology would impact these results. This research demonstrated that students typically enacted the micro-identities the teacher-researcher positioned them into, but this was not always the case. Other researchers seeking to investigate micro-identity development might also consider analyzing when and why students deviate from the micro-identities a teacher is positioning them into.

Our work has implications for teacher-researchers in that it reveals that mathematical identity construction involves teachers as well as students, which points to the need for teachers to use their discretion (Ball, 2018) in ways that uplift students' contributions in the mathematical classroom. Students can interpret how the teacher-researcher responds to them as a reinforcement of the micro-identity they have been positioned into. According to Ball (2018), just and equitable teaching can significantly transform the mathematics education field, especially for students, who behave in response to the moves their teachers make in their classrooms. The decisions mathematics teachers make regarding students' mathematical contributions suggest how their students can essentially play their roles, and they also convey important messaging about how they can behave in both the micro- and macro-mathematical communities. Regardless of how teachers use their mathematical authority, their actions will undoubtedly have a carryover effect in that students may use their roles to construct their micro-identity.

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Appendix

The classroom teaching experiment

Brizuela et al. (2023) used design-based research (Cobb et al., 2003; Collins, 1992; Kelly, 2003) to design and carry out a CTE. This experiment included eight weeks of classroom instruction which was separated into two 4-week cycles, and involved two 40-minute lessons taught per week. In addition, there were select student interviews before, between, and after the two cycles of instruction. The CTE tasks were created based on field-tested tasks which were developed in previous early algebra research by the project researchers. Early algebra research exists to better prepare students for formal algebra by introducing algebraic thinking practices into early mathematics education, and incorporating arithmetical and algebraic content and techniques. In this way, students can begin to engage in many of the fundamental practices inherent in formal algebra, such as representations and reasoning, as tools for success in algebra in later grades. This was not a primary focus of the Brizuela et al. (2023) study, however, it was the intention of the CTE. In particular, the classroom included students who were encouraged to utilize the different tools in early algebra, which the authors claimed may have lent to mathematical discourse which could influence authority dynamics amongst students.

In the focal lesson of Brizuela et al.'s (2023) study, Lesson 12, referenced above, students were discussing how they might use variable notation to depict a relationship between two quantities (two sisters' ages). Recent research has shown that students as young as those in Brizuela et al.'s (2023) study (first grade students), can use variable notation. Moreover, in doing so at an earlier age, it may also help students to represent relationships more generally (Blanton et al., 2015; Brizuela et al., 2015; Carpenter et al., 2003; Fujii & Stephens, 2008). Figure 1 provides a summary of the structure of the CTE.

Activity	Pre-interview	8 activities	Mid-interview	8 activities	Post-interview
Function type	$y = x$	$y = mx$	$y = x + b$	$y = x + b$	$y = mx$ $y = mx + b$

Figure 1: *Sequence of activities during the Classroom Teaching Experiment.*

Chapter 5: Not Just “The Help”: Exploring Black Women Early Elementary Paraeducators’ Narratives About Their Mathematical Identities

Introduction

The National Resource Center for Paraeducators (NRCP) was founded in 1979 with the sole intention of providing support for paraeducators¹², a role that came into existence over 50 years ago (Njuguna, 2016). At the time, the general responsibilities assigned to paraeducators looked drastically different to those of current paraeducators. Then, paraeducators were tasked with office duties and stayed relatively invisible in the classroom as they monitored student behavior. While in many school districts today’s paraeducators fulfill these same duties, the demands of teaching have led to an increase in the responsibilities for paraeducators: their role has shifted as teachers have encountered more complex and pressing teaching situations over the years. Thus, paraeducators have become more valuable in the United States (U.S.) public education systems. No longer are paraeducators unseen in the classroom; now, they often work directly with students and are expected to provide instructional support, if not instruct lessons entirely. This evolution in the paraeducator role is beneficial for both students and paraeducators. Students have additional support with multiple educators present, including more opportunities for different types of learning (e.g., one-on-one support, small group learning, etc.) that might be more difficult to come by with just one educator. This also allows paraeducators to grow in their role as educators as they collaborate with their partner teachers in instructing their students.

The construction of paraeducators’ identities as mathematics teachers is undoubtedly weighty, and has repercussions not only for how they teach, but also for how their students learn. A person’s mathematical identity does not exist in a vacuum, however. Rather, it is the

¹² Paraeducators may also be known as paraprofessionals or instructional aides.

culmination of experiences with mathematics and interactions with others about mathematics, both within and outside the mathematics classroom.

As it relates to mathematics identity construction, narration is a powerful tool to help people make sense of their experiences. Many narratives begin in early childhood, yet they hold momentous power in how mathematics educators identify present-day. For paraeducators teaching mathematics, narration can help them and researchers uncover how a variety of experiences and interactions with mathematics guide how they construct and reconstruct their mathematics identity over time. In this paper, I investigate the following research question: *How do remembered histories of mathematics learning shape Black women paraeducators' mathematical identities?* Note that in this paper, the terms narratives and remembered histories are used interchangeably. I illustrate how there is a common paraeducator experience in that paraeducators are often undervalued in the mathematics classroom despite their frequent interaction with students. This lack of inclusion and perception as educators is amplified for Black women paraeducators, who face additional layers of discrimination because of their social identity. As such, while there are paraeducators who have positive historical mathematical experiences, the development of a strong mathematics educator identity may be stifled due to this discrimination. Nonetheless, paraeducators are well-equipped and knowledgeable to educate students, and investing in the positive construction of their mathematical identity is crucial for their work with students, as they directly impact students' mathematical development and learning and subsequently, their own mathematical identity construction. While much of the research on identity in a mathematics context centers students, educators' mathematical identity is an additional focus for understanding how identity impacts practitioners.

Theoretical Framework

To undergird my research into Black women mathematics paraeducators' identities, I used Martin's (2012) definition of identity and Langer-Osuna and Esmonde's (2017) narrative theoretical approach to mathematical identity. Martin (2012) defined mathematics identity as, "the [momentary] dispositions and deeply held beliefs that individuals develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives" (p. 57-58).

In turn, Langer-Osuna and Esmonde described narrative theory as: "individuals and the people around them create stories about themselves as they make sense of their experiences across social settings" (p. 639). These stories help people develop mathematical identities based on their experiences of achievement and belonging, or lack thereof, which subsequently impact their future mathematics education trajectories. According to Langer-Osuna and Esmonde (2017), a narrative approach focuses on how individuals use experiences to conceptualize stories about themselves and others regarding their relationship to and experiences with mathematics. Further, its focus on individuals' stories helps make identity understandable, because "identit[y] as [human made] stories" (p. 641) is concrete, knowable, socially constructed, and changeable over time and context. There is an agentic aspect to identity in that people tell their own stories about their own experiences and relationships to mathematics. Nonetheless, these stories include stories others tell too. This is due to the fact that the claims one makes about oneself are a manifestation or "reification" (Wenger, 1999) of the stories others tell about them and the mathematical positions they are placed into. As Langer-Osuna and Esmonde concluded: "we shape stories, and stories shape us" (p. 641). The narrative theoretical approach to identity

allowed me to uncover how exactly the stories the paraeducators told about their mathematical selves were developed and changed over time.

Prior Research

Intersectionality for Black Women

I take an intersectional approach to explore this study's participants' narratives because each identity they hold—e.g., their womanhood, their Black womanhood, their role as paraeducators, etc.—all occur at the same time altogether, and impact every aspect of their being, including how they see themselves in the context of doing and learning mathematics. Crenshaw (1989) applied a Black feminist theoretical lens on Black women's intersectionality as it related to antidiscrimination and antiracism in response to race and gender constantly being separated when researchers analyzed Black women's experiences. Crenshaw referred to this as a “single-axis framework” (p. 139) that “distorts” (p. 139) and negates the unique and multidimensional experiences of Black women concerning race and gender. This framework erases Black women in that it only considers the experiences of privileged members of the group, i.e., Black men and White women regarding race and gender, respectively. In doing so, it does not capture the full breadth of how Black women are impacted by these experiences, as the effects encountered by the privileged members are extrapolated onto theirs, rendering their experiences invisible.

Further, Crenshaw (1989) argued that although Black women can encounter discrimination in similar ways as Black men and White women, their experiences can also be different. In some cases, Black women's and Black men's experiences align on the basis of racism; and in others, Black women align with the sexist experiences White women encounter as well. Nevertheless, she posited that the racial context for Black women not only made them

“vulnerable” (p. 162), it delineated a clear difference for Black and White women in that the discrimination they each faced was too complex and nuanced to compare. Intersectionality is thus crucial for Black women because it prioritizes their specific journeys as members of two minoritized groups which cannot be disentangled. In the case of the study described in this paper, although mathematics is oftentimes portrayed as neutral or objective due to its universality, it is not free of bias: There are expectations and norms about what it means to be a member of this community. In this study, I centered on the mathematical experiences of two Black women paraeducators and used intersectionality to reject traditional conceptualizations about who can be a mathematical learner and doer.

Mathematical Identity

There is a robust body of literature on mathematical identity that considers students, teachers, and societal members outside of the classroom and the claims they make about their relationships to mathematics. I employed Martin’s (2012) definition of mathematics identity, as it is rooted in narratives, or the stories people tell about themselves and others, but modified it to include the fact that identities are not static, or unchangeable, or what Wood (2013) categorized as a mathematical macro-identity. Instead, these narratives are contextually dependent, fluid micro-identities (Wood, 2013). People’s interactions with others (van Langenhove & Harré, 1999) and with the mathematics environment itself (Ball, 2018) can seriously regulate the narratives they tell about who they are as mathematics learners. Whether positive or negative, the interpretations of these interactions are significant, as one may craft a personal mathematical identity that is weak or strong, believing they either can or cannot “do well in math” (Cribbs et al., 2021, p. 279), or as Langer-Osuna and Esmonde (2017) assert, the stories one tells about one’s interactions with mathematics are “akin to a self-concept” (p. 641) about one’s

mathematical identity. In taking a narrative theoretical approach to identity work, researchers can not only make sense of how the mathematical experiences people have influence their identity development, but also point to how these experiences and identities can shape people's mathematical trajectories.

