# Comparing different levels of interaction constraints for deriving visual problem isomorphs

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## ABSTRACT

Interaction and manual manipulation have been shown in the cognitive science literature to play a critical role in problem solving. Given different types of interactions or constraints on interactions, a problem can appear to have different degrees of difficulty. While this relationship between interaction and problem solving has been well studied in the cognitive science literatures, the visual analytics community has yet to exploit this understanding for analytical problem solving. In this paper, we hypothesize that constraints on interactions and constraints encoded in visual representations can lead to strategies of varying effectiveness during problem solving. To test our hypothesis, we conducted a user study in which participants were given different levels of interaction constraints when solving a simple math game called Number Scrabble. Number Scrabble is known to have an optimal visual problem isomorph, and the goal of this study is to learn if and how the participants could derive the isomorph and to analyze the strategies that the participants utilize in solving the problem. Our results indicate that constraints on interactions do affect problem solving, and that while the optimal visual isomorph is difficult to derive, certain interaction constraints can lead to a higher chance of deriving the isomorph.

**Keywords:** Interaction, Visual Isomorph, Problem Solving

## 1 INTRODUCTION

In his insightful keynote address to the EuroVis 2009 conference, Pat Hanrahan discussed how visualization plays a role as a system of thought [3]. One remarkable topic of his talk is the use of visual problem isomorphs to make complex problem solving seem simple and often trivial. The examples Dr. Hanrahan provided demonstrated the idea that once the right visual representation of the problem isomorph is found, solving that problem can be as simple as looking at the visual representation and identifying the right answer immediately. Of particular interest to us is the example of using a "magic square" as a visual isomorph to the Number Scrabble game (which is discussed further in section 2.1). It is clear that by transforming the Number Scrabble game into a magic square, this relatively difficult game of finding and adding multiple numbers becomes as simple as playing a game of tic-tac-toe.

This example is compelling because the process of encouraging a user to discover a useful visual isomorph for a problem can be thought of as the primary goal of visualization. However, the obvious question is, how does someone find the right visual isomorph to a problem? Unfortunately, the answer to this question is not trivial or well-understood. Building on work in cognitive science and diagrammatic reasoning [8], we argue that helping a user find a useful visual isomorph is not just a matter of presenting an appropriate

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visual representation. Rather, people can best discover visual solutions to problems through interaction with visual representations. Unfortunately, while visualization researchers understand how to design visualizations to represent data, they have not exploited the relationship between interaction and problem solving to the same extent as cognitive scientists.

The goal of the research presented in this paper is therefore to bridge the gap between the findings in the cognitive science community and the visualization community. Specifically, we acknowledge research in the cognitive science community showing that interaction plays a critical role in problem solving [8]. However, given our emphasis on visualization, we do not simply seek to corroborate their existing findings. Instead, our interest lies at the intersection of these two fields where we look to understand how using interaction to solve problems can lead to the identification of potential visual isomorphs.

First we extend the notion that interaction generally facilitates problem solving. However, we further hypothesize that interactions with different constraints and amount of encoded information will lead to different solutions to the problem. In addition, we hypothesize that during the problem-solving process, the different constraints on interaction will lead to different types of isomorphs, both visual and non-visual. Finally, we hypothesize that these different types of isomorphs have varying degrees of effectiveness in solving the problem, which can be measured quantitatively.

To test our hypotheses, we conducted a user study in which 117 participants were given different types of interaction constraints while developing strategies for the Number Scrabble game. We chose to use the Number Scrabble problem because it is self-contained and is known to have an optimal visual problem isomorph in the form of the magic square [17]. The participants' accuracy and time in playing the game against a computer were logged and tracked, and their strategizing session video-recorded. Based on the data obtained from the study, we find that: (1) different constraints on interactions do affect the participants' performance while playing the game, (2) with more constraints, the participant has a higher chance to derive the optimal visual isomorph (the magic square), and finally, while not all participants were able to derive the optimal visual isomorph, (3) using visual isomorphs in general leads to better performance than using non-visual isomorphs.

We begin by reviewing related work on interaction and problem isomorphs in the context of problem solving. Next, we present our experiment exploring the effect of interaction constraints on deriving visual problem isomorphs. We then discuss the implications of our experimental results and limitations of the study.

## 2 RELATED WORK

Our conception of visualization as providing externalizations for problem solving draws on work in visualization theory as well as cognitive science. In particular, we study how visual representations can provide useful isomorphs of the information they visualize. Two problems or representations are isomorphic if they are informationally equivalent but present that information in different

structures. As an example, we use the Number Scrabble problem and its isomorphic magic square representation.

## 2.1 The Number Scrabble problem

The original Number Scrabble [17] is a game played by two people with nine cards: ace through nine. The cards are placed in a row, face up. The players draw alternately, one at a time, selecting any one of the unselected cards. The objective of the game is for a player to get three cards which add up to 15 before his opponent does. If all nine cards have been drawn without either player having a combination that adds up to 15, the game is a draw.

