one body (*a*) is to the swiftnesse of [the ot]her *b*, as *b* is to *a*[q] and therefore the motion of both bodys shall bee equall.

5. If two [equal?] bodys (*b*, *c*) bee moved by unequall forces, as the force moveing (*b*) is to the force [mov]ing *c*, so is motion of *b*, to the motion of *c*, so is the swiftnes of *b*, to that of *c*.

7. If two bodys [*a* and *b* moving?] against one another the same way towards *O* [Fig. 1], (*a*) overtaking (*b*) none of theire motion shall be lost, for (*a*), presses [(*b*) as much] as (*b*) presses (*a*)[s] and therefore the motion of (*b*) shall increase [as much] as that of (*a*) decreaseth.

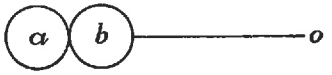

Figure 1.

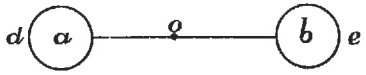

Figure 2.

8.[a] If two quantities (*a* and *b*) [Fig. 2] move towards one another and meete in *O*, Then the difference of theire motion shall not bee lost nor loose its determination. For at their occursion they presse equally uppon one another and[p] therefore one must loose noe more motion than the other doth; soe that the difference of their motions cannot bee destroyed.

9.[b] If one body (*a*) overtake another body (*b*) they both moveing towards *O* then they shall always move together. If the body (*c*) move against an immoveable quantity (*d*) it shall not bee rebounded for *c* having urged *d* with

9.[c] If two equall and equally swift bodys (*d* and *c*) meete one another they shall bee reflected, soe as to move as swiftly frome one another after the reflection as they did to one another before it. For first suppose the sphaericall bodys *e*, *f* [Fig. 3] to have a springing or elastic force soe that meeting one another they will relent and be pressed into a sphaeroidicall figure, and in that moment in which there is a period put to theire motion towards one another theire figure will be most sphaeroidical and theire pression one upon the other is at the greatest, and if the endeavour to restore theire sphaericall figure bee as much vigorous and forcible as theire pressure upon one another was to destroy it they will gaine as much motion from one another after their parting as they had towards one another before theire reflection. Secondly suppose they be sphaericall

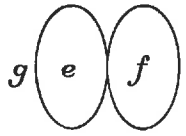

Figure 3.

[q] Def. 3[d]. [s] Axiome 4th. [p] Axiom 4th.

and absolutely sollid[4] then at the period of theire motion towards one another (that is at the moment of theire meeting) theire pression is at the greatest, or rather 'tis done with the whole force by which theire motion is stopt (for theire whole motion was stoped by the force of theire pressure upon one another in this one moment, and there cannot bee/ succeede divers degrees of pressure twixt two bodys in one moment). Now so long as neither of these 2 bodys yeild to one another they will retaine the same forcible pressure towards one another : that is soe much force as deprived the bodys of their motion towards one another soe much doth now urge them from one another and therefore[r] they shall move from one another as much as they did towards one another before theire reflection.

10.[5] There is the same reason when unequall and unequally moved bodys reflect, that they should separate from one another with as much motion as they came together.

11.[d] If a line *df* [Fig. 4] bee moved not with a Progressive but onely a Circular motion its middle point (*n*) shall rest. For if it move let it move towards *r* soe that, when the point (*d*) is in *p* and *f* in (*q*), then (*n*) shall be moved to (*s*).

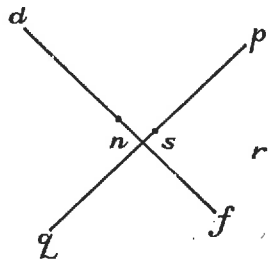

Figure 4.