Educator Mathematical Identity

Beauchamp and Thomas (2009) used narrative identity to explore pre-service and in-service mathematics teachers' self-efficacy regarding teaching mathematics. In describing teacher identity, they cited Sachs (2005):

Teacher professional identity then stands at the core of the teaching profession. It provides a framework for teachers to construct their own ideas of 'how to be', 'how to act' and 'how to understand' their work and their place in society. Importantly, teacher identity is not something that is fixed nor is it imposed; rather it is negotiated through experience and the sense that is made of that experience. (p. 15)

To understand these experiences, mathematical identity researchers have utilized a narrative approach, giving agentive power to mathematics teachers to describe their experiences with mathematics. For instance, Lutovac and Kaasila (2014) conducted a study with 19 pre-service mathematics teachers to investigate how their preparation programs might employ opportunities for the participants to consider their own mathematical identities once they began working as teachers. Through the interviews, Lutovac and Kaasila sought to uncover what stories the pre-service teachers told about their past and future mathematical selves. They found that all 19 participants reported negative views of mathematics, which subsequently influenced the negative mathematical identities they developed. Moreover, the authors concluded that these

results were associated with negative experiences the pre-service teachers had as mathematics students in their adolescence.

Lutovac and Kaasila (2014) argued that interrogating narratives about teacher mathematical identity was crucial for pre-service mathematics teachers, as doing so would allow them to confront and begin reconciling their weak relationship with mathematics in order to orient themselves to their future work as educators. Similarly, taking a narrative approach regarding teacher mathematical identity can assist in uncovering how the past and present connect for Black women paraeducators and in identifying how the claims they make about mathematics relate to the specific mathematical identities they position themselves into. As Battey and Franke (2008) argued, the stories mathematics teachers tell about themselves are undoubtedly linked to what they do in their classrooms.

According to the National Research Council (2001), one goal of mathematics education is to help foster positive dispositions towards mathematics. Although this goal was outlined with students in mind, I contend this applies to mathematics educators as well. In this paper, I consider paraeducators, whom school districts rely on heavily but who are often disconnected from the mathematics classroom and community, when considering the National Research Council's (2001) goal. This study explores what narratives or remembered histories Black women paraeducators use to describe their relationships to mathematics, and what particular mathematical identities these narratives indicate.

Methodology

Participants

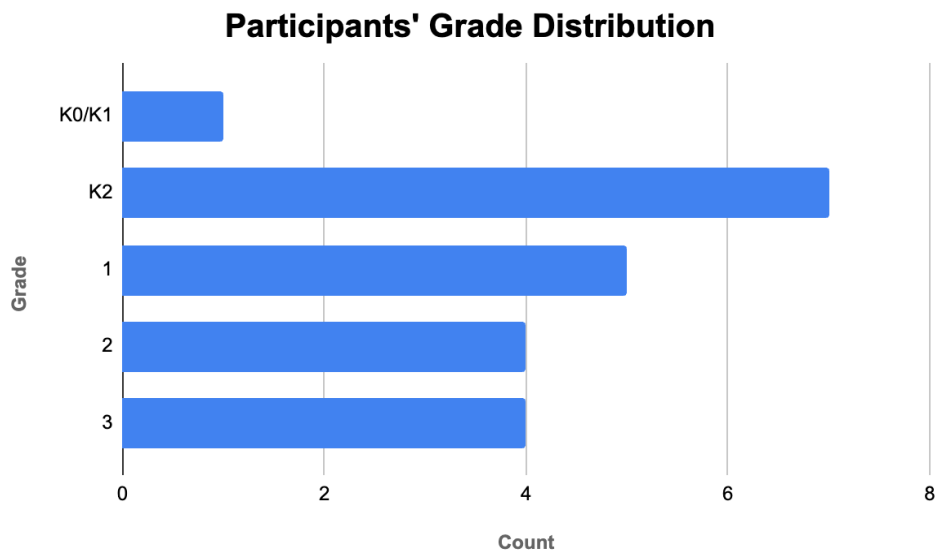
I used a subset of data from the *Doing the Math with Paraeducators: Enhancing and Expanding and Sustaining a Professional Development Model in PreK to Grade 3 Math*

Classrooms (Doing the Math) 4-year National Science Foundation (NSF) project (August 2022 to June 2026). The purpose of this NSF study is to provide mathematics teaching and learning training for paraeducators in Pre-Kindergarten through third grade (PreK-3) over the course of two years through a variety of means (classroom observations, meetings to analyze student work, whole-group professional development sessions, etc.) in order to foster stronger mathematics teaching morale and pedagogical mathematics content knowledge among participating paraeducators. Paraeducators were selected from a large public school system in the Northeast U.S. The researchers began their recruitment process by going to five schools in the partnering district to present the study to potential participants, their principals, and mathematics administrators. Next, they administered an online survey to gather data on demographics, experiences, and educational history among interested paraeducators in PreK-3 grade classrooms. A total of 35 paraeducators completed the survey.

In order to be selected, respondents must not have obtained a degree higher than a Bachelor's degree. Thus, 21 paraeducators were selected as study participants. During the 2022-2023 school year the paraeducator demographic data for the school district was as follows: 46.4% Black, 3.2% Asian, 21% Hispanic, 28.9% White, 0.3% Native American, 0.1% Native Hawaiian/Pacific Islander, and 0.1% multi-racial, non-Hispanic. 76.7% identified as female, and 23.2% as male. Figure 1 below displays the grade distribution for all participants.

Figure 1:

Grade distribution for August 2022 to June 2023 Doing the Math participants.



Data Collection

Of the 21 total participants in the *Doing the Math* study, 12 were Black women paraeducators. Of these 12 participants, six agreed to participate in this study. I interviewed all six paraeducators once over Zoom. See Table 1 in the Appendix for the interview protocol that I created in collaboration with the *Doing the Math with Paraeducators* Principal Investigators. The protocol focused on two main themes: (1) paraeducators' mathematics history and (2) being a Black woman in mathematics. On average, interviews lasted approximately 47 minutes. In addition to the interviews, I used a data artifact from the *Doing the Math* study: All participants were asked to create a drawing outlining their mathematical journey beginning from as early as they could recall to the day the drawings were collected during the August 2022 professional development sessions. Participants were given two days to draw a picture of their mathematics learning journey with labels and brief explanations of major experiences along the way. I used the mathematical journey drawings to create my interview protocol, including targeted questions which would focus on key moments in the paraeducators' drawings.

Data Selection

All six interviews were transcribed verbatim. After reviewing all six transcripts, I sought to identify which narratives seemed to be either more aligned or opposed to one another. As such, I eliminated one of the six interviews, as the paraeducator described a vastly different mathematics journey when raised in another country. I eliminated a second interview from consideration because the participant was fairly new to the paraeducator role relative to the other women. Of the remaining four interviews, it became apparent that there were two paraeducators who had similar backgrounds in terms of age and where they were educated, but also had seemingly opposite narratives about their mathematical identities that stemmed from childhood and professional mathematics experiences. This was compelling due to the fact that from my observation during the *Doing the Math* study, both women seemed to be positioned as strong mathematics educators by their peers. I was motivated to explore how the two appeared to overlap in their educator mathematical identity while having contrasting K-12 mathematics experiences. Thus, I selected these two interviews, Chantel's and Kristina's, in order to explore similarities and differences in their narratives about their relationship to mathematics, and how these stories pointed to their mathematical identities.

Chantel

Chantel was a paraeducator in a second grade, Sheltered English Immersion (SEI) classroom with nine years of experience. She attended the same school district she was teaching in, and obtained both an Associate's and Bachelor's degree in education. From our interactions in the *Doing the Math* project, it was clear that Chantel was very comfortable engaging with mathematics; she participated frequently in whole-group discussions, and often was the first to propose an idea or strategy during small group assignments. Among the project staff, Chantel was generally perceived as a leader among her cohort, with strong mathematical skills.

Kristina

Kristina was a paraeducator in a first-grade inclusion classroom, and worked at the same elementary school as Chantel. Similarly to Chantel, Kristina also worked in the same district that she attended in her youth. At the time of our interview, she had been working as a paraeducator for 23 years in the same school district. She pursued a Bachelor's degree in early childhood education, but dropped out after a year and a half of school. She would later earn a Bachelor's degree in early childhood education 19 years later. While Kristina was also a vocal participant and regarded by the *Doing the Math* researchers as another leader, her interview revealed the depth of her own self-perception as being mathematically incapable.

Despite the overlapping dynamics Chantel and Kristina had with their students, they each had contrasting attitudes towards mathematics. For both of them, their convictions about their own mathematical identities were the result of a culmination of various experiences they had with mathematics in their youth.

Data Analysis

Coding of Interview Transcripts

Once I had selected the two interviews to analyze for this study, I iteratively coded the interview transcripts following steps 1-3 below.

Step 1. I annotated the two interviewees' transcripts, keeping track of any ideas and questions that materialized as I read. This could include a brief summary of responses or highlights of responses which were thought-provoking. I met with the research team to discuss these notes before moving onto my next step.

Step 2. I refined these annotations by collapsing them into broad themes.

Step 3. Lastly, I reread the interview responses and narrowed my themes into codes, mapping each code onto one or more questions from the interview protocol. Wherever pertinent, I also connected these codes to literature. Table 1 outlines the codes I used to analyze the transcripts.