The main reason we chose to use the Number Scrabble game is that there is a known visual isomorph of the problem called the "magic square" (figure 1). Since the magic square visually represents all possible combinations of three numbers that can be added up to 15 in a succinct manner, it can significantly help a player to perform well at the game. In other words, once this visual isomorph is identified, the Number Scrabble problem is turned into a much simpler tic-tac-toe game, which is played by two players who take turns marking the spaces in a 3*3 grid. The number scrabble game represents a large number of well-defined problems that show how visual isomorphs can make evident what was previously true but obscure [17].



Figure 1: 3x3 magic square

## 2.2 Isomorphs and diagrammatic reasoning

Simon defined problem isomorphs as problems whose solutions and moves can be placed in one-to-one relation with the solutions and moves of the given problem [17]. The key to isomorphism is that even when two representations contain the same information, they can still provide very different sets of operations for accessing and inferring about that information, which can make a given problem easier or harder to solve [13]. In our example, the magic square and number scrabble are isomorphs of the same problem in that they both contain all the information needed to play the game. However, in number scrabble, the operations provided to the player to access important information about the game—such as whether your cards contain a winning combination—are mathematical. In the magic square case, that information is contained in a visual operation: seeing whether the cards form a line across the magic square grid. Since the brain processes such visual operations faster than mathematical ones, the visual isomorph is more efficient in this case.

The idea that visual representations make certain operations more efficient to perform is at the core of the theory of diagrammatic reasoning [2, 13]. However, efficiency is not the only measure of interest in visualization; our goal is to make information not just accessible, but understandable. The distinction between these goals is highlighted by Carroll et al. [1], who had participants solve a design problem presented as one of two isomorphs: a spatial arrangement problem and a temporal scheduling problem. The spatial isomorph was easier and faster for participants to solve and led to fewer failures to understand the problem. That is, in the temporal case there were several participants whose solutions did not follow the requirements of the task. Interestingly, when participants in both cases were provided a simple graphical representation (a grid) in which to work on their solution, the temporal case was as easy

to solve as the spatial one, but participants in the temporal case remained more likely to fail to understand the problem requirements. The authors took this to mean that appropriate graphical representations can make problems easier to solve, but not necessarily easier to understand.

Another way to interpret this is that there is more to designing a visual isomorph than making information more efficient to access. Much of the power of visual representations comes from how they set constraints on interpretation and reasoning. Constraints inherent in visual isomorphs can encode constraints on the information they represent, leading to a more direct preservation of information structure [15]. As Stenning and Oberlander [18] argue, these constraints inherent to visual representations help to meaningfully restrict the number and kinds of inferences that can be made about a problem, focusing processing power on only valid cases. In this way, visual isomorphs not only can make operations more efficient, but can also model the constraints of a problem directly. This can affect the difficulty of solving a problem by reducing the cognitive load of remembering rules [12] or by encouraging different types of strategies [4].

## 2.3 Interaction and problem solving

While visual representations can aid problem solving significantly on their own, they gain even more power to model a problem when interaction is introduced. Interaction is increasingly seen as central to the process of reasoning with visualization [14, 16, 20]. Lending weight to the intuition that interaction improves reasoning, Hundhausen et. al [5] found that interacting with an algorithm visualization produces better understanding than viewing an equivalent animation.

We use the term "interaction" in the broad sense defined by Yi et al.: "the dialogue between the user and the system as the user explores the data set to uncover insights" [21]. In this sense, the relationship between interaction and problem solving has been the subject of much research by cognitive scientists in the field of distributed cognition [6]. In particular, David Kirsh has argued extensively that projection and interaction with external representations are fundamental to human reasoning [7, 8, 9, 10, 11]. Kirsh points to the pervasive use of external representations and interaction with the world in everyday problem solving, and identifies several functions performed by interaction in the reasoning process [8]. Of these, most relevant to our work is *reformulation*, or the ability to restate ideas. Kirsh sees reformulation as a process that is frequently too complex to perform entirely in memory, and so is often managed with external tools. Since reformulation is closely related to identifying different problem isomorphs, we argue that this process can also be made easier through certain types of interaction.

## 3 HYPOTHESES

Our research objective is to investigate the question of how constraints on interaction affect problem solving through the derivation of visual isomorphs. We propose that in developing a strategy for playing a game like Number Scrabble, participants will tend to derive an isomorph for the problem that is easier for them to use than the representation in the original game, and that the availability of different levels of interaction while strategizing will lead to different types of isomorphs. If this is the case, it can help to clarify the relationship between interaction with visual representations and reasoning. To what extent does the nature of a visual representation, and the type of interactions a user is allowed to perform upon it, affect the kind of strategy that user develops for solving a problem?

