uniform circular motion while the moon's motion is known to be neither uniform nor circular, so that the tight agreement may to some extent be illusory

b. Worse, vagaries in lunar orbit not just with respect to Theorem 2, but with respect to Keplerian motion too: Kepler's first two "laws" definitely do not hold for it

c. And there is no independent evidence that terrestrial gravity is inverse-square

4. Limitations of using area rule result to conclude that centripetal forces at work elsewhere

a. No mechanism to explain why always directed toward a single point; contrast with Descartes' vortex theory, which rejects idea that forces always directed toward a single center

b. Concern for imprecisions in area rule, vagaries in motion of the moon, and likely in motion of Jupiter and Saturn as well (based on what Kepler had said in his *Rudolphine Tables*), and the possibility of an epochal parochialism of the sort called attention to by Descartes

c. Worse, De Motu itself entails not one center, but four!

d. Would prefer much tighter evidence on how well planets conform with the area rule, or independent evidence for center-directed forces

5. Limitations of using Theorem 2 coupled with Problems 3 and 4 and Theorem 4 to conclude that inverse-square forces at work elsewhere

a. No independent evidence, as from comets: comets as much a challenge (from Cartesians and others) as an opportunity, and hence a lot hangs on that claim of De Motu

b. Vagaries in planetary motion challenge inference by challenging step from circular to Keplerian

c. "Building inference on inference" when prior inferences not secure because of inexactitudes

6. Note also that deceleration from resistance depends on weight of body, as does the centrifugal tension in a string retaining an object in uniform circular motion; but Corollary 5 of Theorem 2 and Theorem 4 treat centripetal force as independent of weight: what justifies the difference?

7. Upshot: loose-ends and imprecisions in phenomena leave openings in suggested evidential reasoning

a. Problem: at a minimum need grounds for concluding that imprecisions come from secondary effects, and not from falsity of claimed primary

b. Key: characterize discrepancies, separating real from observational, and then showing that they can legitimately be treated as arising from secondary effects

c. Newton's situation is just the opposite of resting self-satisfiedly with De Motu Version 1, for it points the way to a huge amount of additional work

II. De Motu, Version 3 (the "Augmented" Version) -- New Preliminaries

A. The New Title, Versus the Old

1. Version 3 the same as Version 1 in the proofs of the 11 propositions, indicating that Newton is not looking for better proofs (i.e. he is not revising as a mathematician)

a. A new first section, recasting hypotheses and then, in a correction, calling them laws

b. An extended scholium following Theorem 4: "Copernican Scholium"

c. A slightly revised figure to Problem 5 followed by an extended scholium, offering more of a transition to Problems 6 and 7: "Resistance Scholium"

2. The manuscript is in Humphrey Newton's hand, save for small corrections and changes in Newton's
   a. Original manuscript lost, and never any reference to this version in letters etc. by Newton or anyone else
   b. Date uncertain: presumably after Version 1 sent, but surely not much after first letter to Flamsteed at end of December, 1684
   c. Presumption: mid to late November, perhaps in response to second thoughts after forwarding Version 1, or early December, following Halley's second visit
   d. First became public in 1893 (in Rouse Ball's *Essay*)!
3. New title: from *De Motu Corporum in Gyrum* to *De Motu Sphaericorum Corporum in Fluidis* -- "The Motion of Spherical Bodies in Fluids"
   a. Sectioned: preliminaries, followed by "On the motion of Bodies in Non-Resisting Mediums" (Theorem 1 through Scholium preceding Problems 6 and 7), followed by "On the Motion of Bodies in Resisting Mediums"
   b. Still a theory of motion under centripetal forces, but now in two distinct domains
4. Unclear what significance ought to be attached to change in title
   a. Why abandon "in Closed Circuits"?; two likely answers: fails to capture full generality of results, and incompatible with conclusions of Copernican Scholium
   b. But why "in Fluids", for fluid in no way enters first half, unless in the form of aether as discussed in the added Scholium following Problem 5
   c. And why "spherical", for nothing in the math requires strict spheres; maybe to support treating celestial bodies as points
5. Perhaps Newton was struggling to identify the domain the theory in De Motu extends across
   a. Some suggestion of a changing conception of the scope of the theory
   b. Titles of manuscripts keep changing: next, *De Motu Corporum in Mediis Regulariter Cedentibus* -- "in Uniformly Yielding Media"
   c. Lucasian lectures: *De Motu Corporum Liber Primus*
   d. Title of overall work remained *De Motu Corporum* until fairly late in composition, when Newton changes to *Principia Mathematica Philosophiae Naturalis*, remarking to Halley that it will stimulate sales

B. The New "Laws" and Definitions

1. "Preliminaries" extensively recast, though in a way that has virtually no impact on any proofs (and therefore not being done to increase rigor)
   a. Exception: now explicit that force is proportional to change of motion
   b. But otherwise Newton has something else in view