Table 1. *Codes developed from interview transcripts.*

Research Question: How do remembered histories of mathematics learning shape Black women paraeducators' identities?				
Code	Definition	Source	Transcript Examples	Applicable Interview Question/s
I can apply mathematics skills outside of the classroom	Describes using mathematics skills outside of the mathematics classroom.	Interview transcripts	“My home life tied into like the math life, like, I had a big responsibility in my home life. And I remember, we were doing measurement... And I remember my classmates being really, really impressed because I was able to peel a really, really long [potato] without like cutting or breaking in half. And it was like, ‘well, that's what happens when you have like, so much responsibility at home, like, look how it’s helping me out in math’” (Chantel interview, line 11).	Q5, Q9a, Q10a

I had/have positive/negative relationships with students in mathematics class or with mathematics teachers	Describes positive or negative interactions with students or mathematics teachers and how they interpret these interactions to make sense of how they view themselves as mathematics learners.	Interview transcripts	“Sometimes when you give a wrong answer in your math class, in high school, you would be laughed at or the teacher [would say] ‘Oh, why are you not getting it?’, you know, in front of everybody” (Kristina interview, line 21).	Q4a-4g, Q6, Q7a, Q8a-8b, Q16a, Q17a, Q20, Q22, Q23, Q24
I use mathematics as a bonding tool	Describes how mathematics is used to strengthen relationships.	Interview transcripts	“My dad is also a math person who just loves numbers or anything like that...I think I feel like we grew closer on our like, love for math” (Chantel interview, line 13).	Q4a-4d, Q5, Q8a, Q9a, Q10a, Q12, Q13,
I see/don’t see myself as being good at math	Describes themselves as being strong in mathematics, finding mathematics easy/challenging, or as being/not being a “math person.”	Laskasky (2018)	When asked if she considers herself to be good at math: “I say no, I still feel slow. Like, you know what I mean? It doesn't come as quick, you know? I can get the same answer you get, but it's not gonna come as quick, you know, so, I guess I can struggle with that a	Q9a, Q9b, Q10a, Q10b, Q11, Q12, Q13, Q22, Q23, Q24

			little bit” (Kristina interview, line 37).	
Others see/don’t see me as a certain type of person in the context of mathematics	Stories or claims people make about the type of person an individual is.	Ingels et al. (2011); Langer-Osuna & Esmonde (2017)	“It's just that people seeing you as a leader, and feeling comfortable to come talk to you when they're struggling or where they don't understand” (Chantel interview, line 25).	Q10a, Q10b
Other people affirm me as a mathematics learner/educator	Description of another person validating an individual on their doing mathematics or in their mathematics teaching practice.	Interview transcripts	“So it's like, I think like just being good in math, it allows people to just like rely on me, when they're struggling in that area, whether it's my daughter, whether it's like, another para in the, in the building or whatever. It's just that people seeing you as a leader, and feeling comfortable to come talk to you when they're struggling or where they don't understand” (Chantel interview, line 25).	Q10a, Q10b, Q11, Q13, Q22, Q23, Q24

I am/am not naturally good at mathematics	Describes their mathematics skills as being due to nature.	Anderson (2007)	“Yes, I feel like math was always one of my favorite subjects, maybe because it just came so easily and naturally to me” (Chantel interview, line 11).	Q9a, Q9b, Q11, Q12, Q13
I like/don’t like mathematics	Describes affinity/lack of affinity for mathematics.	Jaber (2015)	“Everything just was challenging for me. I would say math was easier. But it wasn't like my favorite subject” (Kristina interview, line 15).	Q9a, Q9b, Q11, Q12, Q13
I am/am not confident in my mathematics teaching skills	This can include statements about comfort/discomfort as an educator.	Interview transcripts	When asked if she feels like she is a part of the mathematics community: “I would definitely say yes. Because now, I guess partly because of the [<i>Doing the Math</i> study] I feel like we're more included now. Like as far as the planning, and I do think this year, my principal, he set aside the times for us to plan with our teachers. So it does make us feel more a part of the	Q9a, Q9b, Q10a, Q10b, Q11, Q12, Q13, Q14b, Q18a, Q18b, Q19a, Q19b, Q21a, Q21b, Q21c, Q21d, Q22, Q23, Q24

			community. As far as when it comes to teaching math and stuff like that” (Kristina interview, line 116).	
People perceive me as inferior because I am a paraeducator	Describes not being seen as equal to other general educators because of the paraeducator role.	Interview transcripts	“And we are considered sometimes the help and not seen as equal” (Kristina interview, line 53).	Q14a, Q15, Q22, Q23, Q24
	Describes being afraid to get mathematics questions incorrect.	Interview transcripts	“But I think as it got harder, and older, I feel I would say, I guess scared, scared of always giving the wrong answer, you know?” (Kristina interview, line 19).	Q4a-4g, Q7a, Q7b, Q8a, Q8b, Q9a, Q9b, Q10a, Q10b, Q11, Q12, Q13, Q16a, Q16b, Q17a, Q17b, Q20, Q22, Q23, Q24

In order to better understand Chantel’s and Kristina’s mathematical identities, I coded their interview transcripts. I went through their interviews with the awareness that there was a unifying paraeducator experience, and coding helped me figure out what these shared features were. I acknowledge that researching intersectionality is complex, and coding does not convey all there is to know about Black women paraeducators’ experiences in the mathematical classroom. Nevertheless, coding offers some shape to the intricacies of intersectional experiences to better understand how Chantel’s and Kristina’s social identities as Black women might have impacted their mathematical identities as paraeducators.

After coding each transcript, I began my analysis, using the paraeducators’ response to each question as my unit of analysis. First, I identified which codes emerged from the responses and tracked the frequency of each code. Next, I looked at which codes co-occurred and

calculated the frequency of these co-occurrences as well. After doing this for both paraeducators, I ranked the codes from highest to lowest count for each paraeducator. These rankings helped me identify which themes were the most prevalent and therefore most salient for Chantel and Kristina, and I used these to build my claims.

Results

Chantel's interview was 51 minutes and 40 seconds long, with Chantel speaking for 29 minutes and 19 seconds total. Chantel gave very detailed responses to the questions, which were often centralized around the same code. As such, there were a total of 18 instances across all codes throughout her interview. *Other people affirm me as a mathematics learner/educator* was the most frequent code (six instances). *I am confident in my mathematics teaching skills, I had/have positive relationships with students in mathematics class or with mathematics teachers,* and *Others see me as a certain type of person in the context of mathematics* were all tied for second most frequent (two instances), with the remaining codes each coming up just once (one instance). Figure 2 displays the distribution of codes in Chantel's transcript. The diameter of each circle represents the frequency of each code.

Figure 2:

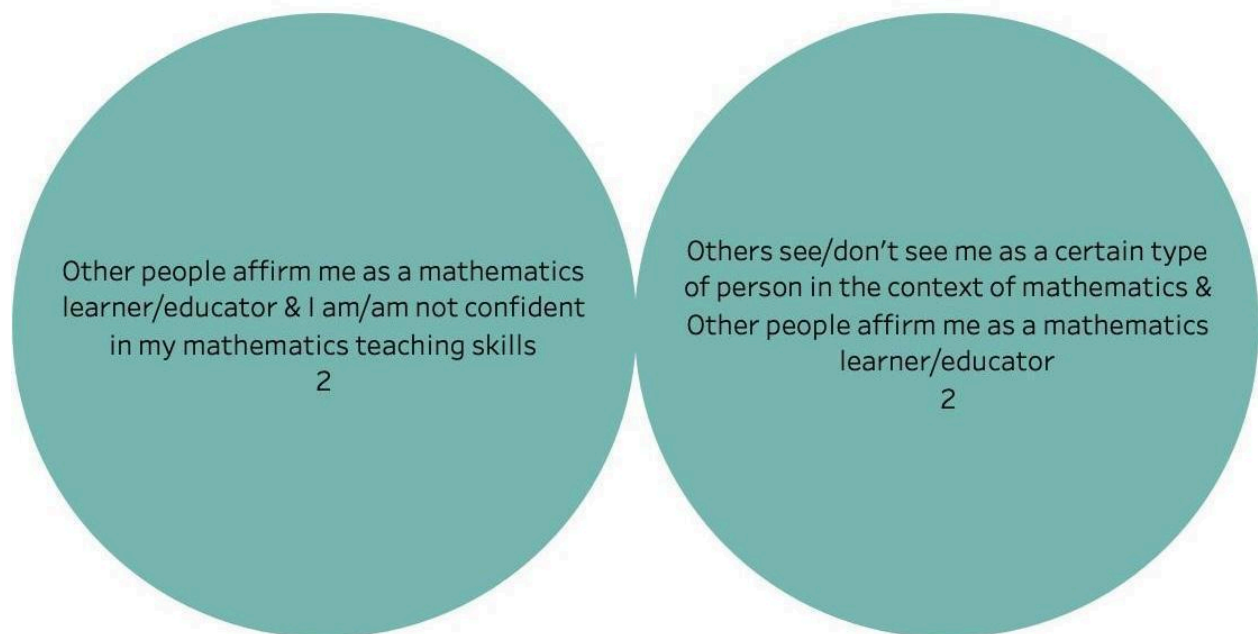
Code distribution in Chantel's transcript.



There were two instances in which *Others see me as a certain type of person in the context of mathematics* and *Other people affirm me as a mathematics learner/educator* co-occurred, and the same was true for *Other people affirm me as a mathematics learner/educator* and *I am confident in my mathematics teaching skills*. The only co-occurring codes for Chantel are illustrated in Figure 3 below.

Figure 3:

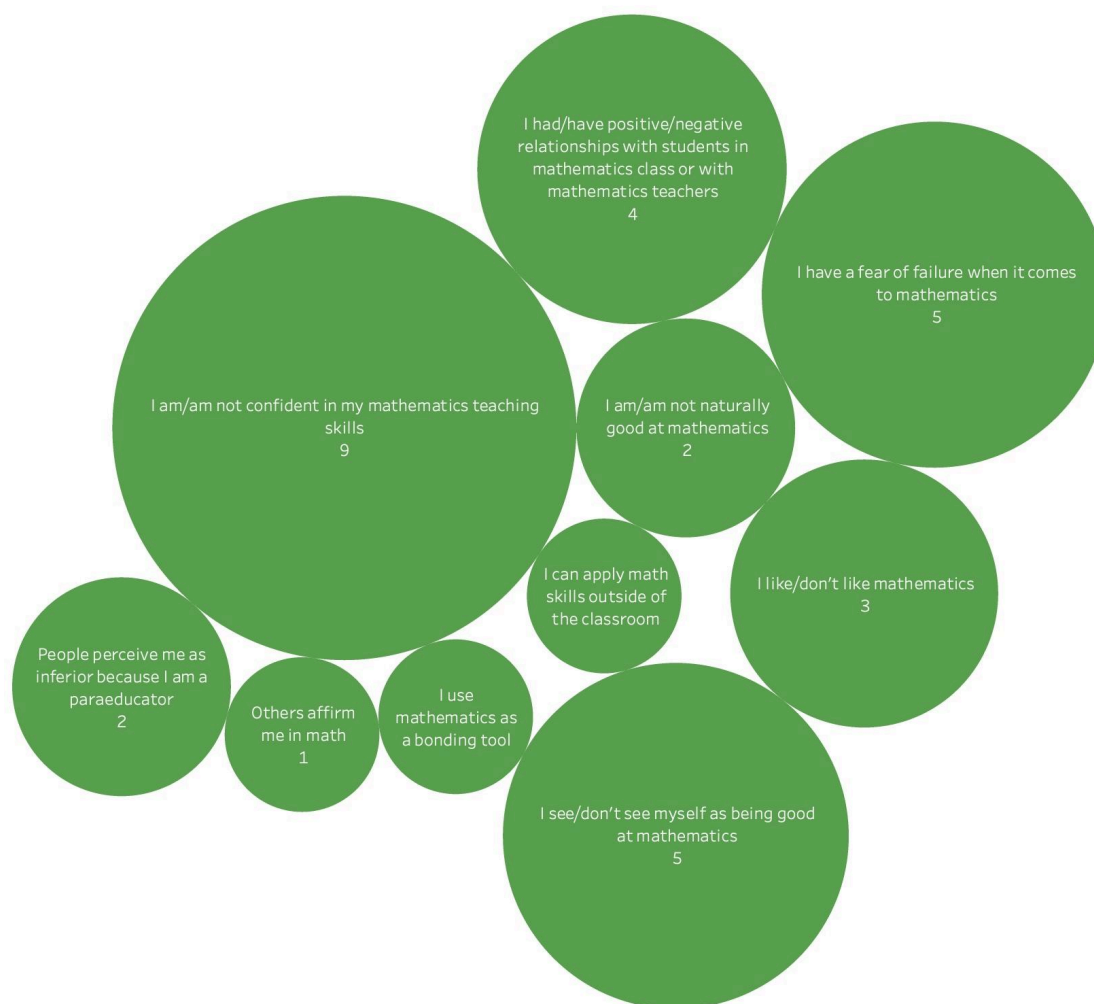
Co-occurring codes in Chantel's transcript.



