able. While this stress on a coherent system served Copernicus very well in the shift to Sun-centered cosmology, it led him astray in technical matters. For it turns out that the planets really do move nonuniformly and that Ptolemy's equant theory was closer to the mark than Copernicus's "improvement" on it.

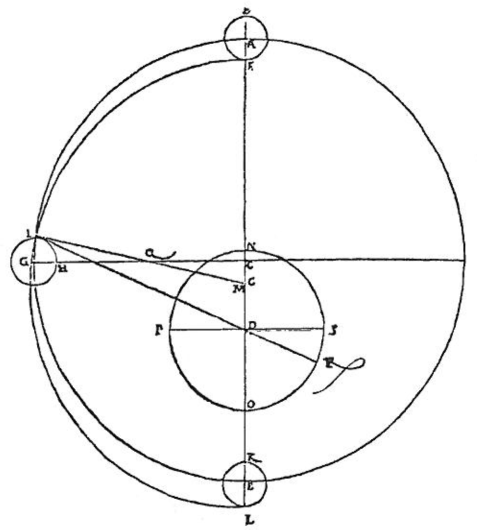


FIGURE 7.63. Copernicus's theory of the superior planets. *NPO* is the orbit of the Earth. *AGB* is the deferent circle of a superior planet, such as Mars. Mars itself moves on a small epicycle which is responsible for producing an anomaly of motion more or less equivalent to that produced by Ptolemy's equant. From *De revolutionibus* V, 4 (Nuremberg, 1543).

## *Copernican Planetary Theory*

A good sense of Copernicus's astronomy can be obtained by examining his theory for the superior planets. Copernicus himself placed a high value on this work, which he believed improved on Ptolemy. Here we must confront not only Copernicus's use of a moving Earth, but also his method of accounting for the planets' nonuniformity of motion.

For the orbit of the Earth, Copernicus chose an eccentric circle: the Earth moves at uniform speed on a circle that is eccentric to the Sun. The model is essentially the same as the solar theory of Ptolemy. For computation of positions it makes no difference whether the Earth or the Sun moves. The essence of the model is uniform circular motion on an off-center circle.

For the superior planets, Copernicus adopted an eccentric circle plus a modified form of the Ptolemaic equant. As we have seen, Copernicus could not abide the equant. But he had, of course, to replace it with something else. He found that a minor epicycle could perform very nearly the same function.

Figure 7.63 is a diagram from the first edition of *De revolutionibus*, illustrating Copernicus's theory of the superior planets. The Earth travels around the annual circle *NPO*, which is centered at *D*. The Sun is therefore located near but slightly displaced from *D*. However, the true Sun does not appear in this figure and plays no part in the theory. For this reason, Copernicus's system has been aptly characterized as merely heliostatic, rather than truly heliocentric. The effective center of the whole system is the center *D* of the Earth's orbit, also called the mean Sun.

In figure 7.63, *C* is the center of the deferent circle *AGB* of a superior planet (let us say Mars). Thus, the center of Mars's deferent circle is eccentric to the mean Sun *D*. So far, this resembles Ptolemy's theory. However, Copernicus does not have an equant point. Rather, he places Mars on a small epicycle, shown in the figure. Further, Mars makes a complete counterclockwise orbit on the epicycle while the epicycle's center travels a complete circle around the deferent. Thus, when the epicycle's center is at *A*, Mars is at *F*. When the epicycle's center is at *G*, Mars is at *I*. When the epicycle's center is at *B*, Mars is at *L*. Finally, the radius *GI* of the epicycle is chosen to be one-third of the eccentricity *DC*.

