- 5. Over the next decade Newton studied classic geometry in more detail than he had before and in lectures rigorously developed foundations of algebra, later published as *Arithmetica Universalis*
 - a. These efforts appear to have convinced him that limits could never be done with proper rigor in symbolic methods, but only by extending geometry to incorporate them
 - b. That view is reflected in the mathematical method used throughout most of his *Principia*, and subsequently in his unfinished treatise *Geometria*, intended to carry out that project on limits
- 6. Newton's first publication of work in the calculus was as an appendix to the 1704 first edition of his *Opticks*, and comparatively little more made it into print until posthumously
 - a. In 1710 the priority controversy over the calculus began, growing into something that then colored all publication of his work in mathematics after that (see Hall, *Philosophers at War*)
 - b. Meanwhile, calculus as we know it continued to be developed in the tradition stemming from Leibniz, with a real explosion in the hands of Euler from 1730 to 1780
 - c. View now is that they discovered the calculus indepedently, but calculus as we know it, including the word itself and much of the notation, derives from Leibniz
- E. Newton's Work in Optics: 1665-1680
 - 1. Much of Newton's initial work on optics and the theory of light occurred during this period, undoubtedly stimulated by Descartes' *Optics* and *Principia*, as well as by Barrow's lectures in geometric optics
 - a. Extraordinary experiments showing that white light is composed of light of different colors, along with a theory of refraction explaining chromatic aberration as a consequence of differing refraction indices of different color light -- Lucasian lectures, 1670-72
 - b. Papers in the form of letters published by the Royal Society in 1671 and 1672 on this work and the reflecting telescope (and then in reply to objections until 1675) made Newton famous and respected throughout the scientific world
 - 2. The controversy they initiated -- in particular, the insistence by Hooke and others that the experiments were predicated on a particle theory of light -- then led Newton to shun further publication
 - a. One source of this insistence was Newton treating rays of light in the abstract, concluding that rays of different color are differently refrangible, and then suggesting that rays are the paths of light particles without in any way using this (see his reply to Pardies in Appendix)
 - b. To some extent the controversy also stemmed from the difficulty of replicating his experiments
 -- though Hooke managed to do so; for, as the key pages (see Appendix) describing his so-called *experimentum crucis* make clear, the experiments were elaborate and required great care
 - 3. But it stemmed even more from differing conceptions of science, where Newton was outspokenly negative toward the "method of hypotheses," especially those concerning underlying processes
 - a. Newton already insistent on a strict distinction between experimentally established theoretical claims, on the one hand, and hypothetical conjectures, on the other

- b. Others, including Huygens, thought this a mere matter of degree: all theoretical claims remain hypothetical, with evidence conveying on them only degrees of probability
- c. This difference between Newton and almost everyone else remained a source of confusion throughout the rest of his life
- 4. To underscore Newton's conception, a sequence of quotations on method in optics from this period is included in the Appendix
 - a. The first is a passage which Oldenburg elected to cut from the first published paper
 - b. The second a translated passage from the Latin in reply to Pardies
 - c. But the most important is from his Lucasian Lectures on optics from the early 1670s, in which he puts forward the idea of combining mathematical methods with those of natural philosophy to "finally achieve a natural science supported by the greatest evidence"

Thus although colors may belong to physics, the science of them must nevertheless be considered mathematical, insofar as they are treated by mathematical reasoning. Indeed, since an exact science [*accurata scientia*] of them seems to be one of the most difficult that philosophy is in need of, I hope to show -- as it were, by my example -- how valuable mathematics is in natural philosophy. I therefore urge geometers to investigate nature more rigorously [*strictius*], and those devoted to natural science to learn geometry first. Hence the former shall not spend their time in speculations of no value to human life, nor shall the latter, while working assiduously with an absurd method, perpetually fail to reach their goal. But truly with the help of philosophical geometers and geometrical philosophers , instead of the conjectures and probabilities that are being blazoned about everywhere, we shall finally achieve a science of nature [*scientiam Naturae*] supported by the greatest evidence [*summis evidentiis*].

- Newton's *Opticks* published in 1704 (in English; second Latin edition in 1706; third edition, in English, in 1717); it includes not just his work on refraction and reflection, but on diffraction too, conducted largely after his *Principia* was published
 - a. The book does not begin to reveal the number of enabling, supporting, and cross-checking experiments he carried out for each experiment he then published
 - b. But his Optical Lectures, first published in 1729, do reveal just how extraordinarily careful he was as an experimentalist (available in Latin and English as Volume 1 of *Newton's Optical Papers*, edited by Alan Shapiro)
- F. Newton's Intellectual Style, versus Huygens's
 - 1. My standard way of explaining Newton's uniqueness: he was one of the two or three greatest mathematicians, one of a handful of the greatest experimental physicists, and one of three or four of the greatest theoretical physicists -- this all in one person
 - a. Huygens was in the same rank as an experimental physicist and not far behind as a theoretical physicist, but, good as he was as a mathematician, he was not in the league of Newton or Gauss
 - b. While this description goes some way toward explaining Newton, it does not begin to address the uniqueness of his style
 - c. Perhaps the most telling remark is by J. M. Keynes, Essays in Biography

