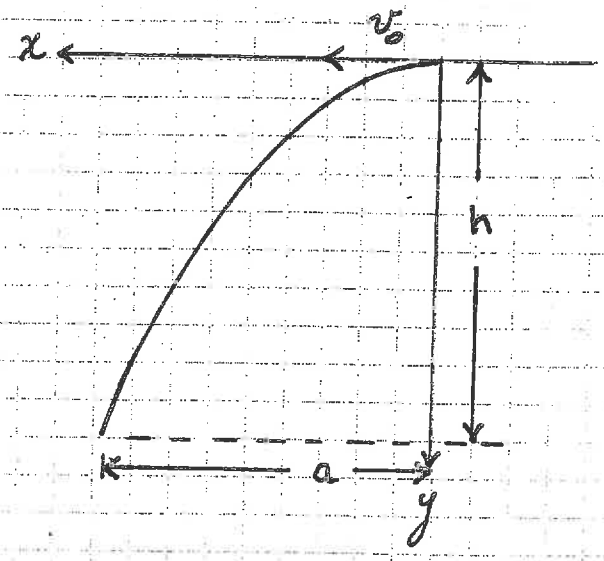
Our Way

$$x = v_0 t \qquad y = \tfrac{1}{2} g t^2$$



$$y = (g/2v_0^2)\,x^2$$

$$\frac{dy}{dx} = (g/v_0^2)\,x = \tan\theta$$

$$v = \sqrt{v_0^2 + 2gy}$$

at impact, $y = h$, $x = a$

$$h = (g/2v_0^2)\,a^2$$

$$\tan\theta_{imp} = \cancel{h/a} = (g/\cancel{2}v_0^2)\,a$$

$$v_{imp} = \sqrt{v_0^2 + 2gh}$$

$$= \sqrt{v_0^2 + (g^2/v_0^2)\,a^2}$$

$$= v_0\sqrt{1 + (g^2/v_0^4)\,a^2}$$

$$v_{imp} = v_0\sqrt{1 + \tan^2\theta_{imp}} = v_0/\cos\theta_{imp}$$

***Sagredo*: The theory of compounding these different impetuses and of the quantity of impetus that results from such mixing is so new to me as to leave no little confusion in my mind. I speak not of the mixing of two equable movements, one along the horizontal line and the other along the vertical, even though unequal to one another; for as to this, I quite understand that a motion results which is equal in the square to both components of it. But I am confused by the mixture of equable horizontal and naturally accelerated vertical [motion].**

***Salviati*: We can reason definitively about movements and their speeds or impetuses (whether these are equable or naturally accelerated) only if we first determine some standard [*misura*] that we can use to measure such speeds, as also some measure of time. As to the measure of time, we already have universal agreement on hours, minutes, seconds, etc.; and just as the measure of time is for us that one in common use, accepted by everybody, so it is necessary to assign some measure for speeds to be commonly understood and accepted by all; that is, one that will be the same for everyone.**

As explained previously, the Author deemed suitable for such a purpose the speed of naturally falling heavy bodies, of which the growing speeds keep the same tenor everywhere in the world. ... To determine and represent this unique impetus and speed, our Author has found no better means than to make use of the impetus acquired by the moveable in a naturally accelerated motion. Any acquired momentum, turned to equable motion, retains its limited speed precisely, and it is such that in another time equal to that of the descent, it will pass through exactly twice the distance of the height from which fall took place.

Prerequisites for *height* of fall from rest to serve as a *proxy* for purposes of representing and measuring *velocity squared*:

1. **Speed acquired in descent from rest is proportional to the time of descent – i.e. descent involves uniformly accelerated motion.**

2. **The same speed is acquired in descent from rest from a given height regardless of the path of descent – i.e. pathwise independence of speed acquired.**

3. **All bodies acquire the same speed in descent from any given height regardless of their weight, shape, composition, etc. – i.e. the only variable that makes a difference to the speed acquired is the height of descent.**

In order for *height* to serve as a uniform, universal measure of *velocity squared*, and not just a local measure:

4. **The increments in speed acquired in equal times in direct vertical fall are the same everywhere – e.g., the distance of fall in the first second is the same everywhere on earth.**