2. Added definition (mistranslated in Hall & Hall): representations of quantities any other quantities proportional to them
    a. E.g. as area represents time, and QR represents both force and displacement
    b. Clarification, but unclear why added since a standard approach
3. Change from "hypotheses" to "laws," presumably with Descartes in mind, though perhaps from fact that material being taken from "Lawes of Motion" paper
    a. In manuscript, 'hypoth' scratched out in 5 places and 'lex' added in Newton's hand, so undoubtedly added upon re-reading
    b. Maybe because of "proof" of Copernican system
4. Newton's second law in basic form it will appear in the *Principia*: force is proportional to change in motion along a specific direction, that of the *impressed* force
    a. From "Lawes of Motion", but implicit in Version 1, where no mention of "*impressed* forces"
    b. Unidirectionality: impressed force a vector quantity
    c. Proportional to displacement from uniform motion in a straight line in a given time: hence neutral between impulse and continuous forces, and fully compatible with $F=ma$
    d. But not rate of change; Newton's focus on proportions allows dimensionality issues to be bypassed (or suppressed)
5. Laws 3 and 4: relative motions same in given space; and inertial motion of center of gravity, sort of from "Lawes of Motion" paper, but also announced for impact generally by Huygens in 1669
    a. Latter explicit In both “Lawes” and Huygens; former, not yet published in Huygens’s full paper, implicit in “lawes” and deducible from velocity of attraction=velocity of repulsion
    b. No derivation of 4 from 3, only a comment that it is derivable; I dispute that the center of gravity principle can be derived from the relativity principle
    c. No relevance to any proofs, only to Copernican Scholium
6. Fairly clear where Laws 3 and 4 came from: concerns over multiple centers raise interaction issue and issue of proper point to which motions should be referred
    a. Newton presumably turns to "Lawes of Motion" for move
    b. But equally could have turned to Huygens, for Law 3 at the base of his work on impact (though proofs unpublished, manuscript in London), and Law 4 published by him in 1669
    c. Law 4 also just a generalization of Law 1 to systems of interacting bodies; this, though not expressly mentioned, doubtlessly behind both Newton and Huygens’s statement of it
    d. In effect, Newton extrapolating from impact, though recognizing the added warrant for doing so
7. Finally, last "law" on resistance clarified by including other factors in the claim
    a. Now more in the form of a law, rather than just a hypothesis whose implications are being explored
    b. But, unlike the others, much more a bare hypothesis

C. The New Lemmas: Deriving Former Hypotheses

1. Two added lemmas, corresponding to Hypothesis 3 and inserted Hypothesis 4 of Version 1
    a. Now deriving from "laws" material previously postulated
    b. Thereby avoiding impact on any proof
2. Lemma 1: the parallelogram rule, derived from Law 2
    a. Crucial need to make directionality in Law 2 explicit
    b. Thus explicit reduction of a feature of the old parallelogram hypothesis to a more fundamental claim; this may be the source of Law 2
3. Lemma 2: old hypothesis 4, now derived from more fundamental considerations
    a. Presupposes mean value theorem, referencing Galileo; unclear whether Newton realized the defects in Galileo's proof (eliminated by Huygens in *Horologium Oscillatorium*)
    b. Let Ab:Ad as AB:AD, with b and d arbitrary, and compare ratios of distances covered (represented by areas) as force changes versus distances covered with uniform forces
    c. ADEG an upper bound: uniform force at end of time AD; ABF a lower bound: uniform force at t=0
    d. Given bounds above and below, now take limits, obtaining result (see Appendix)
4. Comment: key feature in proof is constraint when taking the limits -- i.e. as B, D approach A and e approaches h
    a. Hold the ratios fixed: AB:AD, Ab:Ad, etc.
    b. Conclusion then for general points b and d, via ratios
    c. A feature of the way in which Newton takes geometric limits -- "first and last ratios"
5. Move in "Preliminaries" in part toward more secure foundations, but even more so toward claims that are more truly foundational -- i.e. more appropriate underpinnings of the theory to follow
    a. Toward putatively universal principles that can be said to comprise a simple, straightforward approach to conceptualizing motion
        (1) True even of Laws 3 and 4, for they can be taken to concern the proper point to which to refer motions
        (2) And also of Law 5, for it licenses the treatment of resistance as second-order, induced effects
    b. Making key elements of conceptualization more clear and explicit, as distinct from derived ones
        (1) Parallelogram rule less clearly an element in a way of conceptualizing motion
        (2) Even more so for originally inserted Hypothesis 4
    c. Why doing this unclear, but in spirit of response to Descartes' critique of Galileo's *Two New Sciences*, at least up to a point
    d. Also, though, to some extent being forced to add principles in order to be in position to answer questions, such as questions about true versus relative motions