We therefore designed a study based on the aforementioned Number Scrabble game due to its known optimal visual isomorph, the magic square. In our study, we developed 5 different interaction conditions, ranging from free-form to very restrictive, and studied

Figure 2: Number scrabble game interface

how strategizing under these conditions affects problem solving and the development of isomorphs. In particular, we propose three interrelated hypotheses concerning interaction, problem solving, and isomorphs:

1. **Interactions and Problem Solving**: We hypothesize that different types of interactions will affect the participants' performance in playing the Number Scrabble game. Specifically, we hypothesize that more constrained interactions can encode more information, and will therefore lead to better problem solving.

2. **Interactions and Isomorphs**: We hypothesize that the different constraints on interaction will affect the isomorphs generated by the participants. With higher constraints on interaction, a participant will be more likely to derive the optimal visual isomorph (the magic square).

3. **Isomorphs and Problem Solving**: Based on Larkin's definition on isomorphism [13], we hypothesize that not all isomorphs developed by participants will be visual, but that visual isomorphs will be more effective for playing the Number Scrabble game.

## 4  EXPERIMENT DESIGN

The main factor of interaction constraint had five levels (no interaction, pen and paper, multiple sets of cards, single set of cards, and boundary). Details of each constraint and design rationale will be discussed in section 4.3. We used a between-subjects design with repeated measures. Each subject is randomly assigned to one of the five interaction constraint conditions which determines what interactions are available to them during their strategy session. Qualitative measures in our experiment are the types of isomorphs our subject derived during their strategy session. Quantitative measures involved response time and scores on Number Scrabble games played against a computer, using the game interface shown in Figure 2. The computer was programmed to play the game optimally so that it never loses. While our subjects played the game against the computer, we recorded the number of games tied or lost and the time it took them to figure out the next move for response time. We alternated who makes the first move between the subjects and the computer for every game played.

### 4.1  Participants

We recruited a total number of 117 participants (86 Male, 31 Female) from introduction to computer science courses at our university. Participants' age ranged from 18 to 40 with median of 25. Students were primarily undergraduates, and 80% were in computing-related majors.

### 4.2  Task

The experiment began with investigators introducing the Number Scrabble game to the subjects based on a training script. The investigators were asked to play the game with the participants until they fully grasped the rules. Next, the participants filled out a demographic form on age, gender and experience with mathematical courses through a web interface. The rest of the experiment was divided into four major sessions: pre-test, strategizing, externalizing isomorph, and post-test.

1. **Pre-test**: During the pre-test session, the participants were asked to play the Number Scrabble game six times against the computer. To make sure that our participants did not start developing strategies during the pre-test, we enforced a maximum time limit of 18 minutes to finish all six pre-test games. Failing to meet the time limit resulted in a participant's data being dropped from analysis.

2. **Strategizing**: During the strategizing session, the subjects were given 20 minutes to interact with the materials we provided under the different interaction constraints and were told to look for a strategy that can help them play the game better.

3. **Externalizing isomorphs**: At the end of the strategizing session, all participants were given 2-3 minutes to make a "cheat sheet" out of the strategy they developed so that they could refer to it during the post-test session when they play Number Scrabble again. This cheat sheet was a single sheet of paper onto which participants were told they could write anything they felt would help them play the game (in the case of the pen and paper condition, this was a separate sheet from those they wrote on during the strategizing session.) This gave us a record of the isomorph used by participants in forming a strategy and reduced the cognitive load on participants during the post-test. We gave them a very short amount of time to make their "cheat sheet" so that they could not continue elaborating on it after the end of the strategizing session.

4. **Post-test**: During the post-test session, participants were asked to play the Number Scrabble game six more times against the computer while consulting their "cheat sheet." To be consistent with the pre-test, and also to make sure that the participants do not refine their isomorphs during the post-test, 18 minutes was set as the upper limit for playing all six games. As in the pre-test, failing to meet the time limit resulted in a participant's data being dropped from analysis.

After the post-test session, participants were asked to fill out a questionnaire regarding how they arrived at their strategy and their experience during the strategizing session. The investigators collected all the participants' "cheat sheets" for further analysis of the isomorphs they derived during the experiment. In addition, the strategizing sessions were video recorded, which allowed us to examine how the interaction constraints affected our participants' behavior during the process of searching for an isomorph.

### 4.3  Interaction constraints

We went through multiple rounds of a refining process to design the interaction constraint conditions used in our study. Our goal was to design constraints that ranged from placing no limit on the interaction to restricting the interaction a great deal.

- Constraint #1 (no interaction): The participants were asked to think about the problem in their head during the strategizing session to develop a strategy to help them play the game better. The participants were not allowed to interact with any materials.

- Constraint #2 (pen and paper): The participants were provided with pen and paper to work out their strategy for the Number Scrabble problem.

- Constraint #3 (multiple sets of cards): The participants assigned to this constraint were provided with multiple sets of cards, with each set consisting of the numbers one through nine. Each card is square in shape and made from paper with the numbers printed on them. Within the strategizing session, the participants were encouraged to organize the cards freely.

- Constraint #4 (single set of cards): The participants were further limited to interact with only one set of cards labeled with the numbers one through nine.