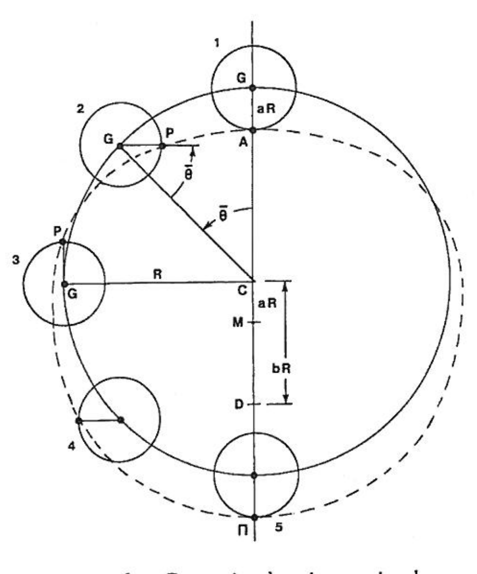



FIGURE 7.64. Copernicus's minor epicycle, a replacement for Ptolemy's equant.

One thing to note is that Copernicus did not eliminate epicycles from planetary theory. However, the large epicycle of Ptolemy *is* gone. Ptolemy's big epicycle was responsible for retrograde motion. In Copernicus's theory of the superior planets (fig. 7.63), this function is taken over by the circle *NPO* of the Earth's annual motion. The minor epicycle *GI* is Copernicus's substitute for Ptolemy's equant point. Let us study this device in more detail.

Refer to figure 7.64, which elaborates on Copernicus's own diagram. The large solid circle of radius *R* is the deferent of Mars, centered at *C*. The deferent circle is eccentric to *D*, the mean Sun, or center of the Earth's orbit. (For simplicity, the Earth's orbit is not shown in this figure.) The dimensionless eccentricity of Mars's deferent circle is $b = CD/R$.

The center *G* of a small epicycle moves counterclockwise and uniformly around the deferent. The planet *P* moves counterclockwise and uniformly on the epicycle whose radius is $aR$. (Thus, $a$ is a dimensionless number less than 1.) Further, the two angles marked $\bar{\theta}$ remain equal to one another while increasing uniformly with time. Consequently, while the epicycle's center moves through 180° from position 1 to position 5, the planet revolves through 180° on the epicycle.

The combination of two uniform circular motions for $P$ in figure 7.64 results in a motion that is neither uniform nor circular. The actual path of the planet is indicated by the dashed line. The effective center of the orbit is not $C$ but $M$, located below $C$ by a distance $aR$ equal to one radius of the epicycle. As Copernicus himself states, the path is not circular but somewhat oblong—the long axis being perpendicular to the line of apsides $\Pi CA$.[143]

Nevertheless, Copernicus's speed rule is virtually indistinguishable from Ptolemy's: the minor epicycle produces a motion that closely approximates equant motion. Refer to figure 7.65. The radius of the epicycle is $aR$. Let us identify point $E$ on the line of apsides at a distance $aR$ above the center $C$ of the deferent. As already remarked, in Copernicus's model, the rotation of $GP$ is such that angle $CGP$ is always equal to the mean anomaly $ACG$: both are equal to $\bar{\theta}$. Since also $CE = GP$, it follows that the quadrilateral $ECGP$ is a trapezoid, with sides $EP$ and $CG$ always parallel. Since line $CG$ turns uniformly, it follows that $EP$ turns uniformly, too. In other words, *E is an effective equant point*. The planet $P$, observed from $E$, appears to move at uniform angular speed.

Furthermore, Copernicus usually makes the radius of the minor epicycle exactly one-third the eccentricity of the deferent. That is, $b = 3a$. Now, from figure 7.65, $EM = 2aR$, and $MD = bR - aR$, so we get also $MD = 2aR$. Thus, the center $M$ of the effective orbit is exactly midway between $D$ and the effective equant point $E$. Copernicus, like Ptolemy, bisects the total eccentricity: $EM = MD$ in figure 7.65, just as $EC = CO$ in figure 7.32. An almost perfect equivalence will be established between Ptolemy's eccentric circle with equant point and Copernicus's eccentric circle with minor epicycle if we identify the radius of Copernicus's epicycle with half the Ptolemaic eccentricity $e_P$; that is, if $a = 1/2\ e_P$. Thus, $b = 3/2\ e_P$.