I believe that the clue to his mind is to be found in his unusual powers of continuous concentrated introspection.... Anyone who has ever attempted pure scientific or philosophical thought knows how one can hold a problem momentarily in one's mind and apply all one's powers of concentration to piercing through it, and how it will dissolve and escape and you find that you are surveying a blank. I believe that Newton could hold a problem in his mind for hours and days and weeks until it surrendered to him its secret. (p. 312)

- 2. Newton had a peculiar mathematical talent, enabling him to zero in on the essence of problems, finding elegant solutions to them and brief proofs (that were generally harder to understand at first); combined with this was a capacity to take so many threads of thought into consideration
 - a. Proofs seemingly based on tricks, but the tricks then turn out to capture the fundamental features of the problem, sometimes by bringing together ideas others had seen as disparate
 - b. While others would struggle to identify and clarify the principal obstacle in a mathematical problem, Newton would see it almost immediately and put all his effort into resolving it
 - c. And his work on the calculus gave him a range of methods well beyond those of Huygens
- 3. Newton had this talent to a far greater degree than any of his contemporaries, even including Leibniz (who had complementary talents)
 - a. Newton's proofs stand in striking contrast with, for example, those of Huygens -- e.g. less than three pages on the isochronism of the cycloid, where Huygens has more than 20 pages using heights to infer velocities, combined with Wren's solution for lengths along the cycloid
 - b. Why it is not surprising that Newton had pulled together the rudiments of the calculus (out of Descartes, Wallis, Barrow and others) years before anyone else did
- 4. This peculiar talent enabled Newton to work by himself, without discussions or contact with others working on similar problems
 - a. Indeed, given his personality, he was probably better off working alone; the point is that he did not pay the usual price for doing so
 - b. 'Alone' here does not mean in complete isolation from the work of others, for Newton read voluminously -- e.g. the Latin edition of Descartes' *Géometrie*, Wallis, Gregory, etc.
 - c. But without the daily give-and-take exchanges of ideas that Huygens enjoyed in his years in Paris (and Leibniz continued in his voluminous correspondence)
- 5. One consequence of Newton's style was that criticisms of his work often seemed poorly motivated or wrong-headed to him
 - a. He had a deeper understanding of what he was doing than anyone around him did -- something of which he was constantly aware
 - b. Hooke's criticism of his experiments on colors was a typical example, for the whole point of the experiments was to avoid begging questions of theory
 - c. Newton thus had a distinct tendency to be contemptuous of his colleagues, with a few notable exceptions (Huygens, Wren, Wallis and later Halley and Gregory's young nephew, David)

- 6. Another consequence of Newton's style was that he often thought of problems in rather different ways, not always being informed of the most current way of thinking about them in, say, London
 - a. We will see this tonight in work paralleling that of Huygens, in which Newton had his own distinctive approach
 - b. Being out of the mainstream put Newton in a position to alter the course of the history of physics
- II. Newton's Early Laws of Motion (vs. Descartes, Huygens)
 - A. Newton's Conceptualization of the Problem
 - Material in the "Lawes of Motion" paper appears to be in response to his reading Descartes' *Principia* in 1664; but the amaneunsis handwriting (John Wickens) in the first half points to its dating between 1672 and 1675, perhaps in response to the *Phil. Trans.* 1669 papers
 - a. Descartes' rules of impact not only difficult to comprehend, but glaringly in conflict with observation, putting the problem at the forefront in both Paris and London in the 1660's
 - c. But, given Newton's distinctive approach, we have no clear evidence of how the "Lawes" relates to work by others
 - 2. His first efforts on impact in the Waste Book are restricted to spherical objects colliding head-on
 - a. He adopts Descartes' Bulk*velocity measure of force, but restricts the conservation of motion to one direction at a time
 - b. And he conceptualizes impact itself as involving elastic deformation of the spheres, with the force tied to the distortion, yielding the principle, separation v = approach v
 - The most striking feature of the way in which he conceptualizes the problem in the "Lawes of Motion" paper is its generality: arbitarily shaped objects at arbitrary angles of attack with respect to one another, not just spheres head-on
 - a. Given Descartes' three kinds of matter, this is the appropriate form in which to pose the problem (though no one else did so)
 - b. The obvious disadvantage is that it makes the problem mathematically much more complicated
 - 4. Newton also conceptualizes the problem within the context of Descartes' three laws of motion, though with some modifications
 - a. Presupposes straight line, constant velocity motion in the absence of forces -- in the form of motion in absolute space
 - b. The force to persevere in a given motion is equivalent to the motion that it can create or destroy in another body -- expressed in terms of $\Delta(B*v)$, just as in Descartes' third law
 - 5. Newton, however, distinguishes components of motion, and adds a parallelogram rule for compounding these components
 - a. No hint of Galileo's parallelogram rule in this addition to Descartes
 - b. Presumably prompted by demands on the problem when he allowed impact other than head-on