Kristina's interview lasted 42 minutes and 31 seconds, nearly nine minutes shorter than Chantel's. Kristina's airtime was 23 minutes and 42 seconds long. She spoke for the majority of the interview, but though her total time speaking was less than Chantel's, just over 40% of her responses were focused on the negative aspects of her remembered mathematical histories, hence there being 33 instances across all codes throughout her interview. *I am not confident in my mathematics teaching skills* was prevalent throughout her transcript (nine instances). This was followed by *I don't see myself as being good at mathematics* (five instances), *I had/have negative relationships with students in mathematics class or with mathematics teachers* (four instances), *I don't like mathematics* (three instances), *I am not naturally good at mathematics* (two instances), and the remaining codes all occurred once. Figure 4 displays the distribution of codes in Kristina's transcript.

Figure 4:

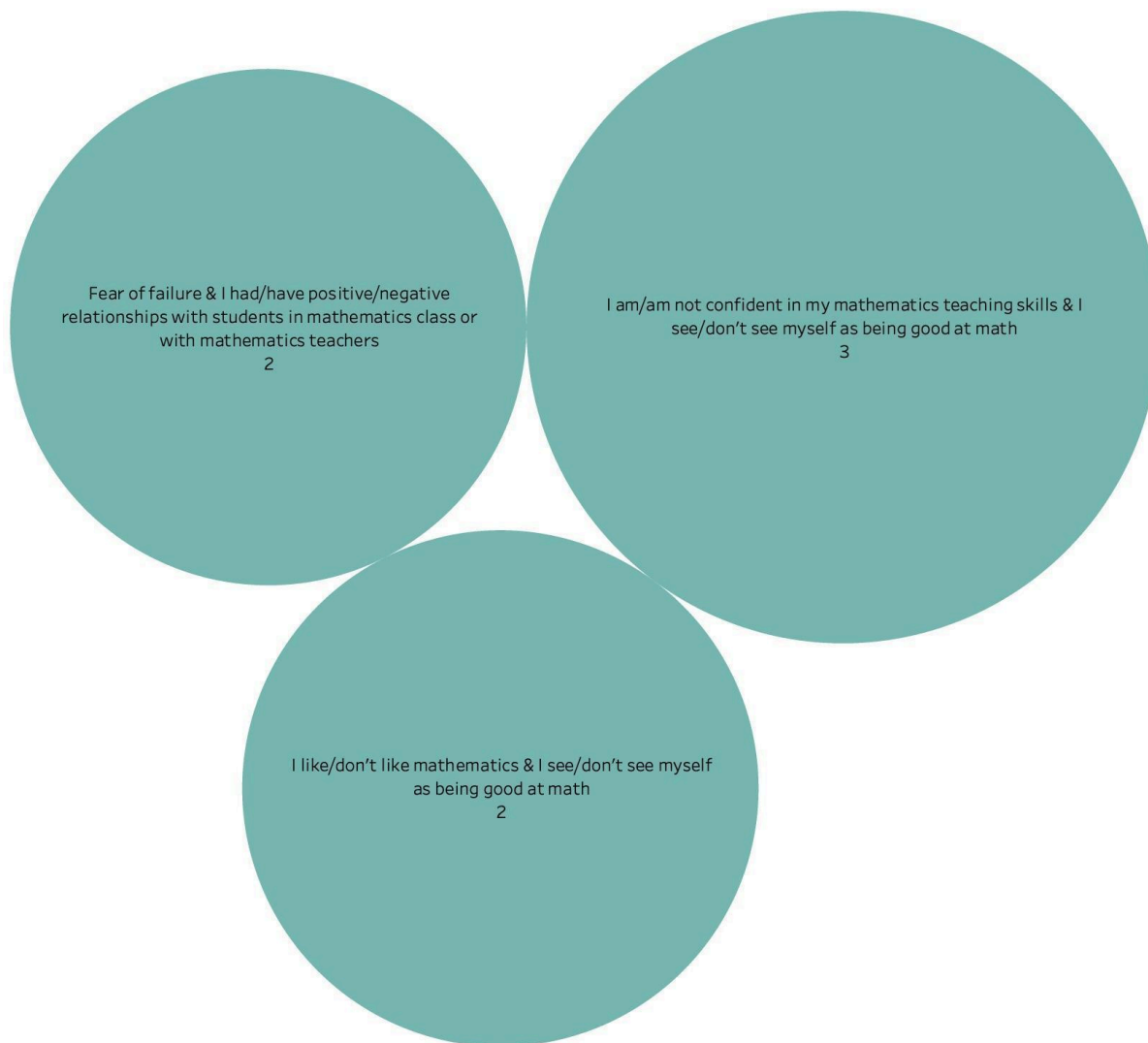
Code distribution in Kristina's transcript.



There were three instances of codes co-occurring in Kristina's transcript. The pairs of co-occurring codes were: *Fear of failure* and *I had/have negative relationships with students in mathematics class or with mathematics teachers* (two instances), *I don't like mathematics* and *I don't see myself as being good at mathematics* (two instances), and *I am not confident in my mathematics teaching skills* and *I don't see myself as being good at mathematics* (three instances). These co-occurrences are illustrated in Figure 5 below.

Figure 5:

Co-occurring codes in Kristina's transcript.



While *Other people affirm me as a mathematics learner/educator* was the most common code in Chantel's data, it was the least common one for Kristina's. These contrasting code rankings speak to the difference in remembered histories and subsequent mathematical identity development for each paraeducator. As you can see in Figure 3, there were nine different codes that came up for Chantel versus 10 codes for Kristina. Although there is only a count difference of one regarding the codes, Figure 4 illustrates that the codes for Kristina were more dense than for Chantel. Both paraeducators had 10 different codes emerge in their interviews but Kristina had four codes with a minimum count frequency of 4, while Chantel only had one code with a

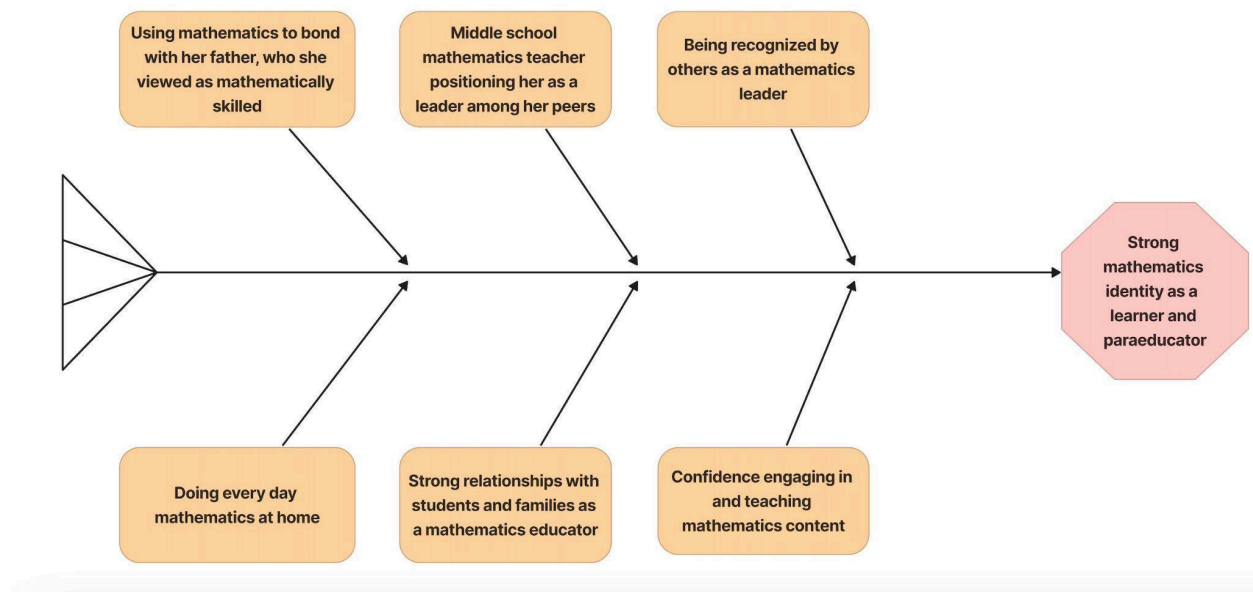
minimum count frequency of 4. I posit this is due to her deeply negative affect about mathematics, which was mediated by the plethora of difficult encounters with the subject across a multitude of contexts (e.g., relationships with teachers, family, application of mathematics skills, etc.).

Claim one: Chantel’s present self-identification as a “math person” is a result of others consistently positioning (van Langenhove & Harré, 1999) her into this identity.