- Constraint #5 (boundary): This is the most restrictive case. Participants were presented with nine cards and a square space only large enough to fit the cards in a grid, and were told to confine their interactions to that space. Figure 3 shows this condition.

In our study, the term constraint refers to not only confinement of interaction space, but also the amount of resource a user can have to externalize his/her mental states. Our conditions are designed so that "no interaction" serves as a control group and "pen and paper" represents no limit on user interaction. Then, based on both the original description of the Number Scrabble problem and the optimal visual isomorph, we derived the other three interaction constraints from "multiple sets of cards" to "boundary" by adding more constraints on interaction each time, all of which encode some information about the optimal visual isomorph of the problem.

## 5 RESULTS

When analyzing the experimental data, we were concerned with the impact of outliers due to random responses. Therefore, we trimmed out the data of four participants whose response times were unusually fast during the pre-test. In addition, 11 of our participants reached the 18-minute time limit during either pre- or post-test, thus their data were dropped automatically since their missing data made it impossible to compare fairly pre-test and post-test scores. As a result, we have valid data from 100 participants with 20 subjects under each interaction constraint.

### 5.1 Isomorph vs. Interaction constraint

Based on the strategies recorded on their cheat sheets, our participants developed a wide range of problem isomorphs during the experiment. Some of these were visual while the others were either mathematical or purely descriptive. We classified these isomorphs into five different categories:

1. **Magic square (Visual)**: The magic square isomorph.

2. **Partial magic square (Visual)**: Same layout as the magic square isomorph with different ordering or numbers.

3. **Other visual isomorph**: Visual isomorph but numbers are not organized in a 3*3 matrix manner.

4. **Permuted isomorph**: All possible combinations of 3 numbers adding to 15.

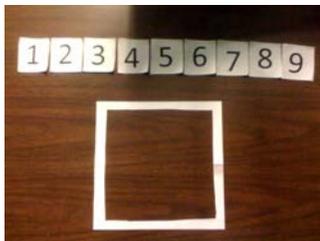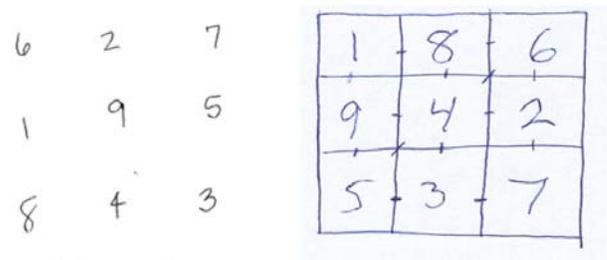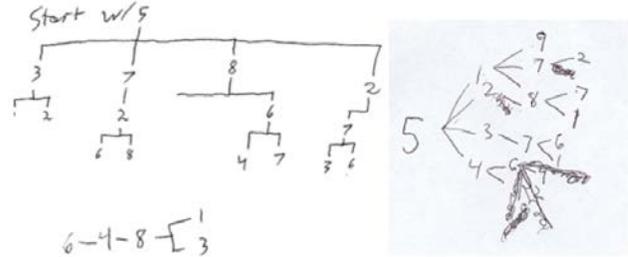5. **Incomplete isomorph**: Strategies that do not involve all 9 numbers.



Figure 3: Cards and Boundary



(a) Partial magic square examples



(b) Other visual isomorph examples



(c) Permuted isomorph examples

Figure 4: Isomorph examples

Note that categories 1–3 are visual isomorphs of the Number Scrabble problem while 4 and 5 are not. In addition, examples of different types of isomorphs are shown in figure 4.

The distribution of different isomorphs developed by our subjects within each interaction constraint is shown in Figure 5. This distribution supports our hypothesis in the sense that as the interactions become increasingly constrained (from pen and paper to boundary), more participants developed visual isomorphs of the number scrabble problem. More importantly, nine out of 20 subjects under the most restrictive constraint (boundary) discovered the optimal visual isomorph (the magic square), while another six subjects developed partial magic square isomorphs. In contrast, only one out of 20 participants in either the no interaction condition or the pen and paper condition discovered any visual solution. A Pearson's chi-square test of independence finds a highly significant interaction between interaction constraint and isomorph, $\chi^2(16, N = 100) = 116.9, p < .001$. Since 15 cells have an expected count of less than five, we performed a Fisher's exact test, which also yielded a probability of $p < .001$.