11. If a line (*ce*) [Fig. 5] be bisected in (*a*) about which the line (*ce*) doth circulate and that point bee fixed. Then the whole line hath noe progressive motion. For making *ab* = *ad*, *bf*, *ag*, and *dh* bee parallel, and perpendic to *fh*, then is *vb* = *dp* and *vf*+*ph* = *bf*+*dh* = 2*ag*. Wherefore the point *c* moveing towards *n* the point *d* shall move soe much towards the line *fh* as the point *b* doth from it, and all the points in (*a*, *c*) or the line *ac* move as much to the line *fh* as all the points in (*ae*) or the line (*ae*) moves from it soe that the whole line *ce* stays *in equilibrio* neither moveing to nor from *fh*, by the 12th Defin.

12. Hence when the center of a line (*a*) is not in the midst of a line (*m e*) [Fig. 5] the whole line moves the same way which the longest parte doth. for supposeing *ca* = *ae* then the line *ce* is *in equilibrio* (par ax. 11) but if (*m c*) moves towards (*fh*) and be added to (*c e*) then (*m e*) moves towards *ce* (by def. 12).

13. When (*ce*) [Fig. 5] moves circularly but maketh noe progression its

[r] axiome 3[d].

Newton's "Lawes of Motion"
Announced Difficulties in Verifying

- **"The bodies here among us (being an aggregate of smaller bodies) have a relenting softness and springiness, which makes their contact be for some time and in more points than one."**

- **"The touching surfaces during the time of contact do slide one upon another more or less or not at all according to their roughness."**

- **"Few or none of the bodies have a springiness so strong as to force them from another with the same vigor that they came together."**

- **"Their motions are continually impeded and slackened by the mdiums in which they move."**

NEWTON ON IMPACT OF SPHERES (ca. late 1670s)

PROBLEM XII.

Having given the Magnitudes and Motions of Spherical Bodies perfectly elaſtick, moving in the ſame right Line, and ſtriking againſt one another, to determine their Motions after Reflexion.

The Reſolution of this Queſtion depends on theſe Conditions, that each Body will ſuffer as much by Re-action as the Action of each is upon the other, and that they muſt recede from each other after Reflexion with the ſame Velocity or Swiftneſs as they met before it. Theſe Things being ſuppoſed, let the Velocity of the Bodies A and B, be a and b reſpectively; and their Motions (as being compoſed of their Bulk and Velocity together) will be aA and bB. And if the Bodies tend the ſame Way, and A moving more ſwiftly, follows B, make x the Decrement of the Motion aA, and the Increment of the Motion bB ariſing by the Percuſſion; and the Motions after Reflexion will be $aA-x$ and $bB+x$; and the Celerities $\frac{aA-x}{A}$ and $\frac{bB+x}{B}$, whoſe Difference is $=a-b$ the Difference of the Celerities before Reflexion. Therefore there ariſes this Equation $\frac{bB+x}{B} - \frac{aA-x}{A} = a-b$, and thence by Reduction x becomes $= \frac{2aAB-2bAB}{A+B}$, which being ſubſtituted for x in the Celerities $\frac{aA-x}{A}$, and $\frac{bB+x}{B}$, there comes out $\frac{aA-aB+2bB}{A+B}$ for the Celerity of A, and $\frac{2aA-bA+bB}{A+B}$ for the Celerity of B after Reflexion.

But if the Bodies move towards one another, then changing every where the Sign of b, the Velocities after Reflexion will be $\frac{aA-aB-2bB}{A+B}$ and $\frac{2aA+bA-bB}{A+B}$; either of which, if they come out, by Chance, Negative, it argues that Motion, after Reflexion, to tend a contrary Way to that which A tended to before Reflexion. Which is alſo to be underſtood of A's Motion in the former Caſe.

EXAMPLE. If the homogeneous Bodies [or Bodies of the ſame Sort] A of 3 Pounds with 8 Degrees of Velocity, and B a Body of 9 Pounds with 2 Degrees of Velocity, tend the ſame Way; then for A, a, B and b, write 3, 8, 9 and 2; and $\left(\frac{aA-aB+2bB}{A+B}\right)$ becomes -1, and $\left(\frac{2aA-bA+bB}{A+B}\right)$ becomes 5. Therefore A will return back with one Degree of Velocity after Reflexion, and B will go on with 5 Degrees.