This claim was built on how frequently Chantel described other people affirming or encouraging her in mathematics (see Figures 2 and 3). In describing her adolescent mathematical experiences, Chantel’s stories led her to present her younger self as mathematically strong. She started the interview with a strong statement: “math was always one of my favorite subjects, maybe because it just came so easily and naturally to me” (Chantel interview, line 11). Later in this interview, she seemingly contradicted herself in stating that mathematics did and did not come “second nature” (line 25) to her. This difference could reflect Chantel presenting as confident and then developing more trust as the interview progressed to subsequently reflect her true beliefs. Nonetheless, I contend that both assertions are true - Chantel did have strong mathematical skills, and this was likely due to others’ narrating stories to her about her being mathematically gifted. Thus, she enacted this identity. Figure 6 displays Chantel’s mathematical journey map that I configured after analyzing her interview. In the *Doing the Math* mathematical journey drawings, Chantel only focused on key moments from her childhood through college. I expanded this journey by connecting these key moments and interactions to recurring themes present in her work as a paraeducator to then point to her current mathematical identity.

Figure 6:

Chantel's mathematical journey map as a learner and paraeducator. The rectangles' location above or below the line have no significance. The arrows indicate that the different experiences are connected to the paraeducator and/or learner mathematics identity.



Immediately after she declared her positive disposition towards mathematics, Chantel highlighted specific memories and interactions with various people in her life who implicitly told the same story about her mathematical adeptness. Her father, who she described as “*also* a math person who loves numbers” (emphasis added, response 6), regularly used mathematics as a bonding tool with Chantel, because the two shared the same “love for math” (response 6). Nowhere before this assertion did Chantel outright claim *math person* as an identity for herself, but in telling this story about her father, she did so implicitly. Her father regularly encouraged and praised her for strong mathematics skills, instilling a sense of confidence that was affirmed by other adults in her life, most notable by her middle school teacher.

In detailing her childhood, Chantel alluded to difficulties at home and high levels of responsibilities she had as a result. Her fifth-grade teacher was aware of the dynamics in her

home and used these difficult moments to affirm Chantel as a mathematics student. The teacher knew she “had a strong point in math” (response 5) and positioned Chantel accordingly during group work, positioning her as the “captain” (response 5) of her group to help her peers who were not as skilled. Chantel interpreted being positioned as a captain to mean she was indeed a commander of mathematics to the point where she could, and often did, lead her group to understanding. She claimed this position made her feel “special” (response 5), and it deepened her positive mathematical identity. When asked to describe what it meant to be considered ‘good at math’, Chantel claimed that it was highly dependent on others seeing her as mathematically capable, or as a resource for solving mathematics problems. “I just know...being good at math, I’m always a person that...just helps, and [that] people always come to. So it just helps...being good at math just puts this role on me” (response 12). This role Chantel spoke of is one of reliance and assurance for others; it is “people seeing you as a leader, and feeling comfortable to come talk to you when they’re struggling or where they don’t understand” (response 12). In addition, in these two excerpts Chantel described how other people recognized and positioned her as a mathematics leader, and she also took up this identity in classifying herself as being “strong in math” (response 12).

Decades later, Chantel was now a participant in *Doing the Math*. One of the purposes of the study was to explore how increasing paraeducators’ access to professional development would impact their self-efficacy when it came to teaching and learning mathematics. Chantel alleged that she did not feel the professional development was rigorous enough for her because she already felt she possessed strong mathematics skills. Nonetheless, she found the professional development was fruitful because she was able to help the other paraeducators who did not feel as positively about or as competent in mathematics. Chantel had so regularly enacted the

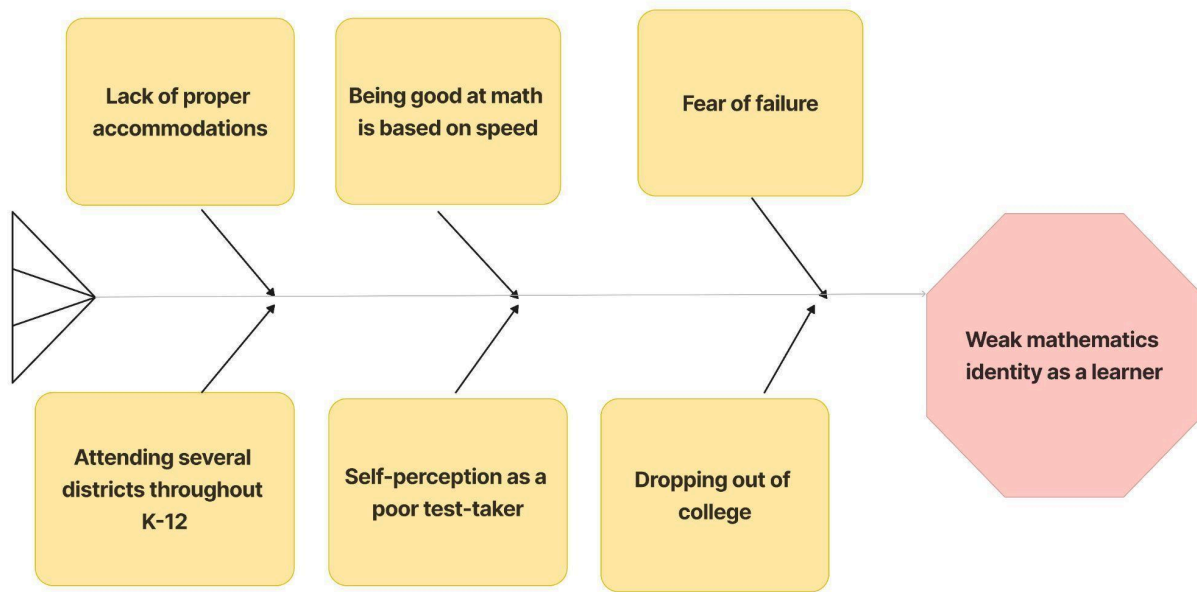
mathematical leader identity that adults had positioned her into as a child that she now positioned herself that way as well. Chantel's interview revealed she had a strong mathematical identity as both a learner and as an educator.

Claim two: Kristina's multiple identities diverge from one another.

Figures 3 and 4 illustrated the overwhelmingly negative history Kristina described about her mathematical journey. Whereas Chantel's past mathematics experiences were filled with positive interactions with adults and peers (see Figures 2 and 3), Kristina's experiences were different. Similar to Chantel, these interactions impressed upon Kristina narratives about her potential to achieve in mathematics which had negative impacts on the mathematics identities she enacted. Kristina's characterization of her mathematics experiences as a student demonstrated that she had a weak mathematics identity as a learner. This identity was seemingly stable when she described herself as a learner, from childhood through adulthood as a paraeducator. Her journey to this identity is depicted in Figure 7, which I configured after analyzing her interview. Kristina's *Doing the Math* mathematical journey drawing included the negative experiences she had with mathematics as a childhood and college student the first time she attended. As I did with Chantel's journeys, I connected the key interactions that Kristina drew in her first mathematical journey drawing with the most salient themes from her interview. Both of these pointed to a weak mathematical identity as a learner.

Figure 7:

Kristina's mathematical journey as a mathematics learner. The rectangles' location above or below the line have no significance. The arrows indicate that the different experiences are connected to the learner mathematics identity.



Kristina’s K-12 journey was rampant with transition and challenge. She was raised by an educator mother whose membership in the education community provided Kristina with access and privilege to resources to help mitigate the struggles inherent in learning for her as a student receiving special education services. She began her schooling in the same district she was currently working in but was then enrolled in a suburban district through a highly regarded voluntary school integration program in her area. Kristina returned back to her original district before being enrolled in two additional districts by the time she graduated from high school. In addition to the disruption that moving schools naturally causes, Kristina’s learning disability added another challenging layer to her education. When asked if she could recall enjoying mathematics in her youth, she quickly responded “no” (Kristina interview, response 7), explaining this was due to the fact that she was on an Individualized Educational Plan for reading, which meant “everything just was challenging for [her]” (response 7). Kristina did not explain why she moved to so many different districts, but in describing her mother’s involvement in her education, including a quip about attending Harvard University tutoring programs, it

seems likely that she advocated for her daughter to receive necessary supports for reading. Nonetheless, in doing so, there was less of a focus on her mathematics skills: Kristina recalled having a lack of access to proper accommodations in mathematics class. Although her mother was active in her education, by focusing on reading rather than mathematics, Kristina did not achieve as highly in mathematics. I argue this began the development of a negative mathematics identity that would follow her throughout her life.

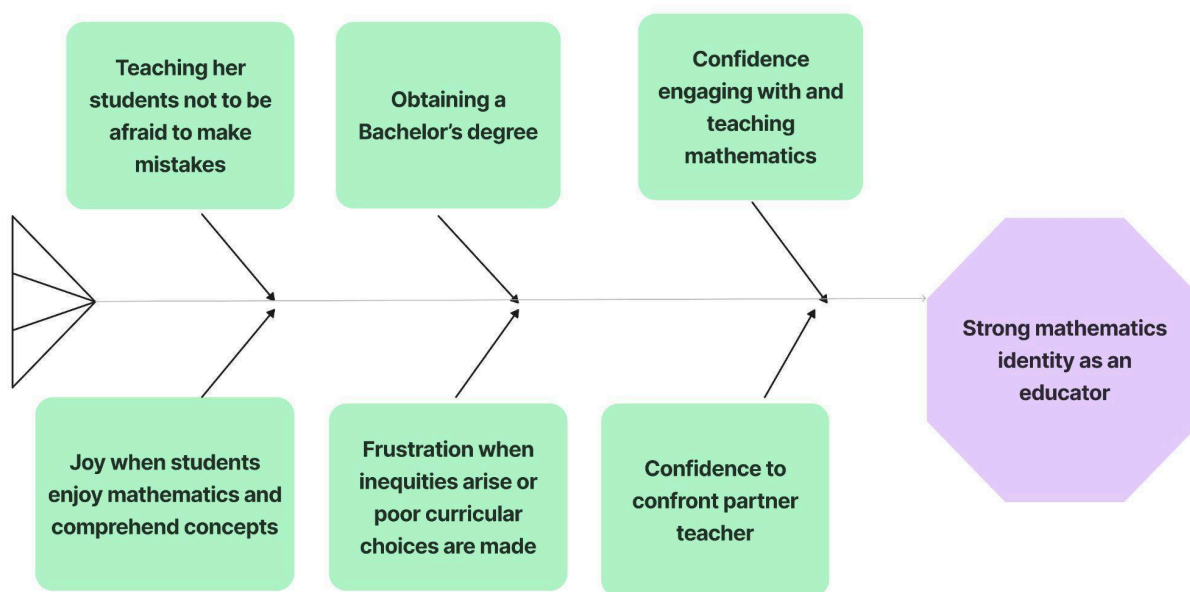
This undermining development was strengthened by further interactions in mathematics classes. Kristina described being fearful in high school because of the culture of the mathematics classes that her teachers had fostered. “Sometimes when you give a wrong answer in your math class, in high school, you would be laughed at or the teacher [would say] ‘Oh, why are you not getting it?’, you know, in front of everybody” (response 10). As a means of protection, Kristina started self-selecting out of classroom discussions, “shut[ting] down” (response 12) completely, a practice which followed her into college. This fear of making mistakes in front of her peers and teachers was further detrimental to her mathematics learning, eventually leading to her dropping out of college because of the mounting challenges. It was no surprise then that Kristina believed that mathematical prowess was synonymous with speed. Even though she claimed this was what she believed in her youth, when asked to answer whether or not she considered herself to be good at mathematics now, Kristina pointed to her lack of speed in answering, saying “no” (response 18), again demonstrating she had a weak mathematics identity as a learner.