### 5.2 The effect of interaction constraints on Response Time and Score

Results regarding time and score were analyzed statistically using an analysis of variance (ANOVA) followed by Tukey's HSD (Honestly Significant Difference) test for pairwise comparisons. The
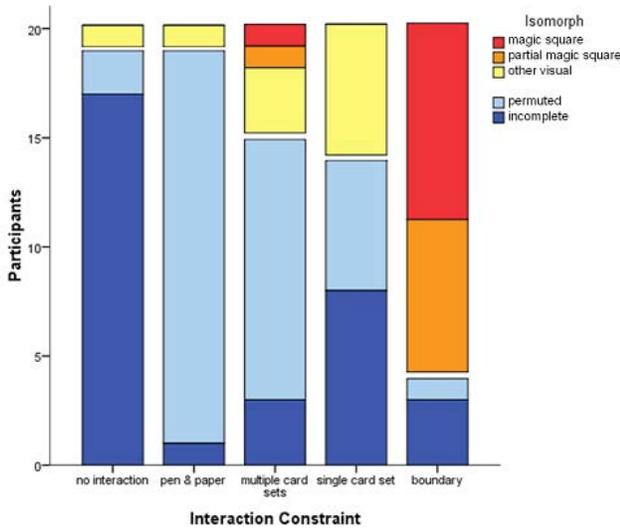
Figure 5: Distribution of isomorphs developed under five different interaction constraints. The gaps divide visual isomorphs (1,2 and 3) from non-visual isomorphs (4 and 5).

factor in our experiment was interaction constraint (five levels) and the dependent variables were difference in response time and improved score.

Difference in response time was derived from the time it took to decide which card to choose next at each move during a game. Response time per game was defined as the average time it took the participants to choose the next card during each game, $T = \sum ResponseTime/n$, with $n$ being the number of cards chosen following the opponent's move during a specific game. Since both the pre-test and post-test sessions comprise six games, difference in response time thus was defined as $IT = \sum_{i=1}^{6} T(i, posttest) - \sum_{i=1}^{6} T(i, pretest)$. In a similar vein, improved score was derived from whether the subjects tied or lost to the computer during each game, with tying counted as 1 point and losing as 0 points. Thus improved score was defined as $IS = \sum_{i=1}^{6} S(i, posttest) - \sum_{i=1}^{6} S(i, pretest)$.

### 5.2.1 Response time

We expected participants to choose the next card faster during the post-test as the interaction constraints increased, since we hypothesized that they would be more likely to derive a better visual isomorph similar to the "magic square". However, we did not observe a significant main effect of difference in response time ($F(4,95) = 1.54, p = 0.097$). Figure 6 (top) shows the difference in response time under different interaction constraints. However, interesting yet surprising findings emerged once we considered response time during pre-test and post-test separately. Figure 6 (bottom) shows the mean response time during both pre- and post-tests under the five interaction constraints. It should be noted that participants in the no interaction condition had an unusually slow average response time in the pre-test, which makes comparisons between that condition and the others problematic. In general, however, we found that most of our participants spent more time deciding which card to choose next during the post-test, and participants under the most confined constraints took the longest time, which ran counter to our expectations. We discuss possible reasons for this in Section 6.4.

### 5.2.2 Score

If we consider mean scores on the pre-test and the post-test separately, it is clear that in general our participants scored higher after the strategizing session under all five interaction constraints ($F(1,1190) = 57.7, \eta_p^2 = 0.046, p < .001$). More importantly, the subjects in the more constrained interaction groups tend to score higher than those in the less restrictive interaction groups.

For improved score (Figure 7), we observed a significant main effect of interaction constraint type ($F(4,95) = 6.5, \eta_p^2 = 0.215, p < .001$). Post-hoc tests showed that the improved scores were significantly different between numerous pairs of interaction constraints. To elaborate, the improved score for participants assigned to interaction constraint #5 (boundary) was significantly larger than that for participants assigned to interaction constraint #1 (no interaction), $p = .001$, constraint #2 (pen and paper) with $p < .01$, and constraint #4 (one set of cards) with $p < .01$. Although the result of other pairwise comparisons were not significant, we saw a clear trend (Figure 7) that as the interaction constraints become more restrictive, the improvement of score increases except in the case of constraint #4. We further analyze this unexpected "dip" in the discussion section.
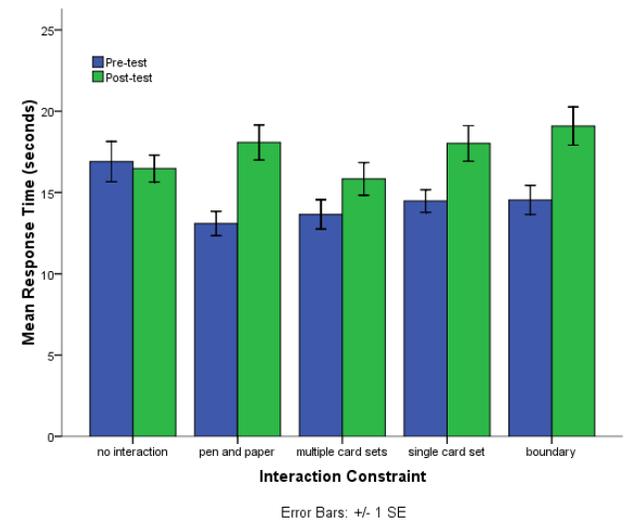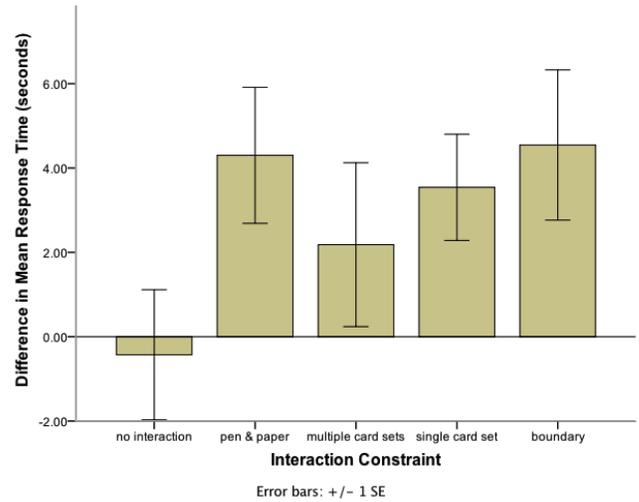