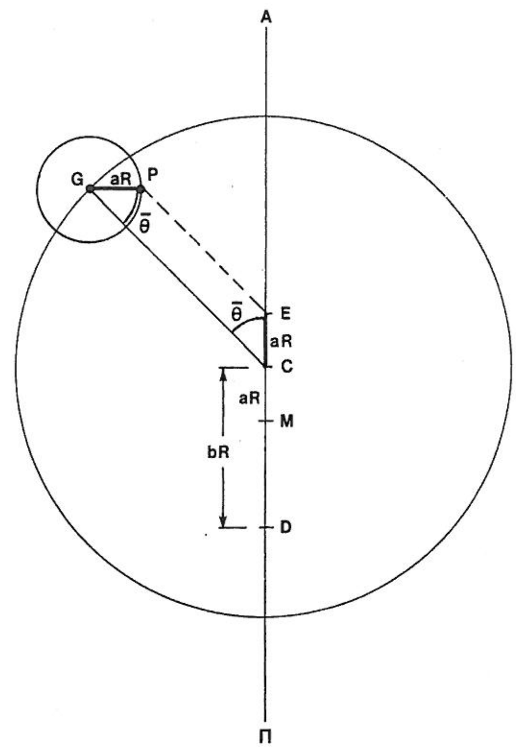



FIGURE 7.65. Copernicus's hidden equant point ($E$).

The combined effect of Copernicus's oblong orbit and hidden equant is illustrated in figure 7.66. $M$ is the center of the solid circle and $E$ represents a Ptolemaic equant point. Thus, if body $P$ moves on the circle according to the law of the equant, $\bar{\theta}$ increases uniformly with time. The dashed curve represents the effective, oblong Copernican orbit. $E$, then, is also the effective equant point of the Copernican orbit. Thus, when the body is at $P$ according to Ptolemaic hypotheses, it will be at $P'$ according to Copernican principles. For an observer at the equant, $P$ and $P'$ could not be distinguished. But, because of the noncircularity of the Copernican orbit, an observer at $D$ (the center of the Earth's orbit) would see $P$ and $P'$ in directions that differ by a small angle $\Delta\theta$. The eccentricity is greatly exaggerated in figure 7.66. Even in the case of Mars, for which Ptolemy's eccentricity $e_P = 0.1$, the maximum difference $\Delta\theta$ between the directions of $P$ in the two models is only about 3′. Before the work of Brahe and Kepler, the observational consequences of Copernicus's modification of the Ptolemaic equant were nil.

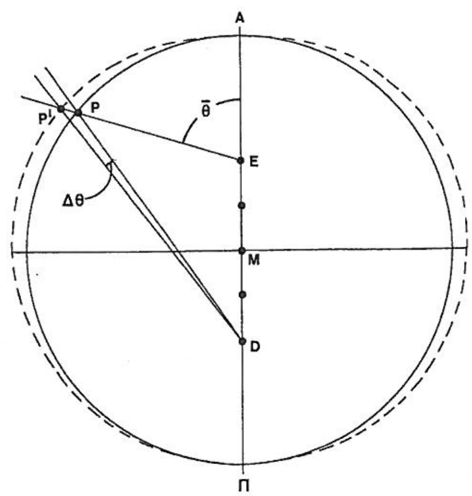



FIGURE 7.66. Comparison of the Copernican model with a Ptolemaic eccentric-with-equant model. The Ptolemaic eccentric circle is drawn in solid line. The oblong Copernican orbit is drawn in dashed line. The Ptolemaic equant point and the hidden, effective equant point of the Copernican model coincide at $E$. At the same moment (and therefore at the same mean anomaly $\bar{\theta}$) the position of the planet in equant theory is $P$ and the position in Copernican theory is $P'$. As viewed from the Sun $D$, there is a small difference $\Delta\theta$ in the directions predicted by the two theories.