Notwithstanding, when describing her work in the mathematics classroom as a paraeducator, Kristina was much more confident about her skills, taking up a strong, positive mathematics identity. Figure 8 shows the complexity in Kristina’s mathematical journey as an educator; it diverges from her journey as a learner, seen in Figure 7. Due to this difference,

Kristina's mathematical identity as an educator is represented as a separate figure.

Figure 8:

Kristina's mathematics journey as an educator. The rectangles' location above or below the line have no significance. The arrows indicate that the different experiences are connected to the paraeducator mathematics identity.



As an educator, Kristina's narratives revealed she employed a strong mathematics identity, which was not revealed in Figures 2, 3, or 7. In describing what it meant to enjoy mathematics, her response was contrary to her response about what it meant to be good at mathematics. Whereas for the latter she claimed being mathematically skilled was related to speed when working through problems, for the former, Kristina did not value it when working with students. This signaled the positive epistemic affect her students might have had when engaging with mathematics. Her students who enjoyed mathematics were "excited because they... want to do the math. Like, there's just like, some of them don't want to go to reading. Some of them love math" (response 22), this made evident by "them smil[ing and being] so

happy [when] they [get] it right” (response 22). To Kristina, seeing her students succeed is “just a joy and it feels good to know that okay, what I’m teaching matters” (response 24). Her joy was tied up in the joy of her students’ positively engaging with mathematics, which is why she, along with Chantel, advocated for their students when confronted with inequities in their respective mathematics classrooms.

Claim three: As Black women paraeducators, Chantel and Kristina rejected the stereotypes associated with being a paraeducator and advocated for their students to be taught mathematics equitably.

Both Chantel and Kristina divulged that their relationships with their partner teachers were precarious such that they believed their teachers perceived them as inferior (see Figures 2 and 4). Each of the paraeducators described specific instances where one of their students was not being treated or taught mathematics equitably. Chantel chronicled various challenges she faced working with her teacher, a teacher who historically had rocky coworking relationships with other paraeducators in Chantel’s school. Because her principal affirmed Chantel’s strong mathematics leader identity, this teacher was intentionally paired with Chantel, which she believed was because her principal knew she would not “just sit back and let certain things happen” (Chantel interview, response 19). According to Chantel, this teacher was “not too fond” (response 19) of some of their Black students, oftentimes ignoring them, or overwhelming them with inappropriate curriculum based on their academic level. Chantel posited that the classroom teacher often failed to make the necessary modifications to help them close gaps in their knowledge and exclaimed that this teacher “was treating them unfairly” (response 22) and not “doing justice for our kids” (response 21). Therefore, she rejected the stereotypes associated with being a paraeducator that would relegate her to the sidelines in the classroom, and instead called

out these insidious behaviors when they occurred.

Chantel not only directly challenged her partner teacher's curricular decisions when she felt it was appropriate to do so, but also brought her concerns about the teacher to her school administrators so that they could hold the teacher accountable and correct her problematic behavior. "Can you imagine coming to work every single day you're working in a classroom, you feel like somebody doesn't like you because of your skin color?" (response 23), Chantel asked? Although she was speaking about herself and the abhorrent relationship she had with her teacher, Chantel is a Black woman, sharing the same racial identity as the majority of her students. Thus, for her, speaking up was paramount.

Sadly, Kristina's situation with her teacher overlapped with Chantel's as her teacher also treated her as though she was inconspicuous in the classroom. She noted the racialized differences in positions at her school, indicating that teachers with privilege and power were White, while the paraeducators tended to be Black. Paraeducators, Kristina observed, were seen as "the help and not seen as equal" (Kristina, response 30). As such, Kristina's teacher regularly neglected to plan with her prior to their joint participation in the *Doing the Math* study, where collaborative planning was a requirement for participation. While Kristina certainly could have enacted the stereotypical, menial behavior of a paraeducator, she did the opposite whenever she had concerns about students being treated unfairly. She relayed an instance of a student being constantly berated by her teacher when she stepped in to not only confront the teacher, but also resolved to be the educator working with the student during mathematics lessons. Kristina's efforts were successful in getting the teacher to cease these negative interactions with the student and, also, the student, according to Kristina, began thriving academically after working one-on-one with her. "It got to be the point where I got angry [hearing the teacher always yelling

at the student], and started crying, like, you get up every day, you have to talk...sometimes I feel like, that's what you have to do" (response 40).

Claim four: Chantel and Kristina relied on mothering as a practice in their role as paraeducators.

I posit that for Chantel and Kristina practicing mothering was rooted in communalism and collective Black identity (Ortiz & Morton, 2022), and was a tool for liberation. Nearly 20% (5 minutes and 51 seconds) of Chantel's responses were extensive responses centered on nurturing her students. Though Kristina's interview did not exactly mirror Chantel's, there was a shift in her responses about her confidence as a math educator when speaking about her relationships with her students. Prior to this turn in the interview, there were six instances of the *I am/am not confident in my mathematics teaching skills* code. The next three instances were of *I am/am not confident in my mathematics teaching skills* code.

Across both interviews, both paraeducators frequently referred to their students, who were predominantly Black, using the possessive determiner pronoun, "our." Chantel and Kristina often made mention of the deep foundations they had built with their students, working with them from as young as Kindergarten, even staying connected with them as they moved up in grades. Moreover, in reflecting on their motivation behind confronting inequitable mathematics teaching, they explicitly classified these instances as acts of racism against their students. Chantel and Kristina recalled conversations they had with their students' families in which their guardians conveyed a sense of comfort knowing these women were working with their children, because, as Chantel noted, they trusted her "because [she's] a Black woman" (Chantel interview, response 28).

As a Black woman, she argued, she had "a connection with the kids" (response 29).

Kristina's experience in defending her aforementioned student affirmed this as well. She described feeling a sense of responsibility to her Black students because as a Black woman herself, she "ha[d] that connection" (Kristina interview, response 45), and was compelled to advocate for her. Chantel's and Kristina's protection and defense of their Black "babies" (Chantel interview, response 29) rights to equitable mathematics education can be linked to the significance of the Black village, something both women spoke of when describing their close bond with their students, their families, and their community. In being these students' "second mom" in school (Chantel interview, response 29), the mothering aspect added to Chantel's and Kristina's mathematics teacher identity.

Discussion

The goal of this study was to use Chantel's and Kristina's remembered histories of their mathematics experiences from youth to their present-day occupation as paraeducators in elementary mathematics classrooms as Black women to explore how these remembered histories shaped their mathematical identities. I have provided evidence that their experiences as children were pivotal in the development of their mathematics identities as students, and now as educators. Part of these developments were the narratives they were either explicitly or implicitly told by people with whom they interacted, both within and outside of mathematics classrooms. Chantel's and Kristina's narratives confirmed Langer-Osuna and Esmonde's (2017) theory of narrative identity, which posited that people's stories about their mathematical selves were a means by which they could understand their experiences. Furthermore, these stories incorporated stories others told about them, which then led to an understanding about who they were as mathematical learners and doers.

For Chantel, this began at home with her father, who instilled a love of mathematics, using it to enhance their bond as father and daughter, as he himself, according to Chantel, was adept at mathematics. This narrative about her aptitude with mathematics was further affirmed by her middle school mathematics teacher, who positioned her as a mathematical leader amongst her peers. Chantel interpreted both of these narratives to mean that she did indeed have a positive affinity to mathematics and was mathematically capable. She brought this particular mathematics identity into her profession as a paraeducator, where this narrative continued to be repeated by others around her. At the time of the study, Chantel presented herself as a leader, not only in front of the students she taught, but also with her fellow paraeducators, teachers, administrators, and parents/guardians who looked to her for support. Chantel had multiple venues of mathematical identity formation that undoubtedly contributed to this strong mathematical identity as a paraeducator, which was significant in her work with students. She could thus rely on her experiences and the mathematical positions others put her into and utilize them as a tool to support her students' mathematical learning and their positive mathematical identity formation.

Likewise, Kristina's mathematics identity was also shaped by the narratives she internalized from her youth. Her mother's laser focus on ensuring she was properly accommodated in reading led to a lot of upheaval in her educational journey. Even while she had an educator mother as an advocate, Kristina expressed regret over not receiving the same support for mathematics, as she struggled to navigate the subject each year in school. Her experience in high school mathematics was especially damaging, as her teachers' explicit narratives about their expectations for students in mathematics were catastrophic for Kristina. To be good at mathematics, one needed to be perfect, and since she found mathematics to be challenging, this narrative ignited such a fear for Kristina that she completely disengaged from discussions in

mathematics class. Fast forward decades later, and while Kristina earned her Bachelor's degree after her second attempt at college, she continued to believe that rapidity in solving problems was indicative of one's ability to learn mathematics. Since she could not perform mental mathematics or solve questions quickly, Kristina established a weak mathematics learner identity.