Figure 6: (top) Difference in mean response time; (bottom) mean response time(pre vs. post test)
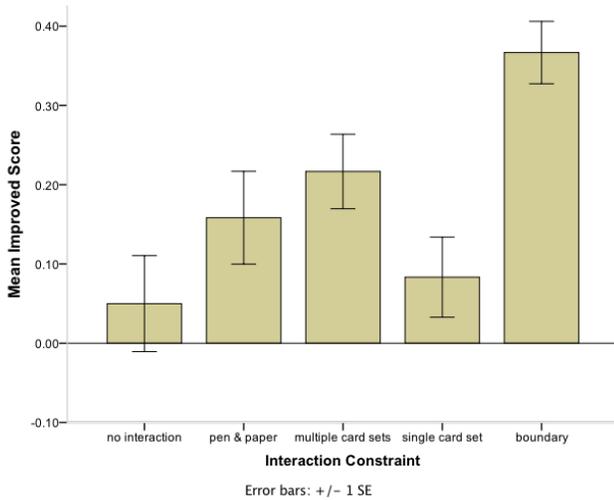
Figure 7: Mean improved score

## 5.3 The effect of isomorphs on Response Time and Score

Overall, the main effect of types of derived isomorph is significant ($F(4, 95) = 8.495, \eta_p^2 = 0.263, p < .001$) on improved score (figure 8). Post-hoc tests showed that the improved scores for participants who derived the magic square isomorph is significantly higher than for participants who derived partial magic squares at $p < .05$, and significantly higher than those of all other participants at $p < .01$. The result supports our hypothesis that the optimal solution does lead to much better performance in terms of accuracy. Although the other pairs were not significantly different on mean improved score, we saw a trend that as the isomorphs were further from the optimal magic square, the mean improved score decreased. We further performed a linear contrast between visual isomorphs (1, 2, 3) and non-visual isomorphs (4, 5) on improved score. The result showed that the mean improved score for participants using visual isomorphs was significantly larger than for those using non-visual isomorphs ($t(95) = 3.822, p < .001$).

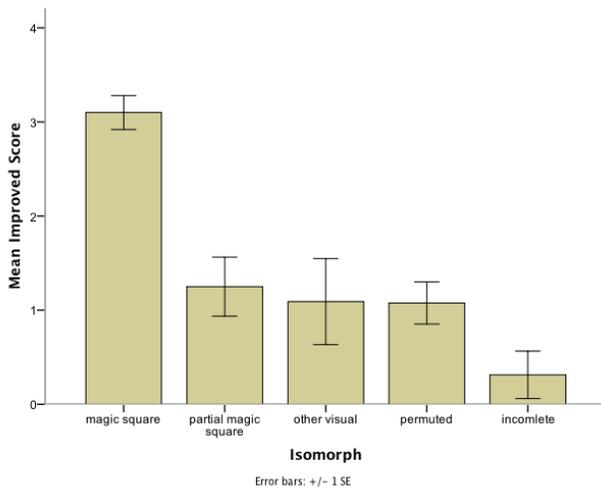Unexpectedly, we did not observe a significant result of iso-



Figure 8: Mean improved score vs. Isomorph

morph type in terms of difference in response time. We provided further explanation in section 6.4.

## 6 DISCUSSION

We start our discussion by addressing the key questions based on our hypotheses:

### 6.1 Do more confined interaction constraints yield a better chance of deriving a visual isomorph?

Yes, based on figure 5 and the chi-square analysis (section 5.1), we observed that as the interaction constraints were increasingly restricted, larger number of visual isomorphs were developed. In addition, the strictest interaction constraints led to the highest number of the optimal visual isomorphs discovered. Nine out of 20 participants under constraint #5 (boundary) discovered the magic square isomorph during the strategizing session and seven participants out of the remaining 11 discovered a partial magic square isomorph. Based on further analysis of feedback about the interaction constraints, most participants under this condition found constraint #5 very helpful in their discovery of the visual isomorphs. Many of them left comments such as, "It helped me visualize the problem and make competitive moves." Similarly, most subjects under interaction constraints #3 (multiple sets of cards) and #4 (one set of cards) felt that being able to manipulate the cards freely was helpful. Thus both statistics and user feedbacks support the hypothesis that interaction constraints significantly affect the types of isomorphs users are able to derive by altering the way participants approach the same problem. In other words, the manipulation of the isomorphs could be embodied in the interaction.