Moreover, Copernicus's values for the eccentricities of the superior planets were borrowed from Ptolemy, as may be seen in the following table:

Eccentricities of the superior planets

| | *Ptolemy* | | | *Copernicus* | |
|---|---|---|---|---|---|
| | $e_P$ | $1/2\ e_P$ | $3/2\ e_P$ | $a$ | $b$ |
| Mars | 0.10000 | 0.05000 | 0.15000 | 0.05000 | 0.14600 |
| Jupiter | 0.04583 | 0.02292 | 0.06875 | 0.02290 | 0.06870 |
| Saturn | 0.05694 | 0.02847 | 0.08541 | 0.02850 | 0.08540 |

Column $e_P$ gives Ptolemy's value of the eccentricity for each planet. The columns headed $1/2e_P$ and $3/2e_P$ give the appropriate fractions of Ptolemy's eccentricity. As shown above, Copernicus's theory for the superior planets

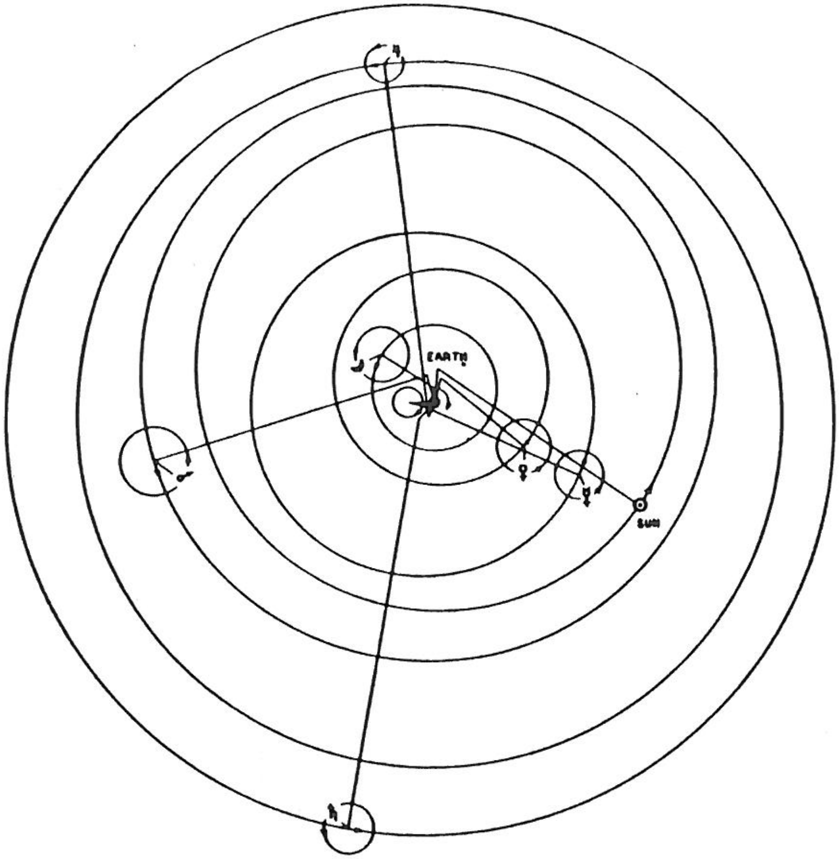



Fig. 1.—The Ptolemaic System

These drawings have been designed to point out how similar in complexity were the Ptolemaic and Copernican systems. Even a cursory glance convinces one that neither system is essentially simpler geometrically than its competitor. Drawings cannot be made accurate in radial dimensions, but special care has been taken properly to orient the centers of the planetary orbits relative to the zodiac. Thus, if one traces in the Ptolemaic diagram the radial line from the Sun to the point under "A" in "EARTH," the point which is the center of the Sun's orbit, it is seen to be between the centers of rotation of Venus and Mars, precisely as Ptolemy's geocentric theory requires. The relative senses of rotation of the epicycles on their deferent circles and the planets on the epicycles are indicated by the arrows. The planetary distances remain arbitrary, which is not so in Copernicus.