In a different social context, this identity did not hold as a paraeducator. I posit this could be due to Kristina recognizing how salient others' stories were about her such that they influenced her mathematical self-concept, leading her to seek to do the opposite for her own students. When it came to advocating for her students, Kristina rejected her weak mathematics learner identity, and instead adopted a strong mathematics teacher identity. She articulated a confidence in comprehending and communicating mathematics concepts to her students and partner teacher, a fact that was observed consistently by the researchers in the *Doing the Math* study. Kristina's ability to move between identities is supported by Wood's (2013) concept of mathematical micro-identities, which are momentary, fluctuating as the context does. Kristina's co-existing mathematical identities are further evidence in support of Wood's (2013) argument that micro- rather than macro-identities provide a more comprehensive perspective of the nuances inherent in the construction of one's mathematics identity.

Although this study involved only two Black women paraeducators who did not always coincide in the mathematics identities they enacted, there was overlap in their teaching experiences. In their teaching experiences, both women told of discrimination they faced with partner teachers or administrators, and of observing their students, particularly students of color, being taught inequitably. These injustices in the mathematics classroom can certainly be barriers to Black women paraeducators being able to execute their teaching responsibilities effectively, which can have negative, far-reaching consequences. This can impact: (1) their ability to teach

their students well and; (2) the positive construction of their mathematics identities. But for Chantel and Kristina, it is possible that their strong mathematics educator identities could have acted as a safeguard against this, as they relied on positive self-efficacy in teaching mathematics to counter the injustice in their classrooms. They were audacious in rebuffing societal norms about being paraeducators, choosing instead to pursue equity when it was absent. Chantel and Kristina used the connections they had with their students and their families to embody *in loco parentis*¹³, stepping in as mothers for their students.

Prior to engaging with the data, I expected to hear more explicitly about how their identities as Black women played a significant role in their experiences as mathematics educators. At times, both Chantel and Kristina referenced how their identity as Black women contributed to their positionality in the mathematics classroom. Other times, they were adamant that this positionality was not due to their Black womanhood, and instead might be due to their professional status as paraeducators. For instance, Kristina stated that being a Black woman did not impact her relationship to mathematics, neither as a student nor as an educator. Nevertheless, she highlighted an interaction in which she called out her partner teacher for not properly teaching their Black students. Similarly, Chantel stated that she did not think the poor relationship dynamic between her and her partner teacher had to do with race. But later on in the interview, Chantel mentioned the lack of respect given to her by her partner teacher that she believed was due to her being a Black woman, as well as the lack of “justice for our kids” regarding how the teacher taught Black students (Chantel interview, response 19). It was unclear how to parse out the impact of the different aspects of their identity on their mathematical identity development. However, one possible way to interpret this data might be that these initial

¹³ Latin for “in the place of a parent.” This refers to a common legal doctrine in which some person or organization has legal responsibility to perform parental functions.

responses that race was not a factor in the classroom may have been due to the fact that the paraeducators were uncomfortable stating this as fact. Another interpretation could be that since race is so salient in their day-to-day lives as Black women, its consistency in their lives may prove difficult to identify explicitly. Moreover, in their descriptions of the poor professional relationships they had with their respective teachers, they may have conflated these negative interactions with their role as Black women paraeducators. It is difficult to parse out exactly if these interactions were due to them being Black women or paraeducators, or both. What is clear, however, is that for Kristina and Chantel, they existed in these classrooms as triply marginalized: as women, as Black people, and as paraeducators. As Crenshaw demonstrated with her theory of intersectionality (1989), these identities cannot be separated - together, they tell a robust and complete story of both women's mathematical experiences.

Conclusions

The purpose of this study was to answer the following question: *How do remembered histories of mathematics learning shape Black women paraeducators' identities?* Chantel's narratives about her mathematical experiences were largely positive, and this positive relationship to mathematics may have been associated with her strong mathematical identity, both as a student and as a paraeducator. Kristina's narratives were different. Her relationship with mathematics was strained, and her overall negative association with the subject may have been decisive in the construction of her mathematical identity. Even beyond schooling, Kristina's mathematical identity as a learner was weak. However, her narratives about her relationship to mathematics teaching pointed to a strong mathematical identity as a paraeducator, supporting the idea that identity is constructed in interaction with the social context.

With the passage of the Every Student Succeeds Act of 2015 (ESSA), paraeducators have seen their role go through a significant transformation. They are not simply “individual[s] employed in school[s] under the supervision of a certified or licensed teacher” (Elementary and Secondary Education Act of 1965) tasked with menial jobs. More and more often, paraeducators are seen as co-educators responsible for instruction alongside their partner teachers. As such, it is vital that researchers begin exploring how their new role as educators influences their educator identity. This is one of the research objectives of the *Doing the Math with Paraeducators: Enhancing and Expanding and Sustaining a Professional Development Model in PreK to Grade 3 Math Classrooms* study: to investigate if there are changes in paraeducators’ self-efficacy as it relates to teaching mathematics. In the substudy presented in this paper, focusing on Black women paraeducators specifically, I sought to narrow in on their entire mathematics experience from childhood to present day to explore the nuances of these participants’ journeys as mathematics students and educators. There is a paucity of research on paraeducators, and even less knowledge about Black women paraeducators. This research is important because of the unique opportunity to add to a growing body of literature using an intersectional approach. For Chantel and Kristina who work in a school where the vast majority of students are students of color as they are, there is a personal connection in which they see their students as their own children, and are especially motivated to support their knowledge and learning. In uplifting paraeducators so they have a strong self-concept when teaching mathematics, students who are most marginalized and at risk for encountering injustice in the classroom could reap the benefit of having multiple, capable mathematics educators teaching them.

In addition to providing a perspective about Black women paraeducators’ mathematical experiences, this study is useful in taking a macro view of how paraeducators are navigating the

changing landscape of the mathematics classroom. Additionally, it can illuminate new opportunities for the supporting roles that paraeducators can play. While ESSA requires that paraeducators have access to professional development to strengthen their instructional skills, oftentimes paraeducators are underutilized. Further, paraeducators should be included in planning time with their partner teacher, which was something both Chantel and Kristina lamented. In fact, their participation in the *Doing the Math* study required collaborative planning time with their partner teachers in order for them to participate, and according to Kristina, this dedicated time only happened because of the study. Because the paraeducator role is still so deeply rooted in its original mission to complete subservient tasks, districts should push for teacher training to clarify the new role of the paraeducator. Both teachers and paraeducators are responsible for instruction, and this shift in their role should be reflected in the classroom. Such training is conducive to ensuring paraeducators have authority in the mathematics classroom in an equitable way and would assist in undoing the notion that paraeducators are inferior to teachers. Lastly, schools should offer opportunities for community for paraeducators. Both Chantel and Kristina pointed to the sense of community with other paraeducators as being motivators for them to continue in their journey as educators through the *Doing the Math* professional development sessions. For Chantel, being a leader amongst her fellow paraeducators bolstered her sense of self as an educator. Similarly, Kristina's second attempt at college was underscored by the kinship she had with paraeducators and other early childhood educators who were enrolled in the same program as her. This sense of community is imperative for paraeducators as they look to find commonality with a cohort of peers who are encountering comparable situations in the mathematics classroom. In providing professional development, planning time for paraeducators and their teachers, training for teachers, and a professional

learning community for paraeducators, districts can directly and positively contribute to the positive development of paraeducators' mathematics teaching identity, as this training will give them time to engage properly with the pedagogical content, thereby increasing their confidence as mathematics educators, and finally, deepening their sense of self as mathematics learners and teachers.

There are several limitations to this study. One major limitation was the small sample size of two participants. While tightening the focus on the interviews that were the most rich, these case studies describe the experience of two participants, and cannot be generalized to the larger Black women paraeducator experience. Further, in exploring how the two paraeducators' mathematics teacher identities were constructed, I learned that Chantel had more teaching opportunities versus Kristina, and this was due to the difference in co-educator relationship they had with their respective partner teachers. This disparity in teaching opportunities may have influenced the development of each woman's mathematics teaching and learning identity. As was mentioned above, it was difficult to delineate how these women's identities as Black women and their professional identities as paraeducators impacted their mathematical identities construction. Future studies should consider exploring how strong partnerships between Black women paraeducators and Black women teachers are developed and established to investigate the relative contribution of their professional roles versus their identity as Black women to their mathematical identity.

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Appendix

Table 1. *Interview protocol.*

Thank you again for agreeing to talk with me. I expect that our interview will last about 45 minutes, and it will be audio and video recorded. This is all about your experience with mathematics, so please be as honest as possible. If at any point you'd like to stop the interview, please let me know and we will end it. Do you have any questions before we get started?
K- 12 Mathematical Background

1. Please state your name and the grade you teach.	
2. Did you pursue any additional schooling? If so, what did you study?	
3a. Where did you grow up and attend school?	<i>3b. For paras with immigrant backgrounds:</i> <i>What are some differences you've observed in your students' math experiences and your own?</i>
I'd like to hear your feelings about and engagement with mathematics throughout your school experience.	<i>4a. Let's start with grades K-3...what do you remember about that period of time?</i> <i>4b. What about upper elementary school?</i> <i>4c. Middle school?</i> <i>4d. High school?</i>

5. Did you learn and apply math at home? If so, can you share some examples of what that looked like? 6. Can you remember what parts of math class you were interested in? 7a. What parts of math were not interesting to you?	4e. Did you have any choice around which math courses you could take? If you did, which sorts of courses did you choose? 4f. Did your school place students into different tracks: standard, honors, advanced? • Which track were you on? • Do you believe this was accurate based on your ability? 4g. Do you think this tracking had any effect on your educational pursuits after high school? 7b. Are you comfortable or uncomfortable in math class? Tell me more.
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	7c. <i>Do you look forward to math class? Why or why not?</i>
8a. What was your attitude towards math in elementary school? Middle? HS?	8b. <i>Can you think of how your attitude changed as you progressed through different grades? Why did it change?</i>
Mathematical Identity	
9a. Do you consider yourself good at math?	9b. <i>What does it mean to be good at math?</i>
10a. Do you think other people in your life see you as a “math person”?	10b. <i>Can you share some examples of this?</i>
11. Is it possible to improve your math ability? How do you know?	<i>If paraeducator speaks hypothetically, prompt them to answer personally.</i> • <i>If “yes”: what can a math learner do to improve their skills?</i>
12. What does it mean to enjoy math?	
13. Do you enjoy math?	
Black Women & Mathematics	