### 6.2 Does a more advanced visual isomorph outperform a non-visual isomorph in terms of score?

Yes. We consider an isomorph as more advanced if it is more similar to the optimal visual isomorph (the magic square). Thus our results summarized in Section 5.3 confirm that visual isomorphs lead a greater increase in score compared to non-visual isomorphs. What is more, within the group of visual isomorphs, the optimal visual isomorph outperforms the other two significantly.

### 6.3 Does more confined interaction constraint always yield larger improvements on score?

The short answer is: not always. As seen in Figure 7, the general trend showed that as the interaction constraints became more restricted, the improved score tended to rise, with the exception of constraint #4 (one set of cards). The low improved score in this condition can be explained by considering Figure 5, which shows that none of the participants under this condition derived a magic square (red) or partial magic square (orange) isomorph. Without more efficient visual isomorphs, it made sense that the subjects did not do much better in their post-test compared to the pre-test. However, when we designed the five interaction constraints, we considered one set of cards as a highly restrictive constraint, thus we expected better scores and more derivation of the optimal isomorph. Based on the comments they left, many participants in this condition felt limited by only being able to interact with one set of cards and wished they were given paper to write down combinations of numbers they found to offload the burden of having to memorize them. After the experiment, when we presented the magic square isomorph to participants, most in this condition thought they were close to discovering the optimal isomorph at some point during the experiment. But without the extra boundary to further constrain their interaction, it was hard for them to find the bridge between one set of cards and the magic square. This finding highlights the fact that more restrictive interaction constraints are not necessarily helpful unless they meaningfully encode information about the

problem. The single set of cards constrained interaction, but without the boundary this constraint did not by itself tell participants anything about the nature of the problem.

## 6.4 Why is response time not a good measure?

Unexpectedly, we did not observe a significant result of isomorph type in terms of both post-test response time and difference in response time. In fact, response times in the post-test were generally longer than in the pre-test, and participants who discovered the optimal isomorph tend to take an especially long time responding during the post-test. We contacted them afterwards about why they made decisions more slowly during the post-test and found out that instead of playing defensively using the magic square, they spent more time thinking about how to beat the computer. Thus we can infer that the bar this particular group of participants set was higher than just "not to lose." Overall, it may have been the case that participants in the post-test took a longer time because they were consulting their cheat sheets or otherwise thinking harder about their strategy, as we encouraged them to do in the strategizing session.

Another reason we did not observe a significant result of different types of isomorphs on difference in response time is that the search time for each of the visual isomorphs our subjects derived to decide the next card might vary drastically. For example, searching through a partial magic square might yield a much faster decision than searching through a 9x9 matrix if the winning combinations are covered by the partial magic square, but this might not necessarily be the case when the partial magic square does not contain the needed combinations of three numbers adding to 15. Overall, since there are many other factors involved in the difference in response time (such as search time and self-expectation of performance), we did not observe a strong causal relationship between types of isomorph and difference in response time.

## 7 A NOTE ON THE VARIETY OF VISUAL ISOMORPHS

In section 5.1, we roughly categorized all the isomorphs our subjects developed during the study into five categories, including three visual and two non-visual isomorph types. In this section we mainly focus on the visual isomorphs discovered by the participants. It is interesting to see that eight participants across interaction constraint #3 (multiple set of cards) and #5 (boundary) developed a partial magic square isomorph, and that 11 participants discovered other forms of visual isomorph across interaction constraints #1, 2, 3 and 4. Within the partial magic square isomorph, there are many variations. Figure 4(a) illustrates a few of them, and we can see that the variations are mainly caused by ordering. There are even more variations under the "Other visual isomorph" category. One type of variation was a decision tree, such as the examples in Figure 4(b); additionally, a few participants built a 9x9 matrix (Figure 9). To see how this isomorph can be used in playing the Number Scrabble game, refer to Appendix A.

In Figure 5, we can see a strong contrast between the types of visual isomorphs the participants generated. Most participants under interaction constraint #5 (boundary) developed magic square-like visual isomorphs during the strategizing session, while there were a relatively larger number of participants under both constraints #3 and #4 who discovered more creative visual isomorphs (such as different forms of decision trees and node-link diagrams). Thus, there seemed to be a trade off between interaction constraint and the creativity of the resulting visual isomorph.