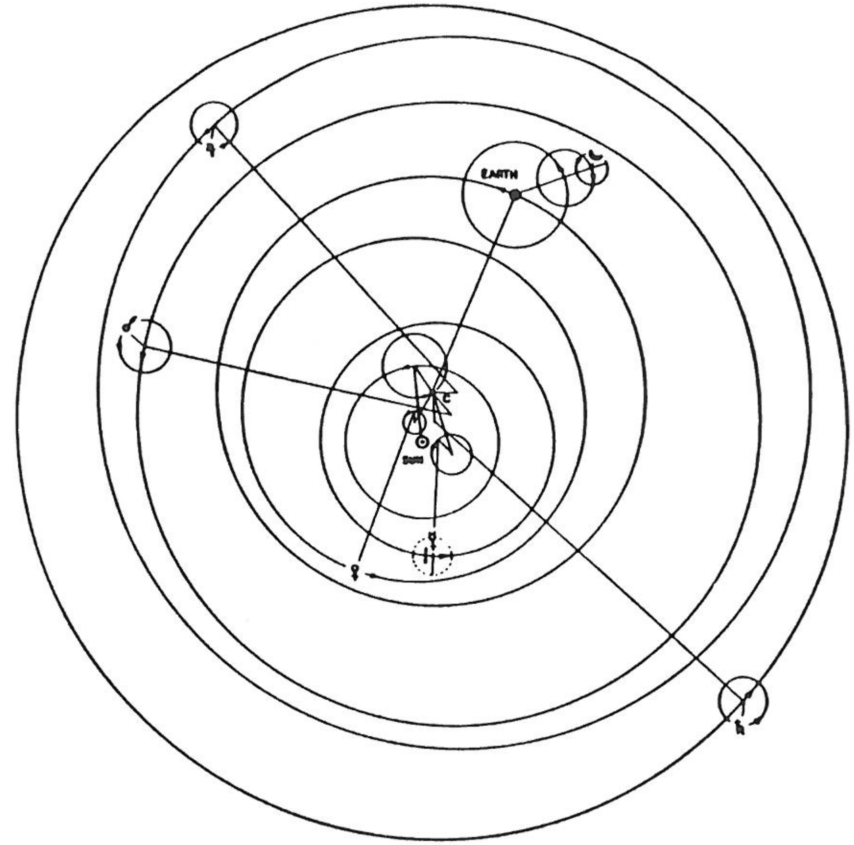



Fig. 2.—The New System as Conceived by Copernicus

In the Copernican system the Sun appears in the center of the stage, but the actual momentary centers of rotation of the planets cluster around the momentary center *C* of the Earth's orbit. In this system Mercury was handled in a unique fashion, librating on the center of an epicycle instead of traveling on the epicycle. The planetary symbols are as follows:

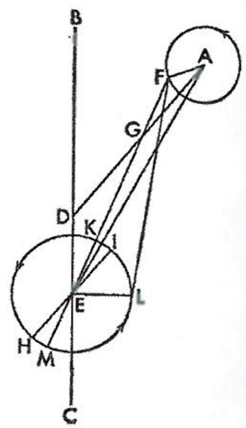
| | | | |
|---|---|---|---|
| ☉ | Sun | ⊕ | Earth |
| ☿ | Mercury | ♂ | Mars |
| ♀ | Venus | ♃ | Jupiter |
| ☾ | Moon | ♄ | Saturn |

Drawings are by the courtesy of Dr. W. D. Stahlman (see also p. 68).

which occur in the case of Saturn are, as we said, parallaxes arising from the annual orbital circle of the Earth, since, as the magnitude of the Earth in relation to the distance of the moon causes parallaxes, so too its orbital circle, in which it revolves annually, should in the case of the five wandering stars cause [parallaxes] which are far more evident in proportion to the magnitude of the orbital circle. Now such parallaxes cannot be determined, unless the altitude of the planet—which, however, it is possible to apprehend through any one observation of a parallax—becomes known first.

We have such [an observation] in the case of Saturn in the year of Our Lord 1514 on the sixth day before the Kalends of May 5 equatorial hours after the preceding midnight. For Saturn was seen to be in a straight line with the stars in the forehead of Scorpio, namely with the second and third stars, which have the same longitude and are at 209° of the sphere of the fixed stars. Accordingly the position of Saturn is made evident through them. Now there are 1514 Egyptian years 61 days 13 minutes [of a day] from the beginning of the years of Our Lord to this time; and according to [149a] calculation the mean position of the sun was at 315°41′, the anomaly of parallax of Saturn was at 116°31′, and for that reason the mean position of Saturn was 199°10′ and that of the highest apsis of the eccentric circle was at approximately 240⅓°.