14a. Why did you decide to be a paraeducator?	<i>14b. Given your math journey beforehand, what sorts of feelings did you have about working in mathematics as a paraeducator?</i>
15. How does being a Black woman affect your career choices?	
16a. Did you ever feel that your race impacted you as a math learner? If so, how?	<i>16b. On a scale from 1-6 where 1 is not at all and 6 is a great deal, how much did your gender influence your sense of yourself as a math learner?</i>
17a. What about your gender?	<i>17b. Using this scale again, how much did being Black influence this?</i>
18a. In what ways does being Black come into play in your math class?	<i>18b. How would you rate this?</i>
19a. What about being a woman?	<i>19b. How would you rate this?</i>

20. Are there specific experiences you've had in mathematics due to being a Black woman? 21a. How does being a Black woman impact your relationship with your students? 21b. What about with your partnering teacher? 21c. Your administrators? 21d. Your district? 22. Have you ever considered being a mathematics teacher? Why or why not? 23. What do you think of when you hear "mathematics community"? 24. Do you feel like you have a place in the mathematics community? Why or why not?	
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Chapter 6: Discussion and Conclusions

Introduction

The central goal of this dissertation was to explore the many factors that are involved in the construction of mathematical identities for K-12 students and educators. In conducting three separate empirical studies, I sought to develop an understanding about how and why people behave in mathematics contexts the way they do, and further, what sorts of mathematical identities they enacted or positioned others into as a result. I used two case studies and data from a CTE to answer this research question. All of the empirical studies supported the idea that mathematical identity construction does not occur *sans* outside influence; it always involves social interactions with others. As I illustrated in the previous chapters, the social influence in the construction of mathematical identities has implications for instruction and research which I will briefly expound on below.

This dissertation addressed the following problem statement: Knowledge about mathematical identities construction is crucial to gain insight into how people view themselves and others relative to their learning and doing of mathematics. In this chapter, I will take a macro view of the dissertation in answering the research question anchoring my work: *What are the factors which interact with each other to influence how K-12 mathematics students and educators take up specific mathematical identities?* Next, I will discuss the intellectual contributions and implications for future research from each empirical study, as well as for the entire dissertation.

Intellectual Contributions and Implications for Future Research from Chapter 3

In Chapter 3, the research question was: *What are the factors associated with a Nigerian-American girl's mathematical experiences, and how do they inform the construction of*

her mathematical identities? For Crystal, these factors were interactions with others that conveyed particular expectations about what it meant for her to be a successful mathematics student as a Black girl of Nigerian descent. The intersectionality of Crystal's social identities as a Black girl hailing from a Nigerian family made it such that her mathematical identity construction was impacted in a complex way. Crystal was not just a Black girl - she attended a public school where the majority of students were White, and her parents were proud immigrants from Nigeria, with their own ideals about attaining success in the United States. While there are certainly commonalities in Crystal's and other Black girls' experiences and self-perceptions as mathematics learners, her experience is her own. As such, this study adds to the literature by showing the nuance and variation in Black girls' mathematical experiences and identity construction. Future research should thus consider how social identity impacts and informs the construction of one's mathematical identity.

In my initial interviews with Crystal, she had very rigid, negative beliefs about mathematics, despite being a high-achieving student. Her affect about mathematics was extremely pessimistic but was nonetheless a motivating factor for her to continue to pursue high mathematical achievement. This study illuminated how epistemic affect can change depending on the context, again dependent on interactions with others. She had positive relationships with her mathematics teachers who made mathematics class enjoyable. Over the course of our interviews, it was evident that Crystal's teachers taught mathematics in ways that led to a changed comprehension of her parents' messaging; in perceiving mathematics as fun, she would find it pleasant and be motivated to seek to increase her mathematical knowledge. While her varying epistemic affect about mathematics did not necessarily lead to a change in academic performance in terms of achievement, there was a shift in Crystal's mathematical learning

experience, which appeared to be associated with the interplay between messages from her parents and her classroom experiences

Traditional literature on affect about mathematics tends to consider it as separate and external from mathematical thinking (Zan et al., 2006). This study confirmed Zan et al.'s (2006) research and showed that affect is indeed involved and a part of the development of students' mathematical learning. This has implications for future research in that researchers should aim to investigate the role of students' emotions while doing mathematics in terms of their beliefs about their mathematical capabilities, and henceforth, their mathematical selves.

Intellectual Contributions and Implications for Future Research from Chapter 4

The research questions for Chapter 4 were: (RQ1) *What do we learn about Wood's (2013) framework of mathematical micro-identities when we explore how it plays out in a new classroom context?* And (RQ2) *What are the connections between Wood's (2013) moment-to-moment positioning and enactment of mathematical micro-identities and how a student enacts influence?* Using data from a CTE, I illustrated how a teacher-researcher's responses to different students' mathematical ideas were connected to certain mathematical micro-identities she positioned them into. This study built off of Brizuela et al.'s (2023) paper, which focused on how one student was able to gain mathematical authority among her peers. In this study, I focused on the same student and their peers to identify a relationship between how a teacher-researcher responded to their ideas, and how these responses pointed to specific mathematical micro-identities she positioned students into.

Given teacher-researchers' discretion in designing CTE lessons, future researchers could utilize other methods in their examination of micro-identity construction. Additionally, while there was a focus on how the teacher-researcher positioned students into numerous

micro-identities, it would be compelling for other research to probe how students position others into micro-identities as well. Lastly, other research into micro-identity construction should consider the role of agency to discern how and why students enact the micro-identities they are positioned into by others.

Intellectual Contributions and Implications for Future Research from Chapter 5

In Chapter 5, I probed the mathematical histories of two elementary Black women paraeducators in order to gain insight into which mathematical identities they enacted in their current role as paraeducators. The research question was: *How do remembered histories of mathematics learning shape Black women paraeducators' identities?* This case study of two Black women paraeducators showed how the social interactions they had with others in their youth held momentous power in the mathematical identities they took up as educators. For Chantel, her recollection of positive relationships with trusted adults in K-12 mathematics who positioned her as mathematically capable was potent in her self-perception as a mathematics educator. Kristina had contrasting experiences as a child, which contributed to the opposite mathematical identities she had as both a learner and educator. I assert that while Kristina's K-12 mathematics education history (which was fraught with poor, negative experiences navigating mathematics) was a dominant factor in her weak mathematical identity as a learner, it was instrumental in her work as an educator. She described approaches she took in supporting her students' mathematical learning and thinking that were antithetical to her own K-12 mathematics journeys. Both Chantel and Kristina conveyed a deeply confident mathematical identity when working with their students, and this stemmed from their past experiences as students themselves. This study adds to research which posits that teachers' childhood mathematical experiences (Kaasila, 2007; Kaasila et al., 2008; Black et al., 2009) have a "central meaning for

their views of mathematics and their mathematical identities” (Lutovac & Kaasila, 2012), which also include teacher mathematical identity.

There has been a burgeoning development of research on teacher mathematical identity, but there is a massive dearth, however, on paraeducators’ mathematical identity. In adding this study to a new body of literature, paraeducators’ often invisible experiences and contributions in the mathematics classroom can finally be revealed. Moreover, in shedding light on their experiences, this study is an opportunity for researchers to learn about the connections between paraeducators’ mathematical teacher identity and practice. Chantel’s and Kristina’s interviews provided evidence that the paraeducator role in the classroom has shifted from the historical role that saw them as responsible only for menial tasks; they were now charged with instructional duties and, as I mentioned above, their personal mathematical histories were central to their teaching. Having a strong mathematical teacher identity influenced the mathematical behaviors of both women, which has implications not only for their students’ mathematical comprehension, but their own mathematical identity development.

Given this dearth of research, future research should expand and focus on a larger data set in order to explore where there are common features and differences in the paraeducator experiences to gain a more comprehensive understanding of the paraeducator experience in mathematics classrooms. Both Chantel and Kristina were veteran paraeducators who were employed at their schools for over nine years each, which adds a layer of experience and privilege that others could inspect to further scrutinize how paraeducators’ mathematics experiences might differ between an expert versus a novice. Lastly, Chantel’s and Kristina’s identities as Black women were indispensable in their role as paraeducators. In some ways, it contributed to deep bonds with their students. In others, it was an obstacle in fostering solid

relationships with their partner teacher. Researchers seeking to answer a similar research question with Black women paraeducators especially could seek out teacher-paraeducator partnerships where both people are Black women to probe how their shared identity as Black women contributes to the development of their mathematical identity.

Answering the Research Question and Implications for Future Research

I end with an answer to the dissertation's research question: *What are the factors which interact with each other to influence how K-12 mathematics students and educators take up specific mathematical identities?* Throughout each empirical study, a repeated finding was how powerful interactions with others were for each participant's mathematical identity construction, and these constructions were contextually situated. In seeking to understand why and how people either take up or position others into specific mathematical identities, it is crucial that researchers center their work on social interactions. As I have established across Chapters 3 through 5, these interactions include direct interactions with others. These interactions occurred both within and outside of the mathematics classroom, with classmates, teachers, and even with societal messaging which, in some cases, proved to be burdensome to overcome. Further, these interactions shifted as the context shifted, confirming Wood's (2013) argument that micro-identity, which fluctuates due to their "enact[ment] in a moment of time" (p. 778), is a better conceptualization of people's mathematical selves than macro-identity.

Additionally, while the context and participants of each empirical study varied, in comprehending how mathematical identities were constructed, their experiences in doing so can be understood as occurring on a time-scale, wherein each construction was affected by time. In the first and second empirical studies, I explored how mathematical identities were developed over the course of a short span of time (i.e., one school year for Crystal, moment-by-moment for

students in the CTE). As such, there was more variation in the mathematical identities these participants enacted or were positioned into. Conversely, for Chantel and Kristina, their narrative histories spanned decades, and as I illustrated, this was associated with a thickening of their mathematical identities as learners, specifically. Mathematical identity research is widely popular and established, but ensuing research ought to concentrate on momentary identity construction and the breadth of mathematical identities one may take up, as well as how these momentary identities may lend itself to more stable, long-term constructions of what kind of mathematical person someone is.

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