## 8 IMPLICATIONS AND FUTURE WORK

Our findings suggest that there is a clear connection between the nature of interactions available in a visual representation and the types of strategies users tend to develop when working with the representation. While we have demonstrated this in the context of a specific problem-solving scenario, we argue that our results have
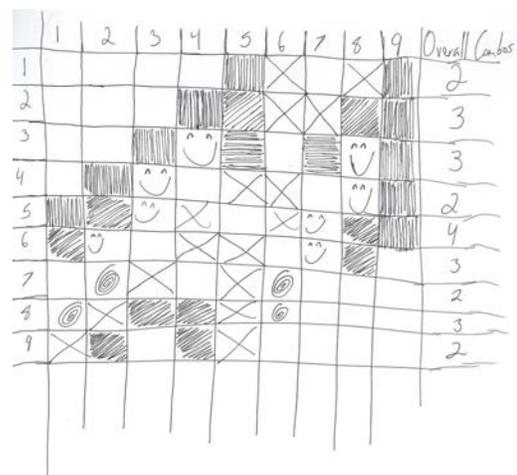


Figure 9: A matrix-like visual isomorph

significant implications for the more general area of interaction with visual representations with which visual analytics concerns itself.

In particular, this research suggests that degree of constraint is an important dimension to consider when designing interactions for visual analytics systems, although this is not a common way of talking about interaction design in visualization. In cases where a task has an optimal solution path—for example, when there is a standardized procedure that analysts are expected to follow—highly constrained interaction is likely to be a good way to guide a user towards this procedure without the need for extensive training. In situations where the designer needs to encourage creative solutions to a problem, some middle ground between constrained and unconstrained interaction is likely to be more helpful. One strong implication of our findings, however, is that complete freedom of interaction may make problem-solving more difficult; encoding some degree of boundaries into the interaction will likely help users to understand the task in a more intuitive fashion.

In particular, when considering the core goals of visual analytics of identifying the expected and discovering the unexpected [19], our findings would suggest that constraints in the user's interactions would have an impact. Specifically, the results of our study imply that highly constrained interactions can impede the discovery of the unexpected, but can also potentially guide the users to consistently identifying the expected findings. In contrast, complete freedom in an interactive visual analytics system has the potential to encourage open-ended explorations that could lead to unexpected discoveries, but users of such systems are likely to find different results in their analysis each time. Although our findings do not directly inform the design decision of the degree of constraints in a user's interactions in visual analytics system, we do believe that the relationship between the degree of constraint in interactivity and creativity should be carefully considered when designing visual analytics systems.

As demonstrated by our results, the optimal visual isomorph indeed makes the Number Scrabble problem easier to solve. But as mentioned in Section 2.2, efficiency is not the only measure of interest in visualization; our goal is to make information not just accessible, but understandable. In this context, it is worth mentioning that we had one participant who discovered the magic square visual isomorph but failed to realize that the nature of the game is just like tic-tac-toe given the optimal isomorph. While one incident does not warrant enough evidence to confirm or counter any existing theory, it is an interesting phenomenon to consider.

Since the problem we considered has a known and clearly defined optimal visual isomorph, our designed interaction constraints were geared towards this isomorph. Realizing the limitations of our

task, we certainly hope that this proof-of-concept could be generalized to more complex problems. The obvious next step is to examine how to design interaction constraints for problems that might not have known optimal visual isomorphs.

## 9  CONCLUSION

We have demonstrated that constraining user interactions indeed affects problem-solving through exploring the relationship between interaction constraints, visual isomorphs, and problem-solving performance as measured by response time and score. Our results showed that more confined constraints lead to better visual isomorphs, and better visual isomorphs result in large improvements in scores on the Number Scrabble game. Our hypothesis is further confirmed by a significant effect of interaction constraints on improved score. Overall, our results indicate that the manipulation of isomorphs can be embodied in user interaction by imposing different constraints, and that certain interaction constraints can lead to a higher chance of deriving a better visual isomorph for a problem. With better visual isomorphs yielding higher performance, our results demonstrate that we indeed can improve the effectiveness of problem solving activities by embodying information in user interaction.

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## APPENDIX A

Figure 9 is a particularly interesting example of how our participants derived creative isomorphs. Here we explain how this isomorph can be used to assist with playing the Number Scrabble game.

The matrix is designed to allow the user to quickly identify three numbers that can add up to 15. For instance, if player 1 (the user) chooses the number 5, the user will then focus on the row in the matrix that starts with the number 5. If player 2 (the computer) chooses the number 3, the user will be able to see that the cell that intersects the row 5 with the column 3 contains a smiley-face. To quickly identify the number that will compliment 5 and 3 for a sum of 15, the user will then follow the same row (row 5) until she finds the other smiley-face. In this case, the user will find the second smiley-face under the column 7.

In addition, Figure 9 also suggests strategies for choosing the best numbers in playing the Number Scrabble game. The right most column, denoted as "Overall Combos" is a count of the number of different types of glyphs in each row. For instance, in row 5, there are four distinct types of glyphs (vertical-stripes, diagonal-stripes, smiley-face, and a cross), which represents the number of combinations of 15 that includes the number 5. The user of this matrix will then choose the number 5 as the optimal starting move, followed by 2, 4, 6, or 8 as the second-best alternative. Note that in Figure 9, the user made a mistake and flipped the values for Overall Combos between the rows 3 and 4. These strategies correspond well to the magic-square isomorph in that the number 5 is always at the center of the square, and the even numbers (2, 4, 6, and 8) at the corners of the square.