Now in accordance with our problem, let $ABC$ be the eccentric circle: let $D$ be its centre, and on the diameter $BDC$ let $B$ be the apogee, $C$ the perigee, and $E$ the centre of the orbital circle of the Earth. Let $AD$ and $AE$ be joined, and with $A$ as centre and ⅓ $DE$ as radius let the epicycle be drawn. On the epicycle let $F$ be the position of the planet; and let




$$\text{angle } DAF = \text{angle } ADB.$$

And through $E$ the centre of the orbital circle of the Earth let $HI$ be drawn, as if in the same plane with circle $ABC$, and as a diameter, parallel to $AD$, so as to have it understood that with respect to the planet the apogee of the orbital circle is at $H$ and the perigee at $I$. Now on the orbital circle let

$$\text{arc } HL = 116°31'$$

in accordance with the computation of the anomaly of parallax; let $FL$ and $EL$ be joined, and let $FKEM$ produced cut both arcs of the orbital circle.

Accordingly since by hypothesis

$$\text{angle } ADB = \text{angle } DAF = 41°10',$$

and

$$\text{angle } ADE = 180° - ADB = 138°50';$$

and

$$DE = 854$$

$$\text{where } AD = 10{,}000:$$

whence in triangle $ADE$

$$\text{side } AE = 10{,}667,$$

$$\text{angle } DEA = 38°9',$$

and

$$\text{angle } EAD = 3°1':$$

therefore by addition

$$\text{angle } EAF = 44°12'.$$



So again in triangle $FAE$

$$\text{side } FA = 285$$

$$\text{where } AE = 10{,}667,$$

$$\text{side } FKE = 10{,}465,$$

and

$$\text{angle } AEF = 1°5':$$

accordingly it is manifest that

$$\text{angle } AEF + \text{angle } DAE = 4°6',$$

which is the total difference or additosubtraction between the mean and the true position of the planet. Wherefore if the position of the Earth had been at $K$ or M, the position of Saturn would have been apparent as if from centre $E$ and would have been seen to be at 203°16′ from the constellation of Aries. But with the Earth at $L$, Saturn is seen to be at 209°. The difference [149b] of 5°44′ goes to the parallax in accord with angle $KFL$. But by calculation of the regular movement

$$\text{arc } HL = 116°31',$$

and

$$\text{arc } ML = \text{arc } HL - \text{add. } HM = 112°25'.$$

And by subtraction[1]

$$\text{arc } LIK = 67°35':$$

hence

$$\text{angle } KEL = 67°35'.$$

Wherefore in triangle $FEL$ the angles are given, and the ratio of the sides is given too: Hence

$$EL = 1{,}090$$

$$\text{where } EF = 10{,}465,$$

$$\text{and } AD = BD = 10{,}000;$$

but

$$EL = 6^p32',$$

$$\text{where } BD = 60^p,$$

by usage of the ancients;

and there is very little difference between that and what Ptolemy gave.

Accordingly

$$BDE = 10{,}854,$$

and, as the remainder of the diameter

$$CE = 9{,}146.$$

But since the epicycle when at $B$ always subtracts 285 from the altitude of the planet, but adds the same amount, *i.e.*, its radius, when at $C$; on that account the greatest distance of Saturn from centre $E$ will be 10,569, and the least 9,431, where $BD = 10{,}000$. By this ratio the altitude of the apogee of Saturn is $9^p42'$, where the radius of the orbital circle of the Earth $= 1^p$; and the altitude of the perigee is $8^p39'$: hence it is quite evident by the mode set forth above in the case of the small parallaxes of the moon that the parallaxes of Saturn can be greater. And when Saturn is at the apogee,

$$\text{greatest parallax} = 5°45';$$

and when at the perigee,

$$\text{greatest parallax} = 6°39';$$

and they differ from one another by 44′—measuring the angles by the lines coming from the planet and tangent to the orbital circle of the Earth. In this way the particular differences in the movement of Saturn have been found, and we shall afterwards set them out simultaneously and in conjunction with those of the five planets.

[1]Arc $MLIK = 180°$.